Indirect Adaptive Fuzzy Control Scheme Based on Lyapunov Approach for a Class of SISO Nonlinear Systems

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Abstract: In this paper we are interested in an indirect fuzzy adaptive control of SISO nonlinear systems in the presence of parametric uncertainties. The plant model structure is represented by a fuzzy system. The essential idea of the on-line parametric estimation of the plant model is based on a comparison of the measured state with the estimate one. The design of adaptive law is based on a Lyapunov approach. The control action comprises two terms: a classical indirect adaptive fuzzy controller and a supervisory controller. The plant state tracks asymptotically any bounded reference input signal. Two examples are used to check performance of the proposed controller.

Keywords: Nonlinear system; Fuzzy system; State estimation; Adaptive fuzzy control; Stability; Supervisory control; Tracking error.

1. Introduction

The basic objective of adaptive control is to maintain constant performance of a system in the presence of uncertainty. We recall that, in the two case of adaptive control, direct and indirect, the parameters of controller will be adjusted into account of the system parameters and disturbances.

In the direct adaptive control, the controller parameters are directly adjusted to reduce some norm of the output error between the plant output and the desired reference trajectory. In indirect adaptive control, the controller is designed assuming that the on-line estimated plant functions represent the true values.

In the fuzzy adaptive control literature, Shaocheng et al. [14] proposed a fuzzy adaptive control scheme based on output feedback for unknown nonlinear systems, which are represented by input-output model. A high gain observer is employed to estimate the state and implement the controller using measurement of the output. An asymptotic analysis is treated to prove that the output feedback controller can guarantee the stability of the closed-loop system. They prove that the designed output feedback adaptive fuzzy control scheme can recover the performance achieved under the state feedback controller.

Chan et al. [12] proposed an indirect adaptive fuzzy sliding mode control. They show that the error dynamics of the Lyapunov synthesis is similar to the sliding surface of the sliding mode control.

Wang et al. [6] proposed an indirect adaptive fuzzy sliding mode control scheme for a class of nonlinear systems where both the equivalent control term and switching-type control term in the sliding mode control law are approximated by fuzzy systems.

Tsung-Chih Lin et al [20] suggested an observer based on indirect adaptive fuzzy neural tracking control equipped with variable structure systems "VSS" and H_∞ control algorithms, which is developed for nonlinear SISO systems involving plant uncertainties and external disturbances. They combine three important control methods to solve the robust nonlinear output tracking problem, adaptive fuzzy control scheme, VSS control design and H_∞ tracking theory. A modified algebraic Riccati-like equation must be solved to compensate the effect of the approximation error via adaptive fuzzy neural system on the H_∞ control. The overall adaptive scheme guarantees the stability of the resulting closed-loop system in the sense that all the states and signals are uniformly bounded and arbitrary small attenuation level of the external disturbance on the tracking error can be achieved.

Chien et al. [2] proposed a fuzzy system-based adaptive iterative learning controller for a class of non-Lipschitz nonlinear plants which can repeat a given task over a finite time interval. Based on the error function, the main structure of the controller is constructed by a fuzzy iterative learning component and a feedback stabilization component. The fuzzy system is used as an approximator to compensate the plant unknown nonlinearity.

Mehrdad et al. [10] proposed a combined direct and indirect adaptive control to construct an adaptive controller using adjustable fuzzy modeling rules to identify and control a class of uncertain structure nonlinear dynamic systems. For adjusting the parameters, they used a hybrid adaptive scheme which combines adaptive fuzzy identification and adaptive fuzzy control.

Wang [19] proposed two indirect adaptive fuzzy controllers based on Lyapunov approach. He used two different representations of fuzzy systems which are an universal approximators for any given real continuous function on a compact set.

This paper deals with an indirect adaptive control law. We use a Lyapunov approach which is combined with a sliding mode to design the fuzzy controller. The unknown plant functions are estimated on-line by fuzzy systems. The control law is based on the estimated functions and a supervisory term.

In this paper, an alternative indirect adaptive fuzzy controller is developed. We use other fuzzy logic systems, called the Takagi-Sugeno fuzzy systems, which are different from those used in [6], [12], [14] and [19].

This manuscript is organized as follows: In section II we recall the continuous fuzzy system. The adaptive control law design method for continuous fuzzy system and the proof of the stability of the proposed control approach are given in section III. In Section IV, some simulation results are given to illustrate the performance of the proposed fuzzy adaptive control algorithm. Finally, a conclusion is given in Section V.

2. Basic fuzzy logic systems

We consider a class of continuous SISO nonlinear dynamic systems which has a following form:

$$\begin{cases} \dot{x} = f(x,t) + g(x,t)\, u(t) \\ Y = Cx \end{cases} \tag{1}$$

Where, $C = [1,0,0, \ldots,0]$, $x \in \Re^n$ is the state vector that is assumed to be observable, $u(t) \in \Re$ is the control input, $f(x,t)$ and $g(x,t)$ are two continuous unknown nonlinear functions. In order for (1) to be controllable, it is required that $g(x,t) \neq 0$.

The continuous SISO non-linear system (1) can be described by Takagi-Sugeno fuzzy logic system, which the basic configuration is shown in figure 1. The fuzzy system is represented by a collection of fuzzy IF-THEN rules whose the ith rule is written as follow: [15].

$$R^{(i)}:\ \ IF\, x_1 \ is\ M_1^i \ \ and \ldots \ and\, x_n \ is\ M_n^i \ \ THEN\ y = y^i \tag{2}$$

The output of the fuzzy system can be written as:

$$y(t) = \frac{\sum_{j=1}^{r} y^j \prod_{i=1}^{n} \mu_j^i(x_i)}{\sum_{j=1}^{r} \prod_{i=1}^{n} \mu_j^i(x_i)} \tag{3}$$

where, $\mu_j^i(x_i)$ is the membership function of the linguistic variable xi , yj is an element of $\Re$ *and r* is the number of fuzzy rules.

By introducing the concept of fuzzy basic function vector $\xi(x)$, the equation (3) can be written as:

$$y(t) = \theta^T \xi(x) \tag{4}$$

where, $\theta = (y^1,...,y^m)^T$ is an adjustable parameter vector and $\xi(x) = (\xi_1(x),..., \xi_r(x))^T$ is a regressive vector with the fuzzy basic function $\xi_i(x)$ defined as:

$$\xi_i(x) = \frac{\prod_{j=1}^{n} \mu_j^i(x_j)}{\sum_{i=1}^{r} \prod_{j=1}^{n} \mu_j^i(x_j)} \tag{5}$$

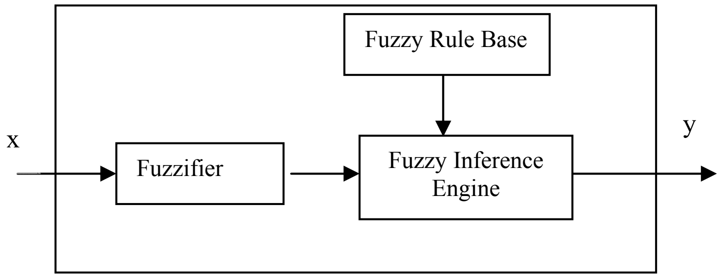

Figure 1: The Basic configuration of a Takagi-Sugeno fuzzy logic system

3 Adaptive Law

Our objective in this paper is to design an adaptive fuzzy controller which guarantees boundedness of all variables for the closed-loop system and tracking of a given bounded reference signal x_d.

The fuzzy feedback linearization method which is based on T-S model can solve this kind of control problem [3].

When the plant parameters contain uncertainties or vary in time, the control law which results in feedback linearization method can not guarantee the tracking of reference signal. So, we propose a controller which comports two terms such that:

$$u(t) = u_c(t) + u_s(t) \tag{6}$$

where: $u_c(t)$ represents the classical controller for nominal values of plant functions and $u_s(t)$ *is* a supervisory term which is used to compensate the uncertainty effects and disturbances of parameters.

If the plant functions $f(x, t)$ and $g(x, t)$ are known, the ideal classical control law will be expressed as:

$$u_c(t) = \frac{1}{g(x,t)}[-f(x,t) - K^T E + x_d^{(n)}] \tag{7}$$

where, the state vector x and the desired state x_d are defined as:

$$x = (x_1, x_2, ..., x_n)^T \ ; x_d = (x_{1d}, x_{2d}, ..., x_{nd})^T$$

the state tracking error is defined as: $E = x - x_d = (e, \dot{e}, ..., e^{(n-1)})^T$, the vector

$K = (k_n, k_{n-1}, ..., k_1)^T \in R^n$ will be chosen such that all roots of the following polynomial

$h(s) = s^n + k_1 s^{n-1} + ... + k_n$ are situated in the open left-half complex plane.

The objective is to find a control law such that the state x of the closed-loop system will follow the desired state x_d, in other words, the tracking error should converge to zero.

In real systems, $f(x, t)$ and $g(x, t)$ are unknown. Thus, it is impossible to generate the control law (7). So to overcome these difficulties, we use fuzzy systems $\hat{f}(x, \theta_f)$ and $\hat{g}(x, \theta_g)$ to approximate respectively $f(x, t)$ and $g(x, t)$. Thus, the control law will be written as:

$$u_c(t) = \frac{1}{\hat{g}(x, \theta_g)}[-\hat{f}(x, \theta_f) - K^T E + x_d^{(n)}] \tag{8}$$

The global control law is given by the following equation:

$$u(t) = \frac{1}{\hat{g}(x, \theta_g)}\left(-\hat{f}(x, \theta_f) - K^T E + x_d^{(n)}\right) + u_s \tag{9}$$

where, u_s will be defined later, θ_f and θ_g are parameters of the approximating fuzzy systems $\hat{f}(x, \theta_f)$ and $\hat{g}(x, \theta_g)$. The two later functions are expressed as follow:

$$\hat{f}(x, \theta_f) = \theta_f^T \xi(x) \tag{10}$$

$$\hat{g}(x, \theta_g) = \theta_g^T \xi(x) \tag{11}$$

We define the optimal parameters of fuzzy systems as:

$$\theta_f^* = \arg\min_{\theta \in \Omega_f} [\sup_{x \in \Re^n} |\hat{f}(x, \theta_f) - f(x, t)|] \tag{12}$$

$$\theta_g^* = \arg\min_{\theta \in \Omega_g} [\sup_{x \in \Re^n} |\hat{g}(x, \theta_g) - g(x, t)|] \tag{13}$$

where Ωf and Ωg are constraint sets for θf and θg . Meanwhile, we define the minimum approximation error as: $\omega = f(x,t) - \hat{f}(x, \theta_f^*) + \left(g(x,t) - \hat{g}(x, \theta_g^*)\right)u_c$ (14)

Applying the control law (9) to the system (1), and after straightforward manipulation, the error dynamic equation can be written as :

$$\dot{E} = A_m E + b_r\left(f(x,t) - \hat{f}(x, \theta_f) + (g(x,t) - \hat{g}(x, \theta_g))u_c + g(x,t)u_s)\right) \tag{15}$$

with:

$$A_m = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ & & & & \\ 0 & 0 & 0 & \dots & 1 \\ -k_n & -k_{n-1} & -k_{n-2} & \dots & -k_1 \end{bmatrix} ;\ b_r = [0, 0, \dots, 1] \tag{16}$$

The Matrix A_m is a stable matrix. It will be chosen such that there exists a symmetric definite positive matrix P which verifies the following Lyapunov equation:

$$A_m^T P + P A_m = -2I \tag{17}$$

A sliding surface can be defined in the error state $S(X, t)$ from the above equation [21].

$$S(x,t) = \left(\frac{d}{dt} + \lambda\right)^{(n-1)} E = e^{(n-1)} + a_1 e^{(n-2)} + \dots + a_{n-1}e \tag{18}$$

where λ is a strictly positive constant which will be chosen such that all roots of the following polynomial $h(s) = s^{n-1} + a_1 s^{n-2} + \dots + a_{n-1}$, are situated in the open left-half complex plane.

The derivative of sliding surface can be expressed as:

$$\dot{S} = x^{(n)} - x_d^{(n)} + \sum_{i=1}^{n-1} a_i e^{(i)} \tag{19}$$

Assumption:

Without loss the generality, we assume that there exist functions $f_H(x,t)$, $g_H(x,t)$ and $g_L(x,t)$ such that $\left|f(x,t)\right| \prec f_H(x,t)$ and $0 \prec g_L(x,t) \le g(x,t) \le g_H(x,t)$.

Theorem

Consider the control problem of the SISO nonlinear system (1). If control law expressed by (9) is applied, $\hat{f}(x,\theta_f)$ and $\hat{g}(x,\theta_g)$ are given by (10)-(11) and the parameters vectors θ_f and θ_g are adjusted by the following adaptive law (20)-(21). Then, the closed-loop signals will be bounded for all bounded input signals, and the tracking error will converge to zero asymptotically:

$$\dot{\hat{\theta}}_f = r_1 \left(S + E^T P b_r\right)\xi(x) \tag{20}$$

$$\dot{\hat{\theta}}_g = r_2 \left(S + E^T P b_r\right)\xi(x) u_c \tag{21}$$

where: r_1 and r_2 are two positive constants and $P = P^T > 0$ is chosen as the solution of the Lyapunov equation (17).

Proof:

We propose the following function (22) as Lyapunov function candidate:

$$V(S,E,\Phi_f,\Phi_g) = \frac{1}{2}\left(S^2 + E^T P E + \frac{1}{r_1}\Phi_f^T \Phi_f + \frac{1}{r_2}\Phi_g^T \Phi_g\right) \tag{22}$$

where: Φ_f and Φ_g are defined as:

$$\Phi_f = \theta_f - \theta_f^* \tag{23}$$

$$\Phi_g = \theta_g - \theta_g^* \tag{24}$$

Then :

$$\dot{V} = \frac{1}{2}\left(\dot{S}S + S\dot{S} + \dot{E}^T P E + E^T P \dot{E} + \frac{1}{r_1}(\dot{\Phi}_f^T \Phi_f + \Phi_f^T \dot{\Phi}_f) + \frac{1}{r_2}(\dot{\Phi}_g^T \Phi_g + \Phi_g^T \dot{\Phi}_g)\right) \tag{25}$$

$$\begin{aligned}\dot{V} = S\dot{S} &+ \frac{1}{2}\left(E^T A_m^T + b_r^T \left(\omega + \hat{f}(x,\theta_f^*) - \hat{f}(x,\theta_f) + (\hat{g}(x,\theta_g^*) - \hat{g}(x,\theta_g))u_c + g(x,t)u_s\right)PE\right) \\ &+ \frac{1}{2}E^T P\left(A_m E + b_r\left(\omega + \hat{f}(x,\theta_f^*) - \hat{f}(x,\theta_f) + (\hat{g}(x,\theta_g^*) - \hat{g}(x,\theta_g))u_c + g(x,t)u_s\right)\right) \\ &+ \left(\frac{1}{r_1}(\Phi_f^T \dot{\Phi}_f) + \frac{1}{r_2}(\Phi_g^T \dot{\Phi}_g)\right)\end{aligned} \tag{26}$$

$$\begin{aligned}\dot{V} = S\dot{S} &+ \frac{1}{2}E^T\left(A_m^T P + P A_m\right)E + \left(\frac{1}{r_1}(\Phi_f^T \dot{\Phi}_f) + \frac{1}{r_2}(\Phi_g^T \dot{\Phi}_g)\right) \\ &+ \frac{1}{2}b_r^T\left(\omega + \hat{f}(x,\theta_f^*) - \hat{f}(x,\theta_f) + (\hat{g}(x,\theta_g^*) - \hat{g}(x,\theta_g))u_c + g(x,t)u_s\right)PE \\ &+ \frac{1}{2}E^T P b_r\left(\omega + \hat{f}(x,\theta_f^*) - \hat{f}(x,\theta_f) + (\hat{g}(x,\theta_g^*) - \hat{g}(x,\theta_g))u_c + g(x,t)u_s\right)\end{aligned} \tag{27}$$

$$\begin{aligned}\dot{V} = S\dot{S} - E^T E &+ \left(\frac{1}{r_1}(\Phi_f^T \dot{\Phi}_f) + \frac{1}{r_2}(\Phi_g^T \dot{\Phi}_g)\right) + E^T P b_r\left(\omega + \hat{f}(x,\theta_f^*) - \hat{f}(x,\theta_f) + (\hat{g}(x,\theta_g^*) - \hat{g}(x,\theta_g))u_c\right) \\ &+ E^T P b_r\left(g(x,t)u_s\right)\end{aligned} \tag{28}$$

Using the equations (10)-(11)-(23) and (24) we can write :

$$\hat{f}(x,\theta_f) - \hat{f}(x,\theta_f^*) = \Phi_f^T \xi(x) \tag{29}$$

$$\hat{g}(x,\theta_g) - \hat{g}(x,\theta_g^*) = \Phi_g^T \xi(x) \tag{30}$$

and:

$$\dot{V} = S\dot{S} - E^T E + E^T Pb_r \left(\omega + g(x,t)u_s\right) - E^T Pb_r \Phi_f^T \xi(x) - E^T Pb_r u_c \Phi_g^T \xi(x) + \left(\frac{1}{r_1}(\Phi_f^T \dot{\Phi}_f) + \frac{1}{r_2}(\Phi_g^T \dot{\Phi}_g)\right) \tag{31}$$

The derivative of sliding surface can be expressed as:

$$\dot{S} = \sum_{i=1}^{n-1} a_i e^{(i)} + f(x,t) + g(x,t)u(t) - x_d^{(n)} \tag{32}$$

$$\dot{S} = \left(\omega + \hat{f}(x,\theta_f^*) - \hat{f}(x,\theta_f) + (\hat{g}(x,\theta_g^*) - \hat{g}(x,\theta_g))u_c + g(x,t)u_s\right) - K^T E + \sum_{i=1}^{n-1} a_i e^{(i)} \tag{33}$$

So:

$$\dot{V} = -E^T E + E^T Pb_r \left(\omega + g(x,t)u_s\right) + S\left(\omega + g(x,t)u_s\right) + \Phi_f^T \left(\dot{\Phi}_f - r_1 (S + E^T Pb_r)\xi(x)\right) + \Phi_g^T \left(\dot{\Phi}_g - r_2 (S + E^T Pb_r)\xi(x)u_c\right)$$
$$+ S\left(-K^T E + \sum_{i=1}^{n-1} a_i e^{(i)}\right) \tag{34}$$

$$\dot{V} = -E^T E + E^T Pb_r \left(\omega + g(x,t)u_s\right) + S\left(\omega + g(x,t)u_s\right) + \Phi_f^T \left(\dot{\Phi}_f - r_1 (S + E^T Pb_r)\xi(x)\right) + \Phi_g^T \left(\dot{\Phi}_g - r_2 (S + E^T Pb_r)\xi(x)u_c\right)$$
$$+ S\left(-K^T E + \sum_{i=1}^{n-1} a_i e^{(i)}\right) \tag{35}$$

The parameter adaptation law should be chosen as :

$$\dot{\hat{\theta}}_f = r_1 \left(S + E^T Pb_r\right)\xi(x) \tag{36}$$

$$\dot{\hat{\theta}}_g = r_2 \left(S + E^T Pb_r\right)\xi(x)u_c \tag{37}$$

Therefore, we have:

$$\dot{V} = -E^T E + E^T Pb_r \left(\omega + g(x,t)u_s\right) + S\left(\omega + g(x,t)u_s\right) + S\left(-K^T E + \sum_{i=1}^{n-1} a_i e^{(i)}\right) \tag{38}$$

$$\dot{V} = -E^T E + (S + E^T Pb_r)\left(\omega + g(x,t)u_s\right) + S\left(-K^T E + \sum_{i=1}^{n-1} a_i e^{(i)}\right) \tag{39}$$

$$\dot{V} = -E^T E + (S + E^T Pb_r)\, g(x,t)u_s + (S + E^T Pb_r)\omega + S\left(-K^T E + \sum_{i=1}^{n-1} a_i e^{(i)}\right) \tag{40}$$

$$\dot{V} \prec -E^T E + (S + E^T Pb_r)\, g(x,t)u_s + \left|S + E^T Pb_r\right| \left(\left|\hat{f}(x,\theta_f)\right| + \left|f(x,t)\right| + \left|\hat{g}u_c\right| + \left|g(x,t)\, u_c\right|\right)$$
$$+ \left|S\right| \left|-K^T E + \sum_{i=1}^{n-1} a_i e^{(i)}\right| \tag{41}$$

Based on $g_L(x,t)$, $g_H(x,t)$ and $f_H(x,t)$ and by observing (44), we choose the supervisory control law u_s as:

If $\left|S + E^T Pb_r\right| \neq 0$ then

$$u_s = -K^* \operatorname{sgn}(S + E^T Pb_r)\ \frac{1}{g_L(x,t)}\left(\left|\hat{f}(x,\theta_f)\right| + f_H(x,t) + \left|\hat{g}u_c\right| + \left|g_H(x,t)u_c\right| + \frac{|S|}{\left|S + E^T Pb_r\right|}\left|-K^T E + \sum_{i=1}^{n-1} a_i e^{(i)}\right| \right) \quad (42)$$

Where :

K^*=1 if $V \succ \overline{V}$ ($\overline{V}$ is a constant specified by the designer),

K^*=0 if $V \le \overline{V}$.

Substituting (41) to (42) and considering the case $V \succ \overline{V}$, we have:

$$\dot{V} \prec -E^T E + |S|\left|-K^T E + \sum_{i=1}^{n-1} a_i e^{(i)}\right| + \left|S + E^T Pb_r\right|\left(\left|\hat{f}(x,\theta_f)\right| + \left|f(x,t)\right| + \left|\hat{g}u_c\right| + \left|g(x,t)\, u_c\right| \right)$$

$$+\left|S + E^T Pb_r\right| \left(-\frac{g(x,t)}{g_L(x,t)}\left(\left|\hat{f}(x,\theta_f)\right| + f_H(x,t) + \left|\hat{g}u_c\right| + \left|g_H(x,t)u_c\right| + \frac{|S|}{\left|S + E^T Pb_r\right|}\left|-K^T E + \sum_{i=1}^{n-1} a_i e^{(i)}\right| \right)\right) \quad (43)$$

It leads to :

$$\dot{V} \prec -E^T E \prec 0 \quad (44)$$

If $\left|S + E^T Pb_r\right| = 0$ then $S = 0$ and $E^T Pb_r = 0$ then $u_s = 0$;

we have yet again $\dot{V} \prec -E^T E \prec 0$

This condition assures the error asymptotic convergence into zero.

4. Illustrations:

To illustrate the performance of the presented approach, we choose two nonlinear mechanical systems which are widely used in the control literature [10] [20]:

- Inverted pendulum
- Mass-spring-damper

4.1 Inverted pendulum

The dynamic equations of the inverted pendulum on a cart are [10]:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = f(x_1, x_2) + g(x_1, x_2)u \\ y(t) = x_1(t) \end{cases} \quad (45)$$

with : $$f(x_1, x_2) = \frac{mlx_2^2 \sin x_1 \cos x_1 - (m+M)g \sin x_1}{m\ l \cos^2 x_1 - {}^{4l}\!/_{3}(m+M)} \quad (46)$$

$$g(x_1, x_2) = \frac{\cos x_1}{m\ l \cos^2 x_1 - {}^{4l}\!/_{3}(m+M)} \quad (47)$$

where:

x_1 is the angle in radium of the pendulum from the vertical axis; x_2 is the angular velocity in rad/s; g is the gravity acceleration; m and M are respectively the masses of the pendulum and the mass of the cart; $2l$ is the length of the pendulum; and u is the force applied to the cart.

The nominal values of the parameters are:

g =9.81m/s^2, m=0.1kg, M=1kg, 2 l =1m.

We propose five membership functions for each variable state, $x_i \in [-\pi/3; \ \pi/3]$.

The membership functions are:

$$\mu_i^1(x_i) = \exp\left[-\left((x_i - \pi/3)/(\pi/12)\right)^2\right] \text{ for } x_i \in [\pi/6; \ \pi/3]$$
$$\mu_i^2(x_i) = \exp\left[-\left((x_i - \pi/6)/(\pi/12)\right)^2\right] \text{ for } x_i \in [0; \ \pi/3]$$
$$\mu_i^3(x_i) = \exp\left[-\left(x_i/(\pi/12)\right)^2\right] \text{ for } x_i \in [-\pi/6; \ \pi/6]$$
$$\mu_i^4(x_i) = \exp\left[-\left((x_i + \pi/6)/(\pi/12)\right)^2\right] \text{ for } x_i \in [-\pi/3; \ 0]$$
$$\mu_i^5(x_i) = \exp\left[-\left((x_i + \pi/3)/(\pi/12)\right)^2\right] \text{ for } x_i \in [-\pi/3; \ -\pi/6]$$

The matrix A_m is chosen as: $A_m = \begin{bmatrix} 0 & 1 \\ -100 & -1 \end{bmatrix}$;

The matrix P, solution of Lyapunov equation (17), is :

$$P = \begin{bmatrix} 101.01 & 0.01 \\ 0.01 & 1.01 \end{bmatrix} ;$$

The sliding surface is chosen as: $S = 5 \ e + \dot{e}$.

The parameters r_1, r_2 and r_3 are chosen as: $r_1 = r_2 = r_3 = 0.005$.

We present in figures 2 and 3 the simulation results of behavior of the inverted pendulum with parameters uncertainties. The parameters uncertainties are: $\Delta m = 0.1$; $\Delta M = 1$ and $\Delta g = 0.3$;

The expression of the desired signal is: $x_{1d} = \pi/10 \ \sin(t) + 0.3 \ \sin(3t)$;

The initial conditions are given by: $x(0) = [-\pi/30 \ ; \ 0]$.

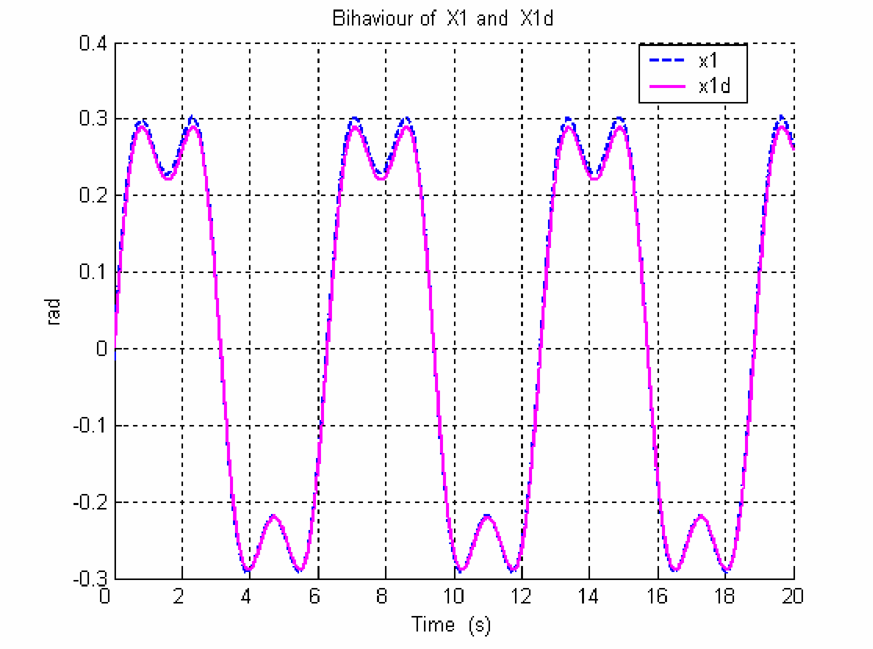

Figure 2: Evolution of x_1 and x_{1d}

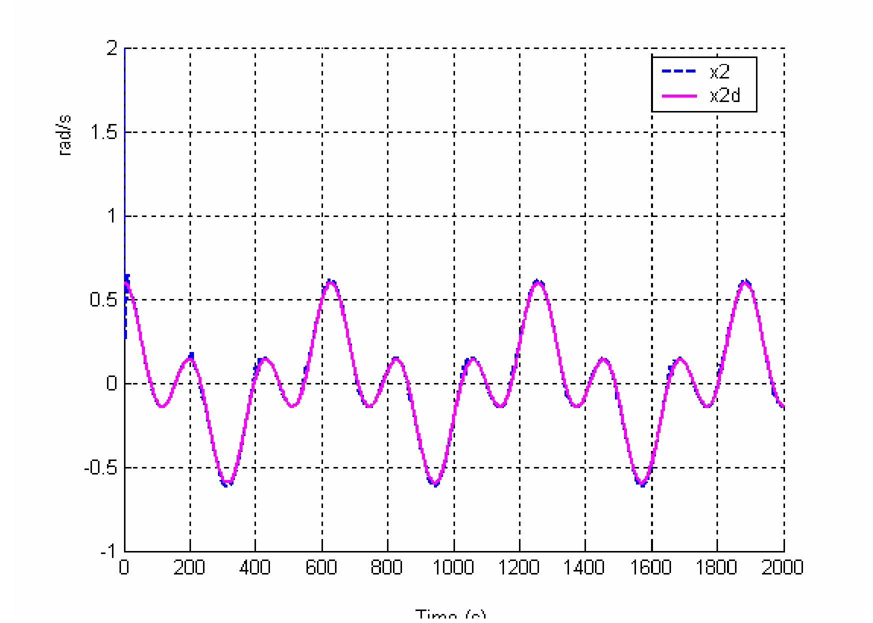

Figure 3: Evolution of x_2 and x_{2d}

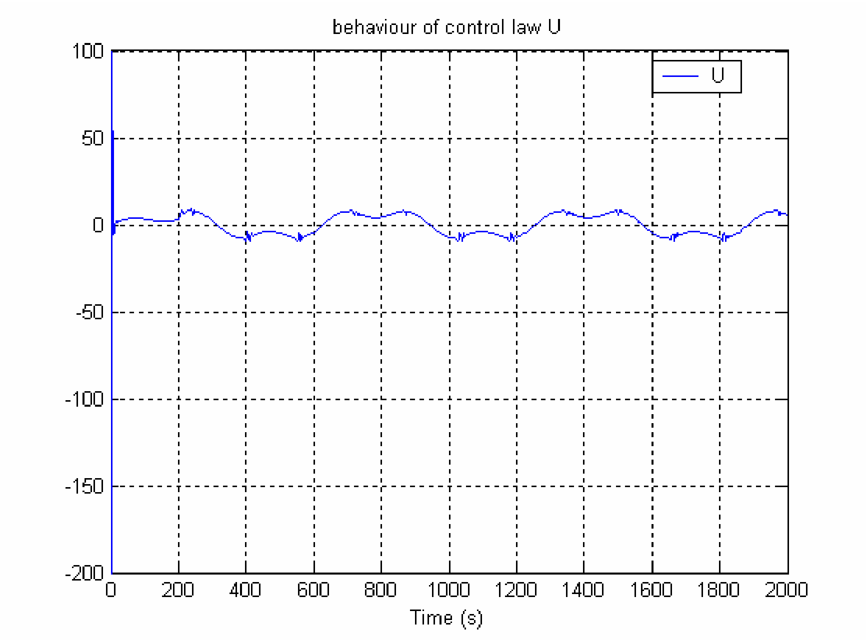

Figure 4: Evolution of the global control law *u(t)*

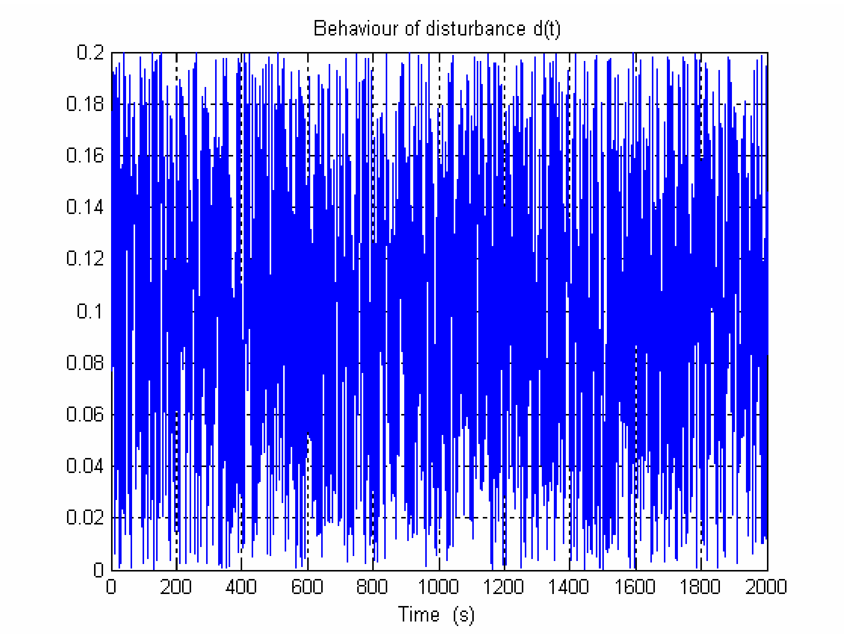

Figure 5: Evolution of disturbance *d(t)*

4.2 Mass-spring-damper

The Mass-spring-damper system is described by the following equation [20]:

$$M\ddot{x}(t) = u(t) - f_k(x) - f_B(x) - f_C(x) + d(t) \tag{48}$$

where, $f_k(x)$ denotes the spring force due to K, $f_B(x)$ is the friction force and $f_c(x)$ is the Coulomb friction force, M is a body mass (kg), K is a spring coefficient (N/m), B is a friction coefficient (N/m/s) and u is an applied torque input (N).

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = f(x_1, x_2) + g(x_1, x_2)u + d \\ y(t) = x_1(t) \end{cases} \tag{49}$$

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = \frac{1}{M}\left(-f_k(x) - f_B(x) - f_C(x)\right) + \frac{1}{M}u + \frac{1}{M}d(t) \\ y(t) = x_1(t) \end{cases} \tag{50}$$

This mass-spring-damper system suffers from plant uncertainties, unmodeled force and external disturbances.

The nominal parameters of the system are given by: $M_0 = 1.0$, $K_0 = 2$ and $B_0 = 2$.

The perturbations of the system parameters are given as: $\Delta M = 0.1\sin(x)$, $\Delta k = 0.5$, $\Delta B = 0.5$. Also the nonlinear spring force and friction force are assumed to be $f_k(x) = K_O x_1 + \Delta K x_1^3;\ f_B(x) = B_O x_2 + \Delta B x_2^2;\ f_C(x) = 0.01\,\mathrm{sgn}(x_2)$

Then, the expressions of $f(x_1,x_2)$ and $g(x_1,x_2)$ are:

$$f(x_1, x_2) = \frac{1}{M_O + \Delta M}\left(-f_k(x) - f_B(x) - f_C(x)\right) \tag{51}$$

$$g(x_1, x_2) = \frac{1}{M_O + \Delta M} \quad ; d_1 = \frac{1}{M_O + \Delta M}d \tag{52}$$

By substituting all parameters into equation (51) and (52), we have to determine the bounds. We obtain:

$$\left|f(x_1, x_2)\right| \le \left(\frac{1}{\left|1 + 0.1\sin(x_1)\right|}\right)\left(0.5\left|x_1^3\right| + 2\left|x_1\right| + 2 + 0.01\left|\mathrm{sgn}(x_2)\right|\right)$$

$$0.9 \le \left|g(x_1, x_2)\right| \le 1.5$$

The premises variables are $x(t) = z(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$

We propose five membership functions for each variable state, $x_i \in [-1; 1]$.

The membership functions are:

$$\mu_i^1(x_i) = \exp\left[-\left((x_i - 1)/(2)\right)^2\right] \text{ for } x_i \in [0.5; 1]$$

$$\mu_i^2(x_i) = \exp\left[-\left((x_i - 0.5)/(2)\right)^2\right] \text{ for } x_i \in [0; 1]$$

$$\mu_i^3(x_i) = \exp\left[-\left(x_i/(2)\right)^2\right] \text{ for } x_i \in [-0.5; 0.5]$$

$$\mu_i^4(x_i) = \exp\left[-\left((x_i + 0.5)/(2)\right)^2\right] \text{ for } x_i \in [-1; 0]$$

$$\mu_i^5(x_i) = \exp\left[-\left((x_i + 1)/(2)\right)^2\right] \text{ for } x_i \in [-1; -0.5]$$

The matrix A_m is chosen as: $A_m = \begin{bmatrix} 0 & 1 \\ -100 & -1 \end{bmatrix}$;

The matrix P solution of Lyapunov equation (17), is :

$$P = \begin{bmatrix} 101.01 & 0.01 \\ 0.01 & 1.01 \end{bmatrix} \quad ;$$

The sliding surface is choused as: $S = 5\ e + \dot{e}$.

The parameters r_1, r_2 and r_3 are chosen as: $r_1 = r_2 = r_3 = 1$.

We present in figures 7 and 8 the simulation results of behavior of mass spring damper with parameters uncertainties.

The expression of the desired signal is: $x_{1d} = \pi/10 \ \sin(t) + 0.3 \ \sin(3t)$.

The initial conditions are given by: $x(0) = [0.15 \quad 0.15]$,

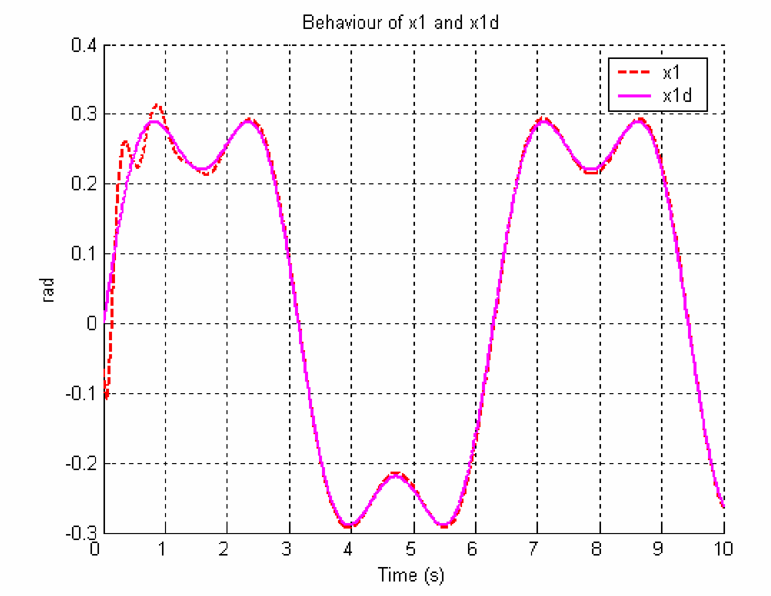

Figure 6: Evolution of x_1 and x_{1d}

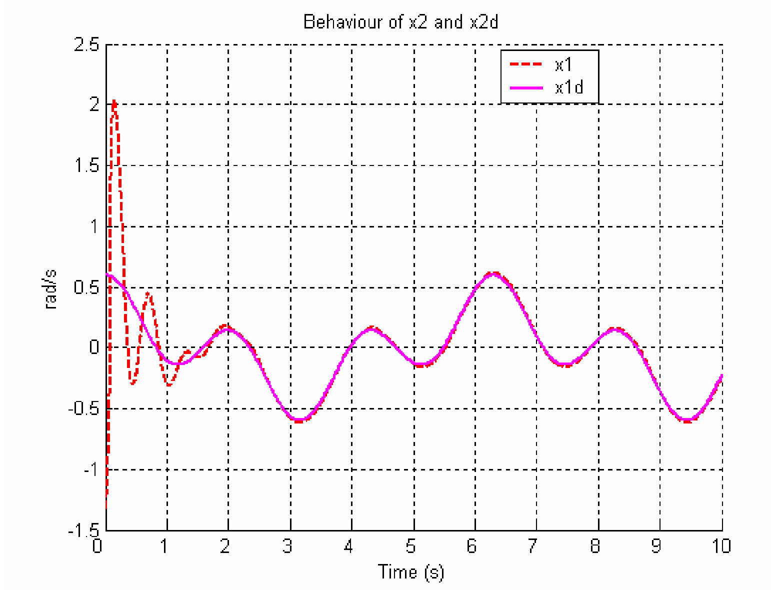

Figure 7: Evolution of x_2 and x_{2d}

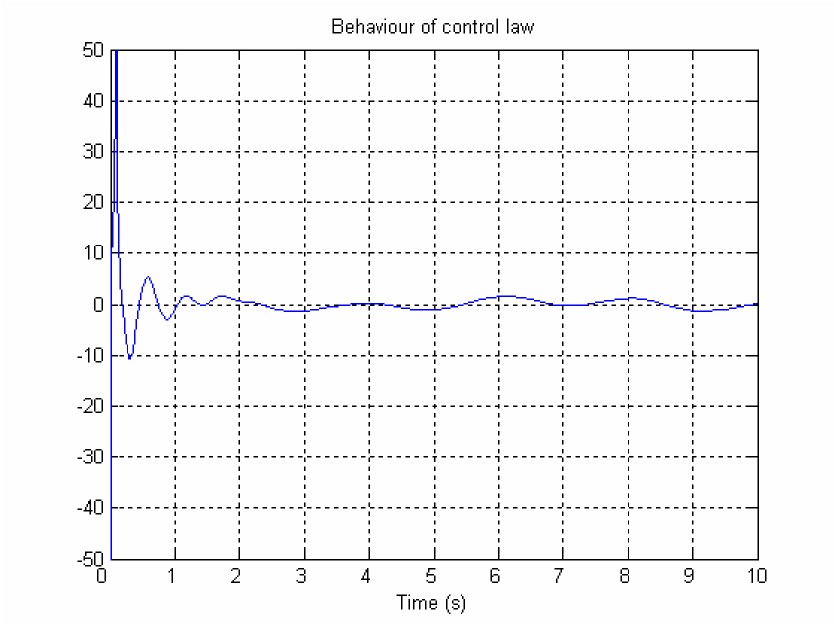

Figure 8: Evolution of global control law *u(t)*

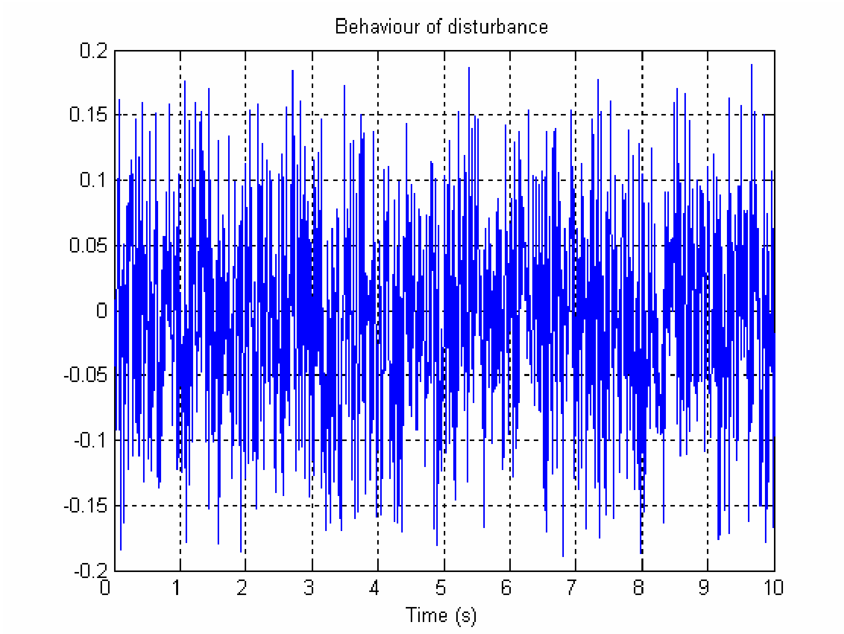

Figure 9: Evolution of disturbance *d(t)*

Commentaries

It is important to recall that, although the uncertainties or the variation of the parameters of plant, the response of system tracks the desired value and the tracking error converge to zero. The error converges asymptotically to zero. We note that there is a slight difference between Gaussian and triangular membership functions.

5. Conclusion

In this paper, an indirect adaptive fuzzy control scheme based on Lyapunov approach for a class of SISO nonlinear systems has been proposed. The functions of plant, which are supposed unknown, are estimated by universal fuzzy system. The adaptive law is based on Lyapunov approach. The control law results in addition of a classical controller with a supervisory controller. The convergence of the estimation algorithm has been proved based of the Lyapunov approach. Simulation results, dealing with the inverted pendulum and mass spring damper systems, show the performance of the proposed approaches.

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