Nonlinear Equations:

Methods, Models and Applications

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Recent advances in nonlinear differential equations are shown in 20 survey and original research papers, presented at the "Workshop on Nonlinear Analysis and Applications", held in Bergamo, Italy, July 9-13, 2001. This workshop was the third edition of the meetings which took place in Campinas in 1996 and 1998, illustrating the fruitful cooperation between researchers in Italy and Brazil. Many new theoretical results addressing elliptic partial differential equations, Hamilton-Jacobi, and Schrödinger equations, and analyses of various models are included. The book is Volume 54 of the Series *Progress in Nonlinear Differential Equations and Their Applications*.

Chapter 1, A Riemann Mapping Type Theorem in Higher Dimensions, Part I: The Conformally Flat Case with Umbilic Boundary, by M.O. Ahmedou, proves that any Riemannian metric on a locally flat manifold with umbilic boundary can be conformally deformed to a flat metric with constant mean curvature; this generalizes known results in the plane.

Chapter 2, Computable Information Content and a Simple Application to the Study of DNA, by V. Benci and G. Menconi, gives computable definitions of information and complexity (which delivers a computable characterization of the Kolmolgorov-Sinai entropy of dynamical systems), and applies the theory to the entropy of complete genomes; numerical results are given. Other asymptotic quantities, measuring the chaotic behavior of dynamical systems, can also be computed.

Chapter 3, Bounded Positive Critical Points of Some Multiple Integrals of the Calculus of Variations by L. Boccardo and B. Pellacci, gives new proofs for some existence results of critical points of a class of integral functionals (with possibly unbounded coefficients) on bounded domains.

Chapter 4, Hilbert Type Numbers for Polynomial ODE's, by M. Calanchi and B. Ruf, investigates the polynomial first order ODE's (in u) with continuous coefficients $a_j(t)$, $j = 1:n$, and right-hand side $f(t)$, $0 <= t <= 1$. For odd n and a_n bounded away from 0 and a_j, $j = 1:n-1$, bounded from above, it is shown that the ODE has at most n closed solutions (i.e., $u(0) = u(1)$), and applies this result to Hilbert's 16-th problem.

In **Chapter 5, S^2-type Parametric Surfaces with Prescribed Mean Curvature and Minimal Energy,** P. Caldiroli and R. Musina study the existence of nontrivial, conformal surfaces in the tri-dimensional Euclidean space parametrized by a sphere, with given (possibly non-constant) mean curvature and minimal energy, with application in capillarity theory.

Chapter 6, Representation of Solutions of Hamilton-Jacobi Equations, by I.C. Dolcetta, investigates first order nonlinear evolution equations of the Hamilton-Jacobi type, where the Hamiltonian function $H(x,p)$ is continuous in x and p, and convex in p. Several methods delivering existence results and representation formulas for the viscosity solution of the problem are described.

Chapter 7, Nonexistence of Global Solutions of Higher Order Evolution Inequalities in R^N, by G. Caristi, proves the nonexistence of global nontrivial weak solutions of evolution inequalities of order higher than a *critical exponent*. Such problems appear in modelling quasi-steady processes in anisotropic continuum electrodynamics.

Chapter 8, Morse Index Computations for a Class of Functionals Defined in Banach Spaces, by S. Cingolani, and G. Vannella, investigates the estimates of critical groups for some functionals associated to a class of quasiliniar equations, containing the p-Laplacian ($p > 2$), extending Morse theory to Banach spaces.

In **Chapter 9, A Global Compactness Result for Elliptic Problems with Critical Nonlinearity on Symmetric Domains,** M. Clapp proposes a description of all G-invariant Palais-Smale sequences for the variational problem associated with an elliptic Dirichlet problem on a bounded domain, invariant under a group G of orthogonal transformations.

Chapter 10, Variational Methods for Functionals with Lack of Strict Convexity, by M. Degiovanni, proposes a suitable nonsmooth approach for a class of functionals of the calculus of variations which are not strictly convex in the gradient of the function.

In **Chapter 11, Some Remarks on the Semilinear Wave Equation**, V. Benci and D. Fortunato study the simplest nonlinear wave equation which is variational and invariant for Poincaré group and for the gauge S^1 (i.e., under the action $\psi \to e^{i\alpha} \psi$, $\alpha \in \mathbf{R}$). Standing waves, travelling solitary waves and solitons, and their interactions with electromagnetic field are discussed.

Chapter 12, Unique Continuation Principles for Some Equations of Benjamin-Ono Type, by R.J. Iorio, Jr., gives conditions for which the solutions of the Cauchy problems associated to the Benjamin-Ono (BO) equations are identically zero. Two unique continuation principles for generalized BO equation are presented, and open problems are indicated.

Chapter 13, Well-posedness Results for the Modified Zakharov-Kuznetsov Equation, by H.A. Biagioni and F. Linares, proves local and global well-posedness for the modified Zakharov-Kuznetsov equation (a two-dimensional generalization of the Korteweg-de Vries equation), using smoothing estimates for solutions of the linear problem and a fixed point theorem.

Chapter 14, A Class of Isoinertial One Parameter Families of Selfadjoint Operators, by O. Lopes, gives conditions under which certain families of selfadjoint operators depending on a real parameter have inertia independent on that parameter. Such families appear when studying the orbital stability of double solitons of integrable equations.

Chapter 15, Traveling Waves in Nonlinearly Supported Beams and Plates, by J. Horák and P.J. McKenna, analyses traveling waves solutions for the nonlinear plate equations in higher dimensions than for the nonlinear beam equation. A variational formulation based on the Mountain Pass Theorem is used, and interesting numerical results are given, illustrating the stability and interaction properties of the solutions.

Chapter 16, Solitary Waves Solutions of a Nonlinear Schrödinger Equation, by A.M. Micheletti and D. Visseti, proves the existence and multiplicity of standing waves solutions for a certain nonlinear Schrödinger equation. These waves are determined by the solutions of an associated nonlinear eigenvalue problem, which, in turn, is considered as a perturbation of the corresponding linear problem. The number of solutions can be as large as one wants. These solutions are characterized by a topological invariant, the *topological charge*.

Chapter 17, Nontrivial Solutions of a Class of Quasilinear Elliptic Problems Involving Critical Exponents, by C.O. Alves, P.C. Carrião and O.H. Miyagaki, deals with a class of quasilinear elliptic problems in radial form, and proves the existence of a nontrivial solution, based on a generalized Mountain Pass Theorem.

Chapter 18, Solutions of Semilinear Problems in Symmetric Planar Domains — ODE Behavior and Uniqueness of Branches, by F. Pacella and P.N. Srikanth, proves an existence and uniqueness result for an initial value problem in the plane, related to a semilinear elliptic equation, whose right-hand side is a smooth convex function.

Chapter 19, Solutions of an Allen-Cahn Model Equation, by P.H. Rabinowitz and E. Stredulinsky, describes the basic and *multibump solutions* for a partial differential equation (PDE) which models the phase transitions in a binary metallic alloy.

Finally, **Chapter 20, Some Equations of Non-geometrical Optics**, by G. Talenti, investigates elliptic and hyperbolic solutions of certain second-order semilinear PDE in optics.

The book is written in the standard theorem-proof formal style. Included are many theoretical results, but there are very few numerical results. Many references to, and comments on the available literature are made, and some ongoing research and open problems are discussed. By its broad coverage and deep analyses, the book is a valuable reference for researchers and professionals in the field of nonlinear PDE's, as well as for advanced graduate students working for a PhD in this area.

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