Delay Dependent Robust Exponential Stability and Stabilization of Uncertain State-delayed Systems

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Abstract: This paper is concerned with the exponential robust stability and robust stabilization for linear time delayed system and subject to real convex polytopic uncertainty. We propose some new necessary and sufficient conditions in terms of Linear Matrix Inequalities (LMIs) for nominal state delayed systems without uncertainties to be stable by estimating the stability bound of delay decay rate and upper bound delay time. Based on these results, the state feedback stability problem is solved and the result is extended to the case of uncertain delayed systems. Examples are provided to demonstrate the effectiveness and applicability of the proposed criteria.

Keywords: Delay systems, exponential stability, robust stabilization, state feedback, uncertain systems, LMI.

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1. Introduction

The stability problem of time delay systems has been the subject of numerous studies and has attracted considerable attention over decades. In the literature, the stability analysis of this time delayed system can be classified as two cases according to their dependence on the size of delays, namely delay-independent stability criterion and delay independent stability criterion. As the name implies, delay dependent results take into account the maximum delay that can be tolerated by the system [3, 8, 10, 11, 12, 16, 17, 19, 20, 21]. Delay independent results guarantee stability irrespective of the size of delay [9, 14].

Most of these existing results of stability were developed based on Lyapunov's second method using either Lyapunov-Krasovskii functionals or Lyapunov-Razumikhin functions. These results are formulated in terms of Linear Matrix Inequalities (LMIs) that have been employed to tackle stability an stabilization problems, and, hence, can be solved efficiently [1, 7, 8, 12].

The exponential stability problem [2, 3, 6] is to find the sufficient conditions such that the solution $x(t,x_0)$ of the system initialized at x_0 satisfy the condition:

$$\exists\ N>0, \alpha>0 : \|x(t)\| \le \|x_0\| N e^{-\alpha t}, \forall\ t \ge 0$$

With α is the stability degree (delay decay rate).

Robust control of linear time delay systems has been the focus of much attention during the past decade and various aspects and approach for analysis and control design for linear uncertain delay systems have been investigated, see for instance [10, 12, 15, 17, 20] and the reference therein.

The state feedback stabilization problem is solved by means of the stability of the closed loop. When the system contains uncertainties, the present method is extended to solve the robust state feedback stability problem.

Motivated by the work of L. Yu *and al.*, 2004, and based on the theorems proposed by Y. Xia *and al.*, 2003, some novel delay sufficient conditions for stability for a class of linear time delay systems are presented. We extend this result when the dynamic matrices are subject to real convex polytopic uncertainty. Based on which, the corresponding state feedback controller is also developed. The maximum bound for the time delay, which ensures that the delayed uncertain system is globally exponential stable, can be obtained by solving a quasi-convex optimization problem. Finally, we provide examples to demonstrate the effectiveness and applicability of the proposed method.

This paper is organized as follow. In section 2, the problem to be solved is formulated and some preliminary definitions are given. In section 3, we present the main result to solve the stability and the stabilization for the nominal time delay system. Then, section 4 deals with uncertain time delay systems and the results of the previous section are extended to robust stability and stabilization conditions. Section 5 presents illustrative examples and finally section 6 concludes the paper.

2. Problem Statement and Preliminaries

Let us consider the following time-delayed linear system:

$$\dot{x}(t) = Ax(t) + A_d x(t-\tau) + Bu(t) \tag{1}$$

with $x(t) = \phi(t), \qquad t \in [-\tau, 0]$

Where $x(t) \in R^n$ is the state vector; $u(t) \in R^m$ is the control vector, $\tau > 0$ denotes the time delay, $\phi(.)$ is the initial condition, and the matrices A, A_d and B belong to a polytope, that is, they satisfy:

$$[A\ A_d\ B] \in \Omega := \left\{ [A(\sigma)\ A_d(\sigma)\ B(\sigma)] = \sum_{i=1}^{p} \sigma_i [A_i\ A_{di}\ B_i], \quad \sum_{i=1}^{p} \sigma_i = 1, \quad \sigma_i \ge 0 \right\}$$

The following lemmas will be used in our main result.

Lemma 2.1. [16]

The following statements are equivalent:

(I) There exist positive-definite matrices $W \in R^{n\times n}$ and $Q \in R^{n\times n}$ such that

$$-W + A_d Q^{-1} A_d < 0 \tag{2}$$

(II) There exist positive-definite matrices $W \in R^{n\times n}$, $Z \in R^{n\times n}$ and matrix $G \in R^{n\times n}$ such that

$$\begin{bmatrix} -W & A_d^T G \\ G^T A_d & -G - G^T + Z \end{bmatrix} < 0 \tag{3}$$

Proof. See [16].

Lemma 2.2. [16]

Suppose that τ is a positive scalar. The following statements are equivalent:

(I) There exist positive-definite matrices $P \in R^{n\times n}$, $Q \in R^{n\times n}$, $W \in R^{n\times n}$ such that

$$(A+A_d)^T P + P(A+A_d) + \tau(A+A_d)^T W(A+A_d) + \tau Q < 0 \quad (4)$$

(II) There exist positive-definite matrices $P \in R^{n\times n}$, $Q \in R^{n\times n}, W \in R^{n\times n}$ and general matrices τ, $F \in R^{n\times n}$ such that

$$\begin{bmatrix} (A+A_d)^T E^T + E(A+A_d) + \tau Q & P + (A+A_d)^T F - E^T \\ P - E + F^T(A+A_d) & -F - F^T + \tau W \end{bmatrix} < 0 \quad (5)$$

Proof. See [16].

Lemma 2.3

For any $x, y \in R^{n\times n}$ and for any positive symmetric definite matrix $Q \in R^{n\times n}$

$$2x^T y \le x^T Q^{-1} x + y^T Q y \quad (6)$$

The above lemmas are used to present theorem 1 in [16, 19].

3. Exponential Stability and Stabilization for Nominal Systems

Consider the nominal state delayed system (1) with $u(t) = 0$ utilizing the following transformations

$$g(t) = e^{\alpha t} x(t) \quad (7)$$

Where $\alpha > 0$ is the stability degree (delay decay rate), to transform (1) into

$$\dot{g}(t) = (A + \alpha I) g(t) + A_d e^{\alpha\tau} g(t-\tau) \quad (8)$$

System (1) is exponential stable with decay rate α.

Now, we apply the theorem 1 in [16, 19] to system (8) in order to obtain the following new stability condition presented below.

Theorem 3.1. Consider system (8) with delay decay rate α. Then given a scalar $\tau > 0$, this system is exponential stable with decay rate α if there exist positive-definite matrices H, P, Q *and* $W \in R^{n\times n}$ matrices E, F *and* $G \in R^{n\times n}$ such that the following LMIs hold:

$$\begin{bmatrix} -Q & A_d^T G \\ G^T A_d & -G - G^T + H \end{bmatrix} < 0, \quad (9)$$

$$\begin{bmatrix} -W & P \\ P & -H \end{bmatrix} < 0, \quad (10)$$

$$\begin{bmatrix} \overline{A}^T E^T + E\overline{A} + \tau Q & P + \overline{A}^T F - E^T \\ P - E + F^T \overline{A} & -F - F^T + \tau e^{2\alpha\tau} W \end{bmatrix} < 0. \quad (11)$$

Where $\overline{A} = A + \alpha I + A_d e^{\alpha\tau}$

Proof of Theorem 3.1

First, we rewrite system (1) in the form

$$z(t) = g(t) + A_d e^{\alpha t} \int_{t-\tau}^{t} g(s)\, ds \quad (12)$$

$$\dot{z}(t) = (A + \alpha I + A_d e^{\alpha\tau}) g(t) \quad (13)$$

Next, construct a Lyapunov functional candidate in the following form:

$$V(t) = V_1(t) + V_2(t), \qquad V_1(t) = z^T(t) P z(t), \qquad V_2(t) = \int_{t-\tau}^{t} \int_{t+w}^{t} g^T(s) Q g(s)\, ds\, dw \quad (14)$$

Where P and Q are positive-definite matrices.

Calculating now the derivative of $V(t)$ in (14) along the trajectory of (12) yields:

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t)$$

$$\dot{V}_1(t) = \dot{z}^T(t)Pz(t) + z^T(t)P\dot{z}(t)$$

$$= [\overline{A}g(t)]^T P[g(t) + A_d e^{\alpha\tau}\int_{t-\tau}^{t} g(s)\,ds] + [g(t) + A_d e^{\alpha\tau}\int_{t-\tau}^{t} g(s)\,ds]^T P[\overline{A}g(t)]$$

$$= g^T(t)[\overline{A}^T P + P\overline{A}]g(t) + \Omega(t)$$

Where $\Omega(t) = 2g^T(t)\overline{A}^T P\int_{t-\tau}^{t} A_d e^{\alpha\tau} g(s)$

Using lemma 2.3, we will have

$$\Omega(t) \le g^T(t)\overline{A}^T PA_d Q^{-1} A_d^T P\overline{A}\tau e^{2\alpha\tau} g(t) + \int_{t-\tau}^{t} g^T(s)Qg(s)\,ds$$

$$\dot{V}_2(t) = \tau g^T(t)Qg(t) - \int_{t-\tau}^{t} g^T(s)Qg(s)\,ds$$

Hence, it follows that

$$\dot{V}(t) \le g^T(t)[\overline{A}P + P\overline{A}^T + \tau Q + \tau e^{2\alpha\tau}\overline{A}^T PA_d Q^{-1} A_d^T P\overline{A}]g(t) \tag{15}$$

$\dot{V}(t)$ is negative definite if the following inequality:

$$\overline{A}P + P\overline{A}^T + \overline{\tau}Q + \overline{\tau}e^{2\alpha\overline{\tau}}\overline{A}^T PA_d Q^{-1} A_d^T P\overline{A} < 0 \tag{16}$$

is satisfied.

In order to turn the inequality (16) into LMI form, we consider:

$$N > A_d Q^{-1} A_d^T \tag{17}$$

$$W > PNP \tag{18}$$

$$\overline{A}P + P\overline{A}^T + \overline{\tau}Q + \overline{\tau}e^{2\alpha\overline{\tau}}\overline{A}^T W\overline{A} < 0 \tag{19}$$

Where P, N, W are some positive definite matrices and $\tau > 0$.

Inequality (16) can be negative if and only if the inequalities (17)-(19) are satisfied. Finally, using Schur complement formula [1], lemma 2.1 and lemma 2.2, (17)-(19) are equivalent to (9)-(11) for some positive definite matrices $H, P, Q \text{ and } W \in R^{n\times n}$, matrices $E, F \text{ and } G \in R^{n\times n}$. Based on the result obtained in this proof, we have $\dot{V}(t) < 0$.

Therefore, system (8) is exponential stable with decay rate α.

Remark. According to the result of [2], the solutions of $g(t) + A_d e^{\alpha t}\int_{t-\tau}^{t} g(s)\,ds = 0$ are uniform asymptotic stable if the following LMI is satisfied:

$$\begin{bmatrix} -\overline{\tau}^{-1} & A_d^T e^{\alpha\overline{\tau}} \\ e^{\alpha\overline{\tau}} A_d & -\overline{\tau}^{-1} \end{bmatrix} < 0 \tag{20}$$

In the following, we extend theorem 3.1 to design a stabilizing state feedback controller $u(t) = Kx(t)$ for the system (1).

The closed loop system becomes:

$$\dot{x}(t) = (A + BK)x(t) + A_d x(t-\tau) \tag{21}$$

with $x(t) = \phi(t),\ t \in [-\tau, 0]$

If we consider the transformation (7), the system (21) can be rewritten as

$$\dot{g}(t) = \overline{A}_c g(t) + \overline{A}_d e^{\alpha\tau} g(t-\tau) \tag{22}$$

with $\overline{A}_c = A + BK + \alpha I$ and $\overline{A}_d = A_d e^{\alpha\tau}$

Theorem 3.2. Consider system (8) with delay decay rate α . Then given a scalar $0 \le \tau \le \overline{\tau}$, this system is exponential stabilizable with decay rate α if there exist positive-definite matrices $\{H, P, Q \text{ and } W\} \in R^{n\times n}$, matrices $E, F \text{ and } G \in R^{n\times n}$ satisfying the following LMIs:

$$\begin{bmatrix} -\overline{\tau}^{-1} & A_d^T e^{\alpha\overline{\tau}} \\ e^{\alpha\overline{\tau}} A_d & -\overline{\tau}^{-1} \end{bmatrix} < 0 \tag{23}$$

$$\begin{bmatrix} -Q & A_d^T G \\ G^T A_d & -G - G^T + H \end{bmatrix} < 0, \tag{24}$$

$$\begin{bmatrix} -W & P \\ P & -H \end{bmatrix} < 0, \tag{25}$$

$$\begin{bmatrix} \overline{A}E^T + E\overline{A}^T + \overline{\tau}Q + BR + R^T B & P + \overline{A}E + BR - E^T \\ P - E + E\overline{A}^T + R^T B^T & -E - E^T + \overline{\tau} e^{2\alpha\overline{\tau}} W \end{bmatrix} < 0. \tag{26}$$

W*here* $\overline{A} = A + \alpha I + A_d e^{\alpha\overline{\tau}}$

Moreover, a stabilizing control law is given by $u(t) = RE^{-1}x(t)$

Proof of Theorem 3.2

The result is immediately gotten by applying Theorem 3.2 to the closed-loop system (22) and setting $E = F^T$ and $R = KE$. The proof is completed.

In the following section, we extend the obtained exponential stability conditions to robust exponential conditions for the uncertain system (1).

4. Robust Exponential Stability and Stabilization for Uncertain Systems

The following theorem provides robust exponential stability analysis of the unforced system (1) with $u(t) = 0$.

Theorem 4.1. Consider system (8) with delay decay rate α . Then given a scalar $0 \le \tau \le \overline{\tau}$, this system is robustly exponential stable with decay rate α if there exist positive-definite matrices $\{H_i, P_i, Q_i \text{ and } W_i\} \in R^{n\times n}$, $i=1,\ldots,p$; matrices $E, F \text{ and } G \in R^{n\times n}$ such that the following LMIs hold:

$$\begin{bmatrix} -\overline{\tau}^{-1} & A_{di}^T e^{\alpha\overline{\tau}} \\ e^{\alpha\overline{\tau}} A_{di} & -\overline{\tau}^{-1} \end{bmatrix} < 0, \tag{27}$$

$$\begin{bmatrix} -Q_i & A_{di}^T G \\ G^T A_{di} & -G - G^T + H_i \end{bmatrix} < 0, \tag{28}$$

$$\begin{bmatrix} -W_i & P_i \\ P_i & -H_i \end{bmatrix} < 0, \tag{29}$$

$$\begin{bmatrix} \overline{A}_i^T E^T + E\overline{A}_i + \overline{\tau}Q_i & P_i + \overline{A}_i^T F - E^T \\ P_i - E + F^T \overline{A}_i & -F - F^T + \overline{\tau} e^{2\alpha\overline{\tau}} W_i \end{bmatrix} < 0. \tag{30}$$

Where $\overline{A}_i = A_i + \alpha I + A_{di}e^{\alpha\overline{\tau}}$

Proof of Theorem 4.1

Replacing A, A_d in theorem 3.1 with A_i, A_{di} respectively.

By virtue of the properties of convex combination, and using characterization (9)-(11) and (20), it is easy to prove the uncertain system (8) is stable under conditions (21)-(24). The proof is completed.

Next, we present a solution to the exponential robustly stabilization problem for system (8).

Theorem 4.2. Consider system (8) with delay decay rate α . Then given a scalar $0 \le \tau \le \overline{\tau}$, this system is robustly exponential stabilizable with decay rate α if there exist positive-definite matrices $\{H_i, P_i, Q_i \text{ and } W_i\} \in R^{n\times n}$, $i=1,\dots,p$; matrices $E, F \text{ and } G \in R^{n\times n}$ satisfying the following LMIs:

$$\begin{bmatrix} -\overline{\tau}^{-1} & A_{di}^T e^{\alpha\overline{\tau}} \\ e^{\alpha\overline{\tau}} A_{di} & -\overline{\tau}^{-1} \end{bmatrix} < 0, \quad (31)$$

$$\begin{bmatrix} -Q_i & A_{di}^T G \\ G^T A_{di} & -G - G^T + H_i \end{bmatrix} < 0, \quad (32)$$

$$\begin{bmatrix} -W_i & P_i \\ P_i & -H_i \end{bmatrix} < 0, \quad (33)$$

$$\begin{bmatrix} \overline{A}_i E^T + E\overline{A}_i^T + \overline{\tau} Q_i + B_i R + R^T B & P_i + \overline{A}_i E + B_i R - E^T \\ P_i - E + E\overline{A}_i^T + R^T B_i^T & -E - E^T + \overline{\tau} e^{2\alpha\overline{\tau}} W_i \end{bmatrix} < 0. \quad (34)$$

Where $\overline{A}_i = A_i + \alpha I + A_{di}e^{\alpha\overline{\tau}}$

Moreover, a stabilizing control law is given by $u(t) = RE^{-1}x(t)$

Proof of Theorem 4.2

As in the proof of Theorem 4.1, replacing A, A_d and B in theorem 3.1 with A_i, A_{di} and B_i respectively. The result fallows immediately by applying Theorem 4.2 to the closed-loop system (26) and setting $E = F^T$ and $R = KE$. The proof is completed.

5. Numerical Examples

In this section, we illustrate the results by using three examples.

Example1: Exponential stability

Consider the same time delay system with delay decay rate α presented in [8] as follows:

$$\dot{g}(t) = (A + \alpha I)g(t) + A_d e^{\alpha\tau} g(t - \tau) \quad (35)$$

Where

$$A = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix}; \qquad A_d = \begin{bmatrix} -0.5 & 0.1 \\ 0.3 & 0 \end{bmatrix}.$$

The goal of this example is to estimate the stability bound of delay decay rate α and upper bound delay time τ to guarantee the stability of time-delay system.

Remark: [4] consider the main problem and he presents a sufficient condition delay dependent asymptotic stability and the decay rates for linear time delay systems. But in his illustrated example, the results shown in table 1 are incorrect. The true results of [8] will be presented in our paper as a comparison with our results.

Table 1: Bound of delay time τ for various stability degree α .

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Liu method [8]	1.546	1.371	1.242	1.142	1.061	0.993	0.936	0.887	0.844	0
Our method	2.970	2.401	1.816	1.611	1.382	1.178	1.078	0.999	0.972	0

By applying Theorem 3.1 and from the above table, it is clear that as α increases, τ decreases. In fact, our method gives much better results than those obtained if we use the theorem 2 presented in [8].

Example2: Exponential robust stability

Consider uncertain time-delay system (1) where the state space data are subject to uncertainties and obey the real convex polytopic model, [16]:

$$[A\ A_d\ B] \in \Omega := \left\{ [A(\sigma)\ A_d(\sigma)\ B(\sigma)] = \sum_{i=1}^{p} \sigma_i [A_i\ A_{di}\ B_i], \quad \sum_{i=1}^{p} \sigma_i = 1, \quad \sigma_i \geq 0 \right\} \tag{36}$$

$$A_1 = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.09 \end{bmatrix}; \quad A_2 = \begin{bmatrix} -2 & -1 \\ 0 & -2 \end{bmatrix}; \quad A_3 = \begin{bmatrix} -1.9 & 0 \\ 0 & -1 \end{bmatrix};$$

$$A_{d1} = \begin{bmatrix} -0.1 & 0 \\ -0.1 & -0.1 \end{bmatrix}; \quad A_{d2} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}; \quad A_{d3} = \begin{bmatrix} -0.9 & 0 \\ -1 & -1.1 \end{bmatrix};$$

Based on Theorem 4.1, we obtain that the uncertain system is exponential robustly stable for any τ satisfying $0 \leq \tau \leq 0,579$ if the delay decay rate $\alpha = 0.1$. If $\alpha = 0.2$, the maximum delay is $\tau = 0,550$.

Then it can be verified that the sharpness of the upper bound delay time τ varies with the chosen decay rate α .

Example 3: Exponential robust stabilization

Consider uncertain time-delay system (1) where the state space data are subject to uncertainties and obey the real convex polytopic model, [16]:

$$[A\ A_d\ B] \in \Omega := \left\{ [A(\sigma)\ A_d(\sigma)\ B(\sigma)] = \sum_{i=1}^{p} \sigma_i [A_i\ A_{di}\ B_i], \quad \sum_{i=1}^{p} \sigma_i = 1, \quad \sigma_i \geq 0 \right\} \tag{37}$$

$$A_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}; \quad A_2 = \begin{bmatrix} 1 & 0.3 \\ 0 & 1.5 \end{bmatrix}; \quad A_3 = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.5 \end{bmatrix};$$

$$A_{d1} = \begin{bmatrix} -1 & -1 \\ 0 & -0.9 \end{bmatrix}; \quad A_{d2} = \begin{bmatrix} -0.7 & -0.6 \\ 0.2 & -1.4 \end{bmatrix}; \quad A_{d3} = \begin{bmatrix} 0.1 & -1.2 \\ -0.2 & -1.1 \end{bmatrix};$$

$$B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; \quad B_2 = \begin{bmatrix} 0.1 \\ 0.8 \end{bmatrix}; \quad B_3 = \begin{bmatrix} 0.2 \\ 1.3 \end{bmatrix}.$$

Applying Theorem 4.1, it has been found that the uncertain system (1) is exponential robustly stabilizable:

- $\alpha = 0.1$ then $0 \leq \tau \leq 0,442$ with state feedback gain $K = -\begin{bmatrix} 0.2272 & 0.4788 \end{bmatrix}$.
- $\alpha = 0.2$, then $0 \leq \tau \leq 0,418$ with state feedback gain $K = -\begin{bmatrix} 0.2412 & 0.5345 \end{bmatrix}$.
- $\alpha = 0.3$, then $0 \leq \tau \leq 0,396$ with state feedback gain $K = -\begin{bmatrix} 0.2587 & 0.5930 \end{bmatrix}$.

From the above results, it is shown that when α increases, τ decreases.

5. Conclusion

In this paper, a new approach to exponential stability and decaying rate of linear time delay systems is introduced. This approach is extended to provide a new robust stability condition when convex polytopic uncertainty is present on the dynamic matrices. Based on which, the corresponding state feedback control law is also developed to obtain the stabilization of delayed systems. It was shown by numerical examples that in the proposed delay exponential stability and stabilization approaches, the sharpness of the upper bound delay time τ varies with the chosen decay rate α .

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