Multimodel: The Construction of Model Bases

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Abstract: In this article is presented, a new approach for the construction of model bases of a complex system. This approach allows to envisage on the one hand the decomposition of a system in several interconnected subsystems, and on the other hand the interconnection of model bases. One can so associate optimized model bases dedicated each one to a particular component (engine, wagon, pendulum, ...) and generate afterward the adequate model bases of a process formed by these components interconnected together. This approach appears very easy to implement when the considered components are linear and when one of the associated model bases is reduced to a single model. Besides, some complex process can be described by interconnected subsets easy to represent by linear models or model bases.

Keywords: Multimodel: Model bases; Control of complex systems; Non linear systems; interconnected systems

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1. Introduction

Several authors were interested in the multimodel representation (1,3,5,6,9,10,12) and the construction of model bases.

Delmotte and col. (3) suggest forming the model bases with linear models obtained by the linearization around various functioning points. These models of the model bases are thereafter run to elaborate the process control. Obtained results show that contrary to what one usually thinks, an increase in the number of models in the bases does not necessarily improve the performances obtained with a limited number of models (4). It arises then two problems, the first one concerning the number of models to choose for the model bases and the other one concerning their choice.

Ksouri Lahmari and col. (7), have developed a systematic generation method of a model base, limited then to 4 or 5 models. In this case it is necessary to have a bounded parameters description of the system. The model base is formed of 4 extreme models to which one can add the average model and/or the local model. This method which leads to very good results when we have strong uncertainties on the process remains dependent on the description of equations of the system in the form of differential equations with linear bounded parameters.

Ben Abdennour and col. propose in (8, 13) a systematic generation of the model bases from experimental measures without using analytic equations. These approaches use classification algorithms of the data using the kohonen self adapting neural network (11, 14) or the Chiu method (2). It is evident that in this last case the reliability of the constructed model bases is strongly dependent on the relevance of the measurements. Further more, the obtained models remain without any physical interpretation in connection with the process.

We propose in this paper a new approach based on the association of model bases and using block diagram simplification techniques.

2. Model bases association

2.1 General case

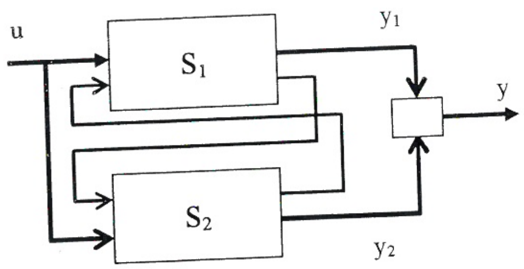


Figure 1: General Case of 2 Coupled Systems

On the figure 1 is represented the general case of 2 coupled systems. Giving the model bases of each of the systems S1 and S2, we propose a method allowing deducing the model bases of the global system, we process at particular cases of bloc diagram associations.

2.2. Blocks in Cascade

Proposition: we consider two systems S_1 and S_2 connected in cascade, as shown in figure 2, and suppose B_1 and B_2 are respectively their model bases having respectively n_1 and n_2 models, a model base for the global system S can be obtained by associating in cascade all models of B_1, to each models of B_2. One obtains thus $n_1 * n_2$ models, composing the global model base.

The validity v_{ij} of a model of the new model base B is given by the product $v_{1i} * v_{2j}$, where v_{1i} is the validity of the model M_{1i} of the model bases B_1 and v_{2j} that of the model M_{2j} of B_2.

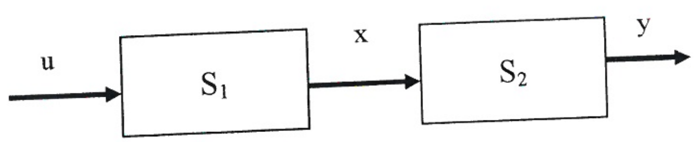


Figure 2: Blocks in Cascade

To illustrate this approach, let us suppose that we have two model bases B_1 composed of 3 models $\{M_{11}, M_{12}, M_{13}\}$ and B_2 composed of 2 models $\{M_{21}, M_{22}\}$, the model bases of the system S noted B includes in this case 6 models $\{M^{ij}\}$. The model M^{ij} is simply the association in cascade of M_{1i} and M_{2j} . For example in the case of a representation by transfer functions, it is simply the product as the shown in the following table :

B_2 \ B_1	M_{11}	M_{12}	M_{13}
M_{21}	$M^{11}=M_{11}*M_{21}$	$M^{21}=M_{12}*M_{21}$	$M^{31}=M_{13}*M_{21}$
M_{22}	$M^{12}=M_{11}*M_{22}$	$M^{22}=M_{12}*M_{22}$	$M^{32}=M_{13}*M_{22}$

The validity of the model M^{ij} is given by $v^{ij} = v_{1i} * v_{2j}$. For example the validity associated with the model M^{21} is $v^{21} = v_{12} * v_{21}$.

One verifies that the sum of validities remain equal to the unity:

$$\sum_{i,j} v^{ij} = \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} v_{1i} v_{2j} = \sum_{i=1}^{n_1} v_{1i}\left(\sum_{j=1}^{n_2} v_{2j}\right) = \sum_{i=1}^{n_1} v_{1i} = 1$$

2.3. Blocks in parallel

If the two systems S_1 and S_2 are connected in parallel, as shown in figure 3, a model base for the global system S can be obtained by associating in parallel all models of the model base B_1, to each models of the base B_2. One obtains thus a model base composed again of n_1*n_2 models. The validity v_{ij} of a model of B is given by the product $v_{1i}*v_{2j}$.

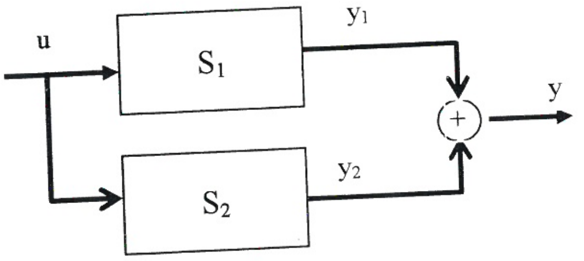


Figure 3: Blocks in Parallel

2.4. Closed loop System

If the system S_1 is in the forward path of the closed loop, as the shown figure 4, a model base for the closed loop system S can be obtained, in the case of a representation with transfer functions (noted F) by:

$$F_i = \frac{F_{1i}}{1 + F_{1i}}.$$

One obtains thus a new model base B composed of $n = n_1$ models. The validity v_i of a model of B is that the associated model of B_1.

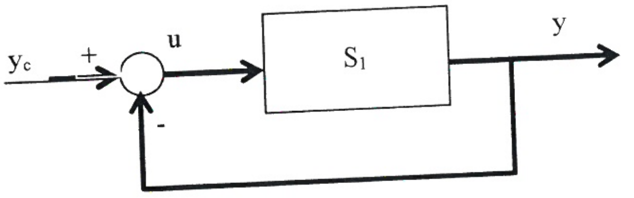


Figure 4: Case of Closed Loop

2.5. Remark:

Suppose that one of the systems S_1 or S_2 is linear (S_1 for example), its model base is reduced to the linear model M_{11} and the model base B of the overall system S is easily obtained by the association of M_{11} to models of the model base B_2, the dimension of B is then $n = n_2$.

Example 1

One considers the benchmark 1 (figure 5) formed by the cascade association of the two systems defined by the following equations:

System 1

$$\dot{y} + y = (-arctg(3y - 7.5) + 5)u$$

System 2

$$(15-10x)\dot{x}+x=(36x(x-1)+10)y$$

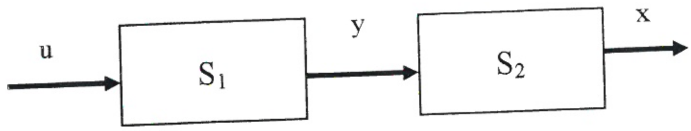
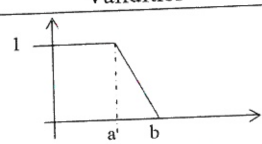
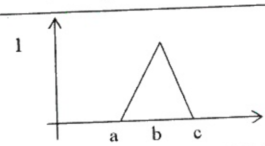
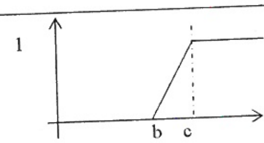


Figure 5: Benchmark 1

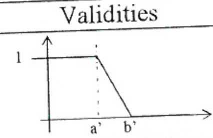
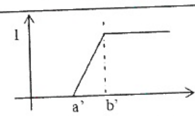
The problem consists to bring the output x of the state 0 to the state 1.

The two systems being strongly non linear, a multimodel approach is proposed. The system 2 is that studied by (4,6), we will adopt the model base proposed by these authors and constituted of linear models corresponding to 3 values of the output x=0; 0.5 and 1 :

Model base B_2	Gains	Time constants	Validities
M_{21}	10	15	
M_{22}	1	10	
M_{23}	10	5	

Validities represented on the above table depend of parameters a, b and c.

We choose for the system S_1, a model base composed of 2 models obtained by the geometrical approach in reference to the curve of the figure 6 that represents the non linear gain of S_1.

Model base B_1	Gains	Time constants	Validities
M_{11}	3.7	1	
M_{12}	6.5	1	

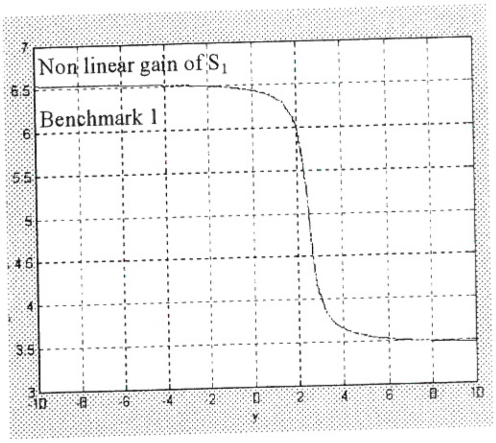


Figure 6: Non linear Gain of S_1

As it has been specified above, we adopt for the global system the following model base :

Model base B	Gains	Time constants
M_1	37	(1,15)
M_2	3.7	(1,10)
M_3	37	(1,5)
M_4	65	(1,15)
M_5	6.5	(1,10)
M_6	65	(1,5)

Figures 7, 8 and 9 show the blocks diagram realization with MATLAB of the benchmark no. 1.

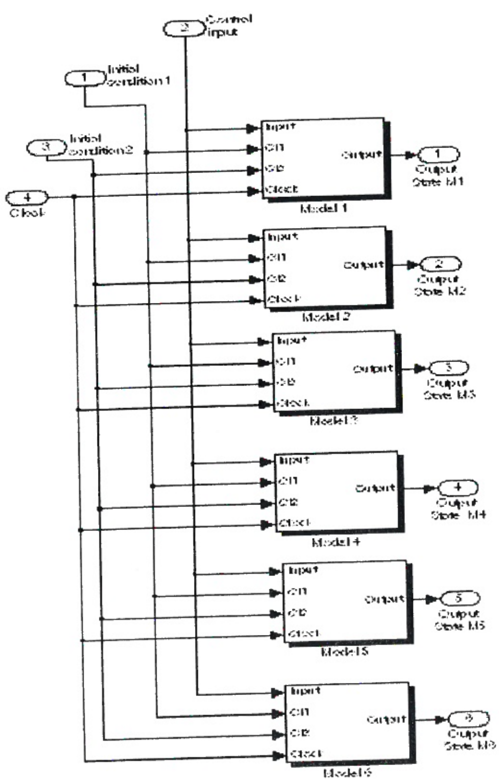


Figure 7 : Model Base

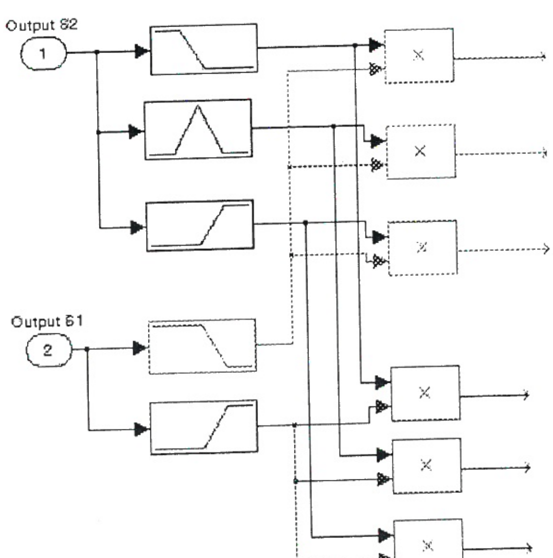


Figure 8 : Validities Calculation

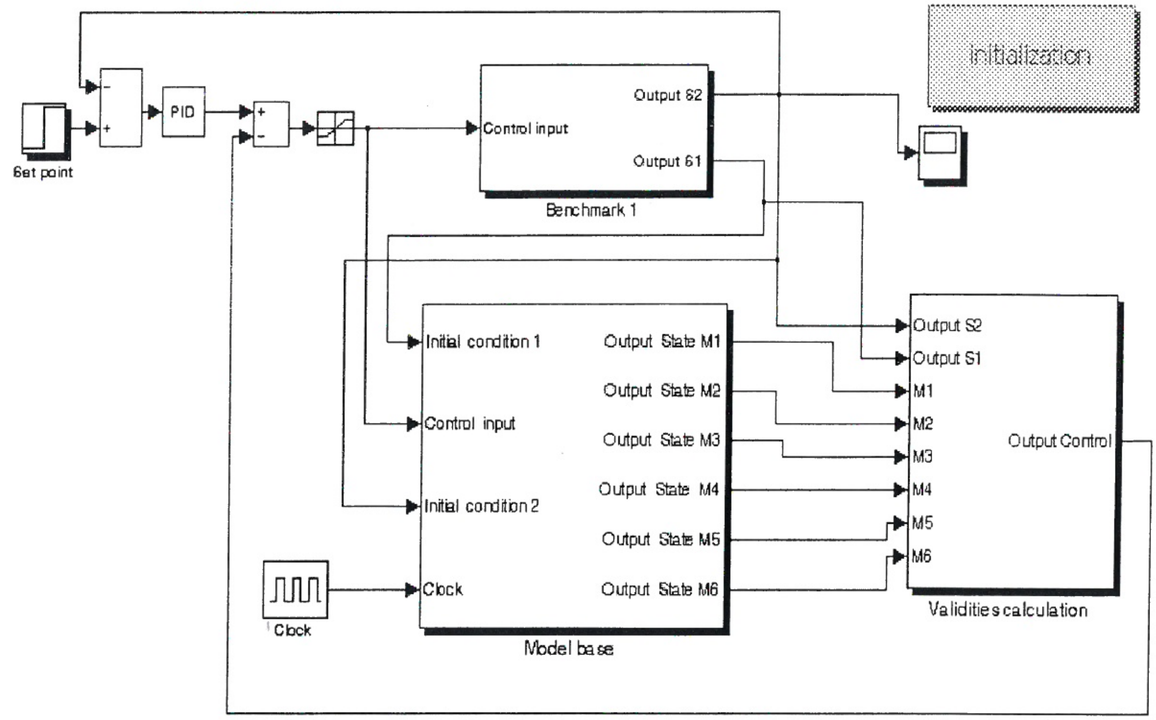


Figure 9: Multimodel Control of Benchmark 1

On the figure 7 are represented 6 blocks corresponding to the 6 models of the model base. Each block have 4 inputs: the control input of the process, two inputs for the initialization of the state of each model and the clock input that determine the period of initialization.

Figure 8 shows the calculation of validities. As shown previously the validity v_k is given by $v_k = v_{1i} * v_{2j}$.

The control signal generated described figure 9 is the sum of a fusion of control signals calculated for each models from the model base and of a PID action. We choose a poles placement (-2, -5) for each of the 6 models of the model base. The obtained results are given on figure 10. We notice the good behaviour of the output of the system; despite the simplicity of the approach, the static error is cancelled.

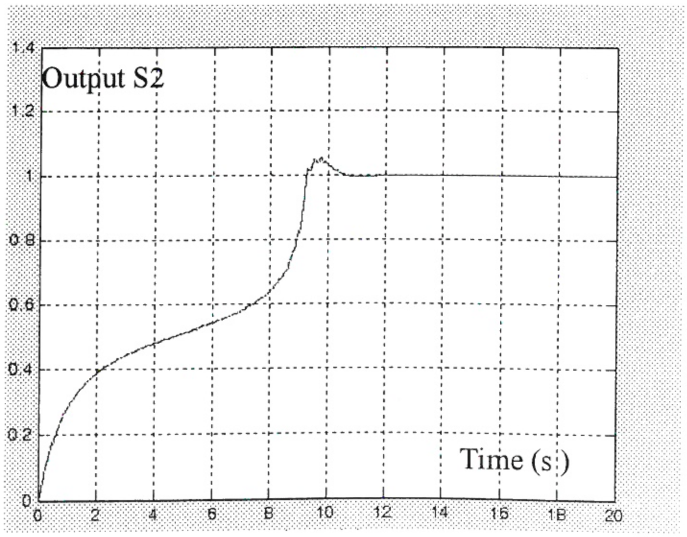


Figure 10: Output of Benchmark 1

Example no.2.

The proposed benchmark no. 2 is characterized by equations:

$$\tau\ddot{x}+\left[1+\dot{\tau}-\left(\frac{\dot{K}}{K}-\frac{1}{\tau_1}\right)\tau\right]\dot{x}-\left(\frac{\dot{K}}{K}-\frac{1}{\tau_1}\right)x=\frac{KK_1}{\tau_1}u,$$

with x the output, u the input control, K_1 a gain factor and τ_1 a time constant:

$$K=(36x(x-1)+10), \qquad \tau=(15-10x).$$

This very complex system at first sight is the result of the interconnection of the following two systems :

$$\begin{cases}\tau\dot{x}+x=Ku_1 & \text{system } S_2\\ \tau_1\dot{u}_1+u_1=K_1u & \text{system } S_1\end{cases}$$

We notice that it concerns the cascade association of the system S_2 of the benchmark 1 and a linear system of the first order S_1, the model bases is then deduced :

Model bases B	Gains	Time constants
M_1	$10K_1$	$(15, \tau_1)$
M_2	K_1	$(10, \tau_1)$
M_3	$10K_1$	$(5, \tau_1)$

The figure 11 represents the block diagram representation of the multimodel control of benchmark 2. We can see 3 blocks : The process (Benchmark 2), the model bases and the calculation of validities and the elaboration of the control signal.

The validities calculation is made in the same way that for the last example (benchmark 1). To generate the control signal we make the fusion of calculated controls using respectively all models of the model bases. For this example, we choose the linear-quadratic regulator design for continuous-time systems to calculate the contribution of each model of the model bases.

Results of simulation as shown in figure 12 confirm clearly the pertinence of the multimodel control algorithm. Finally figure 13 represents evolution of validities of each model of the model bases and the contribution of each of 3 models of the basis accordingly to functions of partition imposed.

The simulation results of figure 12 show the quality of this multimodel approach. The evolutions of the validities are described figure 13

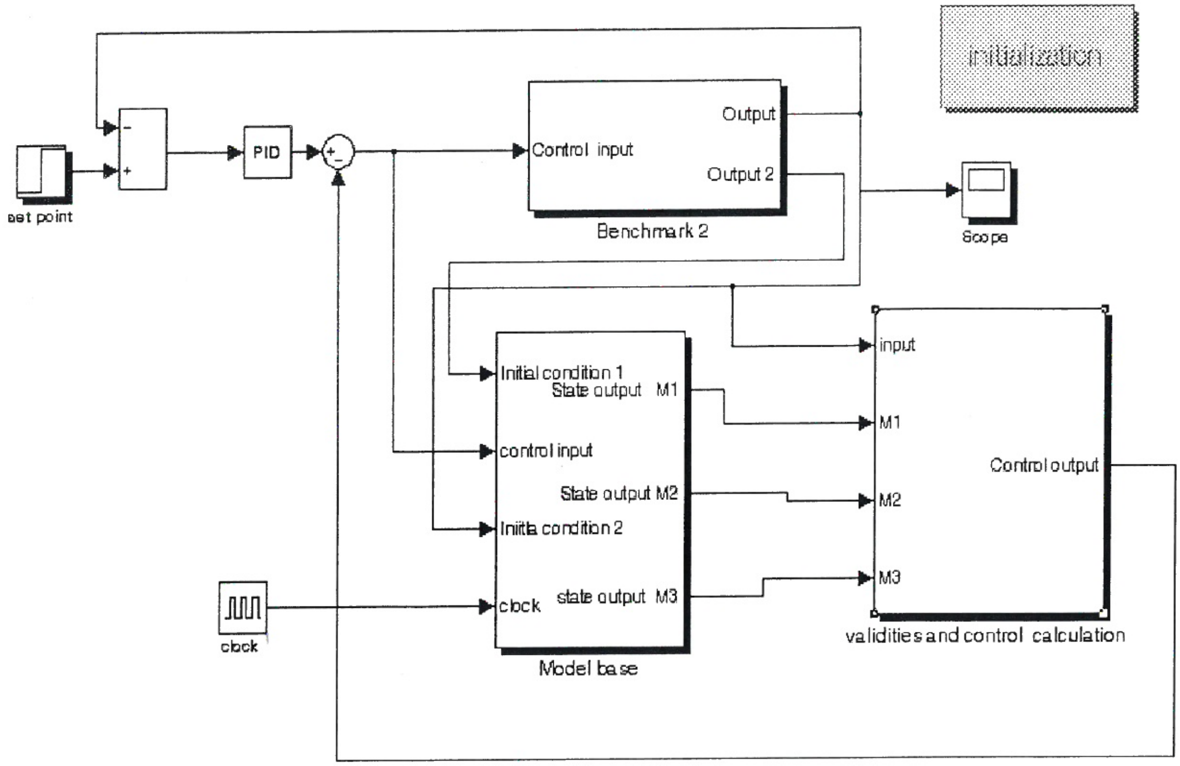


Figure 11: Multimodel Control of Benchmark no.2

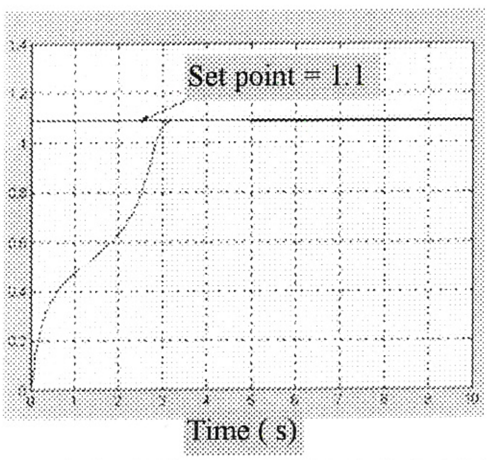


Figure 12 : Output Benchmark 2

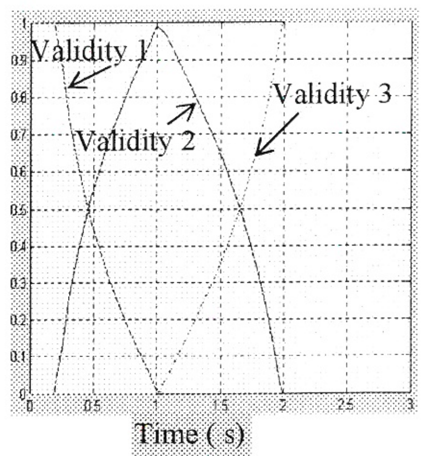


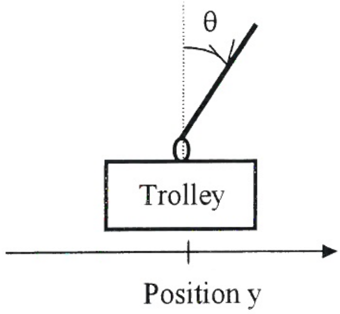
Figure 13: Validities, Benchmark 2

Example 3: the inverted pendulum

The equations of the inverted pendulum are:

$$\begin{cases} (M+m)\ddot{y} = -F\dot{y} - mL\ddot{\theta}\cos(\theta) + mL\dot{\theta}^2\sin(\theta) + Gu \\ \left(J + mL^2\right)\ddot{\theta} = mLg\sin(\theta) - mL\cos(\theta)\ddot{y} - C\dot{\theta} \end{cases}$$

With θ (rad) the angular position of the pendulum, y (m) the position of the trolley, m (Kg) the mass of the pendulum, M (Kg) the mass of the trolley, L (m) the outstrip which separates the connection pivot of the gravity centre of the pendulum, J (Kg m ^{2}) the inertia moment , C (Kg m^2 s^{-1}) the friction coefficient of the connection pivot, F (Kg s^{-1}) the friction coefficient of the trolley, G (N V^{-1}) the coefficient of conversion of the engine and g (m s-2) the gravity acceleration.



This complex fourth order non linear system admits an unstable equilibrium point. The problem is then to realize a control that allows to maintain the pendulum in this position by action on the trolley.

The figure 14 shows that the inverted pendulum can be decomposed in two interconnected subsystems, the pendulum and the trolley.

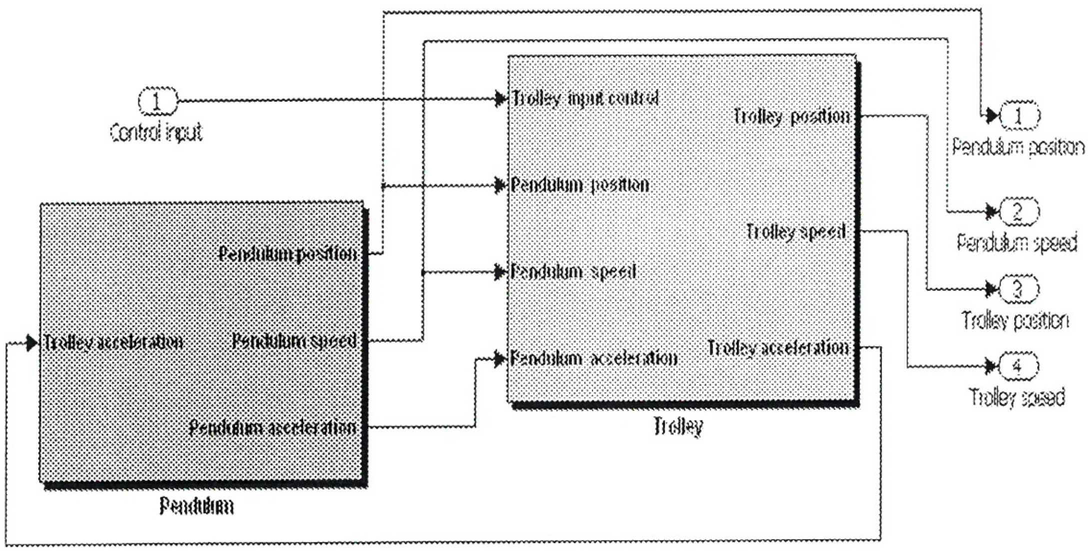


Figure 14: Inverted Pendulum

One notices that if we consider the trolley only, we obtain a linear differential equation of second order, it suffices then to construct the model base only for the pendulum and then to use the composition of the model bases described in the paragraph 2.4.

The model base for the pendulum has been constructed with 3 models obtained by linearization of the equation of the alone pendulum around from points $-\pi/4$, 0 and $+\pi/4$. The symmetry presented by this process around the unstable position allows to limit the base to two models as shown in figure 15 that gives the complete diagram of the proposed control of the inverted pendulum.

Figures 16 and 17 represent successively the evolution of the positions of the pendulum and of the trolley evolving from arbitrary initial conditions. These results are relevant and confirm once more the validity of the approach that we propose.

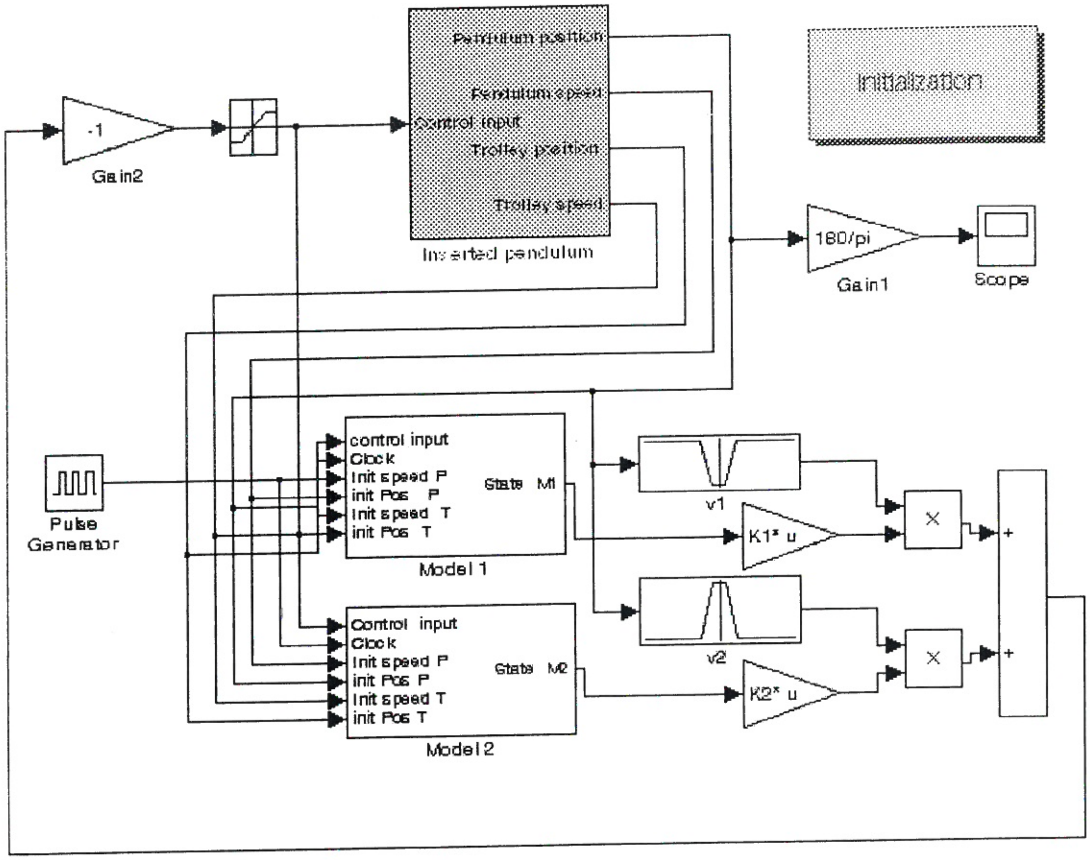


Figure 15: Multimodel Control for Inverted Pendulum

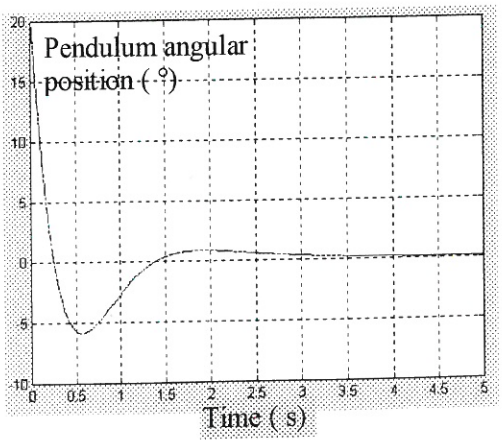


Figure 16: Pedulum Angular Position

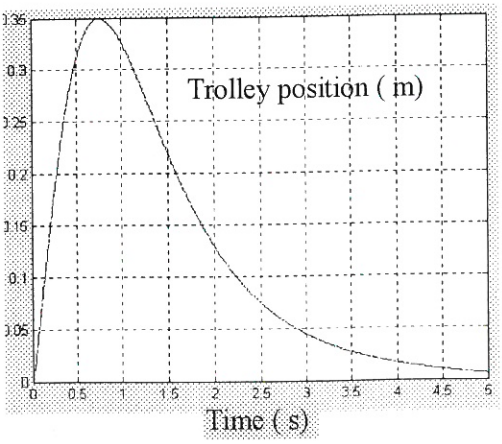


Figure 17: Trolley Position

3. Conclusions

In this paper we proposed an original approach allowing to build the model base for complex non linear systems which can be described as interconnections of several subsystems. In that case, the use of simple rules of block diagram transformations allows to elaborate the model bases of the global system.

We illustrated this approach by examples and we conceived specially two benchmarks allowing to show the relevance of this method.

The first benchmark corresponds to the interconnection of two subsystems represented successively by two model bases, the first one composed of 2 models, and the other of 3 models. The final model base is formed of 6 models. The elaborated multimodel control shows in simulation the good behaviour of the system.

The second benchmark corresponds to a second order system strongly non linear, for which an adequate variable change allows to considerer this benchmark as the interconnection of two subsystems, the first one is linear and the second being able to be represented by a model base. On this example also the multimodel control has given very good results.

The last example of application is the inverted pendulum control. In this case, the fact of envisaging each subsystem alone (pendulum and trolley), allowed to obtain a representation by a model base reduced to two models. The results obtained confirm the validity of the proposed approach.

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