Nonlinear Accommodation of a DC-8 Aircraft Affected by a Complete Loss of a Control Surface

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Abstract: The occurrence of failures in the control surfaces of an aircraft may become a serious threat to the safety of both aircraft and passengers. That is, using fault-tolerant control (FTC) is vital for such critical systems. The nonlinear progressive accommodation (NPA) is a FTC strategy that consists in solving the State-dependant Riccati Equation (SDRE) in an iterative way. The aim of this work is to study the efficiency of the NPA, in the case of a non-classical stabilization problem and a different modelling of a nonlinear (NL) system, affected by a total actuator failure. That is, a modified NPA strategy is proposed by combining the SDRE control and a feed-forward compensator, derived from the Forward-Propagation-Riccati-Equation (FPRE). The system considered in this paper is a full NL model of a DC-8 aircraft with coupled dynamics. The modelling of the aircraft as well as the simulation of the healthy, affected and accommodated system are presented. The proposed NPA method allows the aircraft to achieve a trajectory-following mission despite the complete failure of its actuator. Most importantly, by preserving the system's stability, the developed approach can be considered as a good alternative to the use of redundant actuators in aircraft.

Keywords: Aircraft, Nonlinear fault accommodation, Failure, Control surface, Trajectory following.

1. Introduction

In critical systems, security is the major concern of manufacturers in their various fields. Thus, having FTC is crucial, particularly for aircrafts where human lives are involved. Indeed, aircrafts may encounter a wide range of unexpected failures such as actuator, sensor, airframe damages. Even the occurrence of a minor fault during the flight may sometimes cause human losses. Therefore, to preserve the safety of both aircraft and passengers, it is crucial to actively deal with these failures and "tolerate" them (Amin & Hasan, 2019).

The different studies dealing with the problem of FTC are based either on analytical or hardware redundancy. Hardware redundancy consists of replacing the failing component with a healthy-one, capable of performing the same task. This last strategy is necessary in safety-critical systems. It is advantageous especially in the case of a complete loss of an actuator or a sensor. However, it is evident that this solution cannot be used in all industrial processes since it requires multiplying the number of actuators and sensors. This makes hardware redundancy financially expansive, regarding the costs of instruments and maintenance. Thus, using analytical redundancy can significantly reduce these material costs as well as the dependence on hardware redundancies.

The analytical FTC strategies are divided into two types: passive and active. In passive approaches, controllers are already designed to be robust against a predefined set of failures. This strategy relies mainly on robust control methods (H_∞ , disturbance rejection, etc...) (Ijaz et al., 2022). However, these methods are very restrictive, since not all the faults and their effects on the system can be known in advance. Active methods, however, rely on diagnosis process and react actively to the faults by redesigning the control law (Dong et al., 2021). Compared to passive FTC approaches, it is obvious that active methods are more realistic and preferable since they can deal with a large number of faults even with the unforeseen ones. Moreover, the real time aspect makes these methods more appropriate to be implemented in aircrafts with the ability to accept degraded performance in fault conditions.

In the specialized literature, there are different active techniques such as Neural Network (NN) (Lin et al., 2020), applied for the accommodation of actuator faults in affine NL systems. Although NN is a very attractive technique for various fields and has many advantages, it has the disadvantage of being computationally expensive and hardware dependent since it requires a processor with parallel processing power. The Eigenstructure Assignment (EA) (Brizuela-Mendoza et al., 2020) is also a widely used technique which consists in making the eigenstructure of the faulty system coincide with that of the nominal system. Despite that stability is always guaranteed with this technique, the major drawback is that full state feedback is

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needed which reduces the applicability to high-degrees-of-freedom systems. In the specialized literature (Zhang et al., 2021), the predictive control was used for the reconfiguration of an aerial vehicle system. This approach can easily manage the different control constraints (such as the physical limitations of actuators). But it induces high computational burden since it requires online optimization. The Linear Quadratic Regulator (LQR) (Wang & Sun, 2022) is another efficient FTC technique that was applied to different aeronautical systems such as airplanes, helicopters, etc. It consists of synthesizing a feedback control, based on the Algebraic-Riccati-Equation (ARE), while minimizing a cost function that depends on the control and state variables. Besides its simplicity, the main advantage of the LQR is that the settling time of the state variables and the amplitude of the control inputs can be taken into account. Another major advantage of this optimal technique is that it can be extended to the case of NL systems (the SDRE theory), which is of interest to this present study.

Concerning aircraft applications, several studies from the specialized literature have investigated the aircraft model using linear equations (Brizuela-Mendoza et al., 2020; Liu et al., 2022) since they are simple to implement and particularly adequate to put in the state-space form. Contrarily, NL equations are more complex and slower to simulate. However, they are closer to reality because the aerodynamic behaviour of the aircraft is itself nonlinear. Moreover, the existing works from the specialized literature considered either longitudinal dynamics (by acting on the elevators) (Guo et al., 2022; Liu et al., 2022), or lateral dynamics (by acting on the ailerons) (Chang et al., 2021). Assuming that longitudinal and lateral motions are uncoupled makes the control system much easier as it avoids complex NL relations between state variables. However, the separation of longitudinal and lateral equations means separated gravitational and inertial forces, while in "flight reality", the six degrees of freedom are strongly coupled.

Moreover, these techniques do not take into account the delay associated with the computation of the FTC, i.e. they apply the new reconfigured control just after the diagnosis which is not correct and supposes that the system is "ideal". During that time, the faulty system is still under the nominal control law and can risk instability or control inadmissibility. Therefore, it is useful to shorten this period as much as possible to avoid the damages that can be caused by the failure. The progressive accommodation (PA) was conceived to minimize these risks by applying an iterative algorithm in real-time. It was first proposed by Staroswiecki et al. (2006) and Staroswiecki et al. (2007), based on the LQR control. The authors proved the efficiency of the PA compared to the direct approach, where it was applied to a linearized model of a Boeing 747. However, the validity of this linear PA is limited to a certain range of initial conditions. To overcome these problems, a few existing works from the specialized literature (Ghachem, Benothman & Benrejeb, 2013; Ghachem, Benotman & Benrejeb, 2017) used the NPA, but the application was limited to first- and second-order equations and never involved a real NL system. Lately, the NPA was used for the stabilization and the recovery process of a drone (Mlayeh et al., 2020; Mlayeh et al., 2021). A comparison between the NPA and the classical accommodation was presented.

However, all the aforementioned studies considered only the partial actuator failure. The case of a complete loss of an actuator was never investigated. Moreover, a unique linear-like-form of NL systems was considered to allow the application of the SDRE. Additionally, the control problem considered in the previous studies concerned the stability of the system around an equilibrium point (such as the drone case (Mlayeh et al., 2020)), but never a trajectory-tracking problem.

This paper proposes a modified NPA strategy to deal with a non-classical stabilization problem and a different model of a NL system, affected by a complete loss of one actuator. The system considered in this study is a full NL model of a DC-8 aircraft, where longitudinal and lateral motions are coupled. This NL model includes a mismatch term that is due to state-independent terms. The objective is to perform a straight flight with steady longitudinal velocity while there is a complete loss of one control surface. The proposed control accounts for both the tracking problem of the aircraft, and the mismatch term in the NL model. Different simulations are presented to prove the efficiency of the proposed strategy.

The rest of this paper is organized as follows. Section two presents the concept of the NPA, based on the SDRE technique. In section three, the NL model of the DC-8 aircraft is built. Section four presents the problems related to the control objectives and to the form of the NL system. Then, a solution is presented based on the modified control law. Before conclusion, section five sets forth the different simulations.

2. The Iterative Strategy: NPA

In this section, the NPA scheme, based on the SDRE, is presented. The SDRE methodology consists in modifying the NL system in order to put it in a "linear-aspect" form. The matrices depend on the state and can be calculated at different time intervals. The control is computed by solving the ARE. Let the aircraft dynamics have the following form:

$$\dot{x} = F(x,u) \tag{1}$$

where F is a NL function. Equation (1) should be modified to obtain a "linear-aspect" (2). This parameterization is called the state-dependent-coefficient (SDC) form.

$$\dot{x} = A(x)x + B(x)u \tag{2}$$

where $A(x) \in \mathbb{R}^{n\times n}$ and $B(x) \in \mathbb{R}^{n\times m}$ are the state matrix and the input matrix, respectively. There is not a unique selection of the matrices $A(x)$ and $B(x)$ which results in more levels of freedom and so, the performance of the controller is enhanced. Then, the SDRE (3) can be resolved at each sampling interval.

$$\begin{aligned} &A(x)^T T(x) + T(x)A(x) - \\ &T(x)B(x)R_p^{-1}B(x)^T T(x) + Q_p = 0 \end{aligned} \tag{3}$$

where Q_p and R_p are used for penalizing the control input and the state, respectively. Finally, the control is expressed in (4).

$$u = -R_p^{-1}B(x)^T T(x)x \tag{4}$$

where $T(x) > 0$ is the solution of SDRE.

Remark. To obtain a solution $T(x)$ that ensures the asymptotic stability of the system, the following conditions must be satisfied (Yucelen, Sadahalli & Pourboghrat, 2010). For any state $x \in \Re^n$ $(n > 1)$ and given $Q_p(x) \geq 0$ and $R_p(x) > 0$, the choice of the pair $\{A(x), B(x)\}$ must ensure the system (2) controllability, i.e. $rank\left(C_{trb}(x)\right) = n$ where C_{trb} defines the controllability matrix.

$$C_{trb}(x) = \left[B(x) \quad A(x)B(x)\ldots A(x)^{n-1}B(x)\right] \tag{5}$$

If the above condition is not fulfilled, a stabilizable solution $T(x)$ does not exist. It is worth noting that a suitable selection of the pair $\{A(x), Q_p(x)\}$, that makes the system (2) observable, can also ensure the existence of a stable estimation law. However, in this study, it is supposed that states are known and available for measurement and consequently, no observer is used.

As mentioned in the introduction, the NPA is used to shorten the delays related to the computation of the FTC (4) and to avoid the critical damages that can be caused by failures. It consists in applying an iterative algorithm in real-time, (the Kleinman Newton (KN) (Kleinman, 1968)), that converges to the final FTC. Each control produced by this iterative scheme guarantees the stability of the system and gets better as calculation time passes. Let one define t_{fc} as the application time of the FTC using the direct SDRE method, and t_{i0} as the computation time of the first iteration using the NPA. The time required to begin the accommodation process of the direct method is $\Delta = t_{fc} - t_{fdi}$ (where t_{fdi} is the diagnosis instant). However, for the NPA, it takes $\Delta = t_{i0} - t_{fdi}$ where ($t_{i0} < t_{fc}$). This proves the efficiency of the NPA with respect to convergence speed and stability especially when the initial solution is close to the final one. The theory of the KN algorithm consists in the following steps.

- Select a starting matrix $T_0(x) \in \mathbb{R}^{n\times n}$.

Get the subsequent iterate $T_{k+1}(x)$ by solving the Lyapunov equation (6) (k is the number of iterations).

$$\begin{aligned} &T_{k+1}(x)\left(A(x) - B(x)G_k(x)\right) + \\ &\left(A(x) - B(x)G_k(x)\right)^T T_{k+1}(x) = \\ &-Q_p(x) - G_k^T(x)R_p(x)G_k(x) \end{aligned} \tag{6}$$

- Determine the gain (7):

$$G_k(x) = R_p^{-1}(x) B^T(x) T_k(x) \tag{7}$$

- The control is computed online using equation (4) each time the iterate $T_k(x)$ is generated and the associated gain $G_k(x)$ is determined. The algorithm terminates once $T_{k+1}(x) = T_k(x)$.

It is worth noting that the condition $T_{k+1}(x) = T_k(x)$ is theoretical. It may never be satisfied or may require too many iterations. Therefore, it is common in real-time simulations to use a small enough tolerance ε such that $\left\| T_{k+1}(x) - T_k(x) \right\| \le \varepsilon$. In this study, one considers $\varepsilon = 1.0e-5$.

Let $T_\infty(x)$ be the final solution to which the algorithm converges. It also represents the unique solution of the SDRE (3) which leads to the control law (4). It was proved in (Kleinman, 1968) that all the produced iterations $T_{k+1}(x)$ in (6) guarantee the closed-loop stability and converge uniformly to $T_\infty(x)$ such that:

$$0 \le T_\infty \le \ldots \le T_{k+1} \le T_k \le \ldots \le T_0. \tag{8}$$

One can also stress that the choice of an accurate starting matrix makes the convergence faster.

3. Aircraft Modelling

As mentioned in the introduction, simulations will be performed using the NL coupled dynamics of the DC-8 aircraft. To achieve this, one needs to build the appropriate model describing the system behavior and to put it in the form of equation (2) in order to apply the SDRE method. The aircraft dynamics (McLean, 1990) are described using the equations of translational and rotational motions (9), where X, Y, Z denote the forces according to (x, y, z) axis, respectively and L, M, N are the moments according to the roll, pitch and yaw axis, respectively:

$$\begin{aligned}
X &= m\left(\dot{U} + QW - RV + gsin\theta\right) \\
Y &= m\left(\dot{V} + RU - PW - gcos\theta sin\varphi\right) \\
Z &= m\left(\dot{W} + PV - QU - gcos\theta cos\varphi\right) \\
L &= \dot{P}I_x - I_{xz}\left(\dot{P} + PQ\right) + \left(I_z - I_y\right)QR \\
M &= \dot{Q}I_y - I_{xz}\left(P^2 - R^2\right) + \left(I_x - I_z\right)PR \\
N &= \dot{R}I_z - I_{xz}\dot{P} + PQ\left(I_y - I_x\right) + I_{xz}QR
\end{aligned} \tag{9}$$

U, V, W represent the forward, side and vertical velocities respectively. P, Q, R are the roll rate, the pitch rate, and the yaw rate, respectively. θ and φ are the pitch angle and the roll angle, respectively. m is the mass of the aircraft. I_{xz} denotes the product of inertia. I_x, I_y, I_z are the moments of inertia around $x-axis$, $y-axis$ and $z-axis$, respectively. By solving the six equations in (9), one can obtain the derivatives of velocities as well as the derivatives of pitch and roll angles.

Let $x = \begin{bmatrix} U & V & W & P & Q & R & \theta & \varphi \end{bmatrix}^T$ be the state vector. In order to conceive the SDC form, one needs to eliminate the *cos* and *sin* terms (10) by doing the small angle approximation. This implies that the model is valid for small angles and for small angular rates which is actually the case of large aircrafts that cannot generate large angular rates. One must stress that the approximation of the *tan* and *sin* functions have a percentage error higher than the one of cosine. Thus, *sin* and *tan* functions were expanded up to the 2^{nd} term, in order to reduce the error and to make results more accurate.

$$\begin{aligned}
&\sin x = x - \frac{x^3}{3!},\ \cos x = 1 - \frac{x^2}{2!}, \\
&\tan x = x + \frac{1}{3}x^3.
\end{aligned} \tag{10}$$

If all the calculations are correct, one obtains the model in (11) of the form: $\dot{x} = A(x)x + Bu + d$ with the controllability condition (5) satisfied $rank\left(C_{trb}(x)\right) = 8\ \forall x \in \mathfrak{R}^8$. The inputs of the system are δE, δA and δR which represent the command on the elevator, the ailerons and the rudder, respectively. B is the input matrix that depends on the dimensional stability derivatives and on the inertial parameters which vary according to the flight conditions of the DC-8. The matrix $d \in \mathbb{R}^{8\times 1}$ illustrates the discrepancy between the SDC (2) and the NL dynamics (9). It depends on the stability derivatives of the aircraft, and on U_0 which is the steady-state velocity along the x-axis. All variables and parameters in (11) as well as in A1 and A2 are defined in (Mclean, 1990).

$$\begin{bmatrix} \dot{U} \\ \dot{V} \\ \dot{W} \\ \dot{P} \\ \dot{Q} \\ \dot{R} \\ \dot{\theta} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} A1 & A2 \end{bmatrix} \begin{bmatrix} U \\ V \\ W \\ P \\ Q \\ R \\ \theta \\ \varphi \end{bmatrix} + \begin{bmatrix} X_{\delta E} & 0 & 0 \\ 0 & Y_{\delta A} & Y_{\delta R} \\ Z_{\delta E} & 0 & 0 \\ 0 & C_1 & C_2 \\ C_3 & 0 & 0 \\ 0 & C_5 & C_6 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta E \\ \delta A \\ \delta R \end{bmatrix} + \begin{bmatrix} -X_u U_0 \\ 0 \\ -Z_u U_0 \\ 0 \\ -I_6 U_0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{11}$$

where:

$$A1 = \begin{bmatrix} X_u & 0 & X_w & 0 & -W \\ -R & Y_v & P & 0 & 0 \\ Z_u & -P & Z_w & 0 & U \\ 0 & I_1 & 0 & I_2 + I_4 Q & -I_5 R \\ I_6 & -M_{\dot{w}} P & I_7 & I_8 P & M_{\dot{w}} U + M_q \\ 0 & K_4 & 0 & K_5 - K_6 Q & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & \theta\varphi \end{bmatrix}$$

$$A2 = \begin{bmatrix} V & -g + \frac{g}{6}\theta^2 & 0 \\ 0 & \frac{g}{12}\theta\varphi^3 + \frac{g}{2}\theta\varphi & g - \frac{g}{6}\varphi^2 \\ 0 & -\frac{g}{2}\theta & \frac{g}{4}\theta^2\varphi - \frac{g}{2}\varphi \\ I_3 & 0 & 0 \\ -\left(I_8 R + I_9 P\right) & -M_{\dot{w}}\frac{g}{2}\theta & M_{\dot{w}}\frac{g}{4}\theta^2\varphi - M_{\dot{w}}\frac{g}{2}\varphi \\ K_7 - K_8 Q & 0 & 0 \\ 0 & 0 & -\frac{1}{2}Q\varphi \\ -\frac{1}{2}\theta\varphi^2 + \frac{1}{3}\theta^3 + \theta - \frac{1}{6}\theta^3\varphi^2 & -\frac{1}{6}Q\varphi^3 & \frac{1}{3}Q\theta^3 - \frac{1}{18}Q\theta^3\varphi^2 \end{bmatrix}$$

$I_1 = K_0 I_x L_v + K_0 I_{xz} N_v$

$I_2 = K_0 I_x L_p + K_0 I_{xz} N_p$

$I_3 = K_0 I_x L_R + K_0 I_{xz} N_R$

$I_4 = K_0 K_1 (I_y - I_x + K_0 I_{xz})$

$I_5 = K_0 K_1 I_{xz} + K_0 (I_z - I_y)$

$C_1 = K_0 I_x L_{\delta A} + K_0 I_{xz} N_{\delta A}$

$C_2 = K_0 I_x L_{\delta R} + K_0 I_{xz} N_{\delta R}$

$C_3 = M_{\dot{w}} Z_{\delta E} + M_{\delta E}$

$C_4 = M_{\dot{w}} Z_{\delta F} + M_{\delta F}$

$C_5 = K_3 I_z N_{\delta A} + K_3 I_{xz} L_{\delta A}$

$C_6 = K_3 I_z N_{\delta R} + K_3 I_{xz} L_{\delta R}$

$K_0 = 1 / (I_x - I_{xz}^2 / I_z)$

$I_6 = M_u + M_{\dot{w}} Z_u$

$I_7 = M_w + M_{\dot{w}} Z_w$

$I_8 = I_{xz} / I_y$

$I_9 = \left(I_x - I_z\right) / I_y$

$K_1 = I_{xz} / I_z$

$K_2 = I_{xz} / I_x$

$K_3 = 1 / (I_z - I_{xz}^2 / I_x)$

$K_4 = K_3 I_z N_v + K_3 I_{xz} L_v$

$K_5 = K_3 I_z N_p + K_3 I_{xz} L_p$

$K_6 = K_3 \left(I_y - I_x\right) + K_2 K_3 I_{xz}$

$K_7 = K_3 I_z N_R + K_3 I_{xz} L_R$

$K_8 = K_2 K_3 \left(I_z - I_y\right) + K_3 I_{xz}$

The nonlinearity in (9) makes it difficult to obtain analytical solutions (Mclean, 1990). Therefore, for simplification purpose, every motion variable is assumed to have 2 components (12): a steady-state or mean motion, denoted by the subscript 0 that represents the equilibrium or trim condition, and a dynamic or perturbed motion denoted by the lower case letter. For instance:

$$U = U_0 + u, \quad V = V_0 + v, \\ W = W_0 + w, \ldots \tag{12}$$

These trim conditions are specific to the aircraft type and the flight conditions. They are used with the stability derivatives in order to calculate the perturbed forces and moments x', y, z, l, m', n. For instance, the derivative of speed along the x-axis is:

$$\dot{U} = \frac{x'}{m} - QW + RV - g sin\theta \tag{13}$$

where:

$$\frac{x'}{m} = X_u \left(U - U_0\right) + X_w W + X_{\delta E} \delta E \tag{14}$$

By substituting equation (14) in (13), one can obtain the expression of $\dot{U}$ (15) as a function of state variables, control input and the steady-state term.

$$\dot{U} = f_1\left(U, V, W, Q, \theta\right) + f_2\left(\delta E\right) + f_3\left(U_0\right) \tag{15}$$

where f_1, f_2, f_3 are real functions. The rest of the derivatives can be found in (McLean, 1990).

4. Control Problem

In the sequel, the control objectives associated with the flight case are first presented, as well as the control problem that the aircraft model imposes. Then, the new control strategy is proposed as a solution to the aforementioned problems.

4.1 Problem Position

In flight control systems, it is common to consider different flight cases with simple trim conditions. For instance, a common interesting case is when the aircraft flies straight with its wings level, in symmetric, steady flight. Straight flight is when the components of the angular velocities are zero. In this study, one is particularly interested in the case of a straight flight with steady longitudinal velocity. That means that angular velocities must be zero, and the linear velocity along x axis is constant.

One can notice that here, the control problem differs from that of the quadrotor (Mlayeh et al., 2020), where the major concern was the stability of the drone around an equilibrium point. However, for the aircraft, one is rather concerned with the stability of a motion, i.e, whether the aircraft can keep a specific desired trajectory, if perturbed away from it. The control procedure must then include the trajectory-following problem.

Another important aspect to consider is that the form of the aircraft model in (11) differs from that of the SDC parameterization in equation (2). Indeed, the mismatch (represented by matrix d), between the aircraft dynamics and its SDC factorization must be handled in the control design. The solution to the aforementioned control problems is given in the following subsection.

4.2 Proposed Solution

In the sequel, a NL control law that accounts for the tracking problem of the aircraft, as well as the mismatch due to the state-independent terms in the aircraft dynamics are presented. The control law is a combination of the SDRE control and a feed-forward compensator, and is derived from the FPRE (Prach, Tekinalp & Bernstein, 2016). In the aforementioned works from the specialized literature, the FPRE was applied for disturbance rejection and command following of a NL system. The "Forward" term is used because the method relies mainly on the solution of differential equations, integrated forward in time.

The FPRE method is motivated by the classical tracking problem which consists in the resolution of two differential equations. The first one is a Riccati differential equation and the second one is a vector differential equation. Both equations are propagated backward in time and need a prior knowledge of system dynamics. In FPRE strategy, one replaces the backward-propagating differential equations by equations that propagate forward. The main advantage about this forward propagation is that no prior knowledge of the command signal, nor the system dynamics, is required. This last property, in addition to the simple structure of the FPRE, make it an attractive method for real-time implementation.

In this study, the minus sign in the feedback regulator is kept as in the case of SDRE method, and a feed-forward compensator is added, derived from the vector differential equation. Let one consider the following NL system (16).

$$\begin{aligned} \dot{x}(t) &= A(x)x(t) + B(x)u(t) + f(t) \\ y(t) &= C(x)x(t) \end{aligned} \tag{16}$$

where: $A(x) \in R^{n\times n}$, $C(x) \in R^{l\times n}$, $B(x) \in R^{n\times m}$, $x(t) \in R^n$, $y(t) \in R^l$, $u(t) \in R^m$ and $f(t) \in R^n$ is a signal that varies slowly and that might be constant for a period of time. In this study, $f(t)$ is constant and it represents the mismatch term due to the independent-of-state terms. It can also represent a known disturbance (Prach, Tekinalp & Bernstein, 2016). The control objective is to make the output signal $y(t)$ follow a desired reference signal $z(t)$ as close as possible, while minimizing the infinite horizon cost function (17).

$$J = \int_{t_0}^{\infty} \left\{ e^T Q_p(x) e + u^T R_p(x) u \right\} dt \tag{17}$$

where: $e(t) = z(t) - y(t)$ is the tracking error. The optimal control is obtained from the derivation of the Hamiltonian:

$$\begin{aligned} u(t) &= -R_p^{-1}(x) B^T(x)\left[T(x)x(t) - g(x)\right] \\ &= -G(x)x(t) + R_p^{-1}(x)B^T(x)g(x) \end{aligned} \tag{18}$$

where the feedback gain $G(x)$ is as follows:

$$G(x) = R_p^{-1}(x)B^T(x)T(x) \tag{19}$$

$T(x) \in R^{n\times n}$ and $g(x) \in R^{n\times 1}$ represent the solutions of the forward-propagation equations (20) and (21), respectively.

$$\begin{aligned} \dot{T}(x) &= T(x)A(x) + A^T(x)T(x) - \\ & T(x)E(x)T(x) + V(x) \end{aligned} \tag{20}$$

$$\begin{aligned} \dot{g}(x) &= \left[T(x)E(x) - A^T(x)\right]g(x) - \\ & W(x)z(t) + T(x)f(t) \end{aligned} \tag{21}$$

where:

$$\begin{aligned} E(x) &= B(x)R_p^{-1}(x)B^T(x) \\ W(x) &= C^T(x)Q_p(x) \\ V(x) &= C^T(x)Q_p(x)C(x) \end{aligned} \tag{22}$$

It is noteworthy that for slowly varying reference signals $z(t)$, as it is the case in this study, the derivative in (21) can be set to 0 and the solution of the differential equation is as follows:

$$g(x) = \left[T(x)E(x) - A^T(x)\right]^{-1}\left(W(x)z(t) - T(x)f(t)\right) \tag{23}$$

Then, the final expression of the optimal control law is given in (24):

$$u(t) = -G(x)x + K_z(x)z(t) + K_f(x)f(t) \tag{24}$$

where equation (25) defines the different controller gains.

$$\begin{aligned} G(x) &= R_p^{-1}(x)B^T(x)T(x) \\ K_z(x) &= R_p^{-1}(x)B^T(x)\left[T(x)E(x) - A^T(x)\right]^{-1}W(x) \\ K_f(x) &= -R_p^{-1}(x)B^T(x)\left[T(x)E(x) - A^T(x)\right]^{-1}T(x) \end{aligned} \tag{25}$$

5. Simulations and Analysis

In this section, the NPA is applied for the recovery process of the aircraft affected by an actuator failure. To achieve this, one uses the NL model of the aircraft defined in equation (11). First, one starts applying the control method developed in subsection 4.2 to observe the response of the NL system before the occurrence of failure. Then, one simulates the affected system with and without the correction of the NPA.

Flight condition

A class III large transport DC-8 aircraft with the category B flight phase is considered (Teper, 1969). One assumes the case of the cruise flight condition where the aircraft is flying in a uniform atmosphere at an altitude of $33,000$ ft and 0.84 Mach number. The corresponding weight is $230,000$ lbs and the position of the center of gravity is $(0.15,0,0)$. The steady-state speed along the x-axis is $U_0 = 824.2$ *ft/s*.

5.1 NL Healthy System

Based on the model (11), the aircraft is described by the following state-space representation:

$$\dot{x} = A_n(x)x + B_n u + d, \qquad y = \begin{bmatrix} x_1 & x_4 & x_5 & x_6 \end{bmatrix}^T. \tag{26}$$

The subscript n denotes the nominal system. As stated in subsection 4.1, the main control objective is to make the aircraft fly straight with a steady cruising speed (ideally $U = U_0$) and steady angular velocities ($p = q = r = 0$). In order to apply the FPRE control in normal conditions, the penalization matrices $Q_p(x) \in R^{8\times 8}$ and $R_p(x) \in R^{3\times 3}$ need to be defined. For the system presented in this study, both matrices are chosen independent of states (27).

$$R_p = diag(1,1,1), \qquad Q_p = diag(10^{-6}, 10^{-10}, \ldots, 10^{-10}) \tag{27}$$

Figure 1 shows the output responses and the actuators positions of the healthy system using the FPRE technique. One should notice that short period oscillations last for only $5s$. Then, U converges to the steady speed of 825.5 ft/s and the angular velocities as well as the deflections of the control surfaces are maintained at $0\ rad/s$ and $0\ rad$, respectively. The absolute steady-state error related to the longitudinal and angular velocities is expressed in (28).

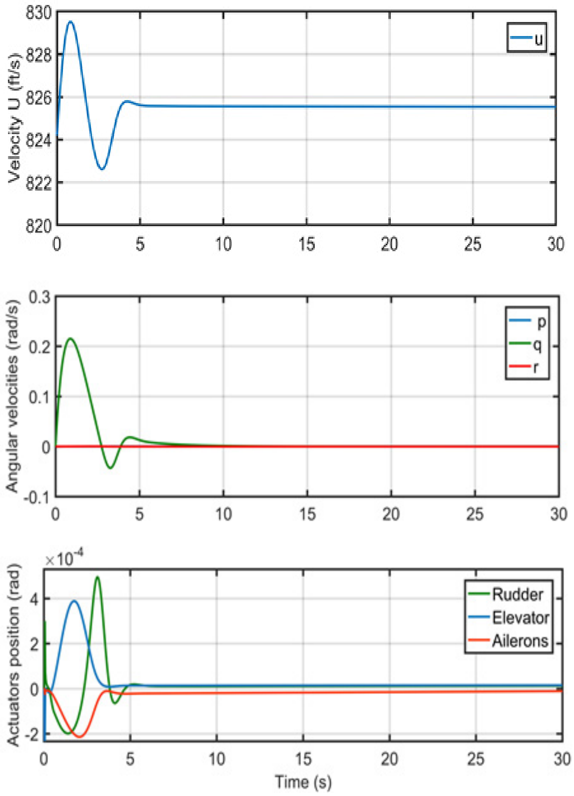

Figure 1. Output responses and actuators positions of the healthy system

$$e_n = (|U - U_0|; |p|; |q|; |r|) = (1.3;\ 0.0002;\ 0.0077;\ 0.0001) \tag{28}$$

Although some slight tracking errors can be seen (which is normal during the cruising of the aircraft), the overall performance of the controller is considered satisfactory. From equation (29), one can obtain the exact actuators positions evaluated after the convergence of the system, for $(x(t > 10s))$:

$$u_n = \begin{bmatrix} 1.5\times10^{-5}, & -1.1\times10^{-5}, & 1.2\times10^{-5} \end{bmatrix}^T \tag{29}$$

5.2 Faulty System

In this subsection, one considers that there is a total loss of the rudder effectiveness at time t_f. The reduction of control effectiveness can be caused by the control surfaces impairment such as stuck, floating, etc. Such failure is modelled by a multiplicative coefficient $\eta \in [0,1]$ that indicates the control effectiveness, i.e. $\eta = 1$ stands for 100 % efficiency, and $\eta = 0$ indicates a total loss of the actuator which is the maximum damage condition. Depending on the type of fault, the diagnosis module provides a pair of matrices $\left(A_f(x), B_f\right)$ describing the affected system where f stands for faulty. In this case, $\left(A_f(x) = A_n(x)\right)$ and the faulty control matrix becomes (30):

$$B_f^T = \begin{bmatrix} X_{\delta E} & 0 & Z_{\delta E} & 0 & C_3 & 0 & 0 & 0 \\ 0 & Y_{\delta A} & 0 & C_1 & 0 & C_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (30)$$

In order to see the impact of the failure, one simulates the behaviour of the affected system using the nominal control law, i.e., using the same control from subsection 5.1 without any accommodation procedure. As it can be seen in Figure 2, the velocity U is continuously increasing after t_f (it reaches $1112 \; ft/s$ at $t = 30s$). The rudder deflection increases as well, from (0 to 0.3) *rad* in 20 s. The pitch rate becomes negative in the interval $[10 \; 20]$s and reaches -0.116 *rad/s*. Similarly, the elevator switches to the negative position to reach -0.083 *rad* at $t = 30 \; s$. One should remark that the failure of the rudder has significantly affected the elevator deflection because of the coupling effects between longitudinal and lateral channels.

5.3 NL Accommodation Procedure

In the previous subsection, the performance of the system under the nominal control u_n was not acceptable. The failure caused the system to deviate from its normal behavior. Thus the necessity to use a corrective procedure: the NPA. As explained in sections 3 and 4, one starts applying the initial gain at time t_{i0}, then one solves the Lyapunov equation and applies the obtained control u_i online until the convergence of the algorithm at time t_{fc} (when $G_i(x) = G_{(i+1)}(x)$). This final iteration corresponds then to the final corrective control u_{fc}. Each time one obtains a solution $G_i(x)$ of the Lyapunov equation, it is applied to equation (25) to compute the iterative control u_i. For illustration purposes, one assumes the delay time $t_{fdi} = 1s$ for the diagnosis, while the time needed to compute each control law u_i is assumed to be $t_e = 0.5s$. One should remind that the main objective of FTC is to ensure that the post-fault system can still achieve the control objectives of the healthy system. Some limited performance degradations can be allowed but not at the cost of stability. Simulation results have shown that the NPA has successfully accommodated the faulty system and has driven its overall behavior very close to the healthy one.

As it can be seen in Figure 3, the stability of the system and the convergence to the steady states is ensured. The rudder deflection resumes to 0 *rad* but at the cost of some steady tracking error (31). In addition to the deflection of the elevator (1.8×10^{-3} *rad*), the longitudinal speed U has decreased to $814.2 \; ft/s$. On the other hand, the angular velocities perform similarly to the nominal case.

$$\begin{aligned} e_{fc} &= [10; \;\; 0.0002; \;\; -0.0001; \;\; -0.0001] \\ u_{fc} &= \left[1.8\times10^{-3}; \;\; -0.1\times10^{-3}; \;\; 0\right]. \end{aligned} \quad (31)$$

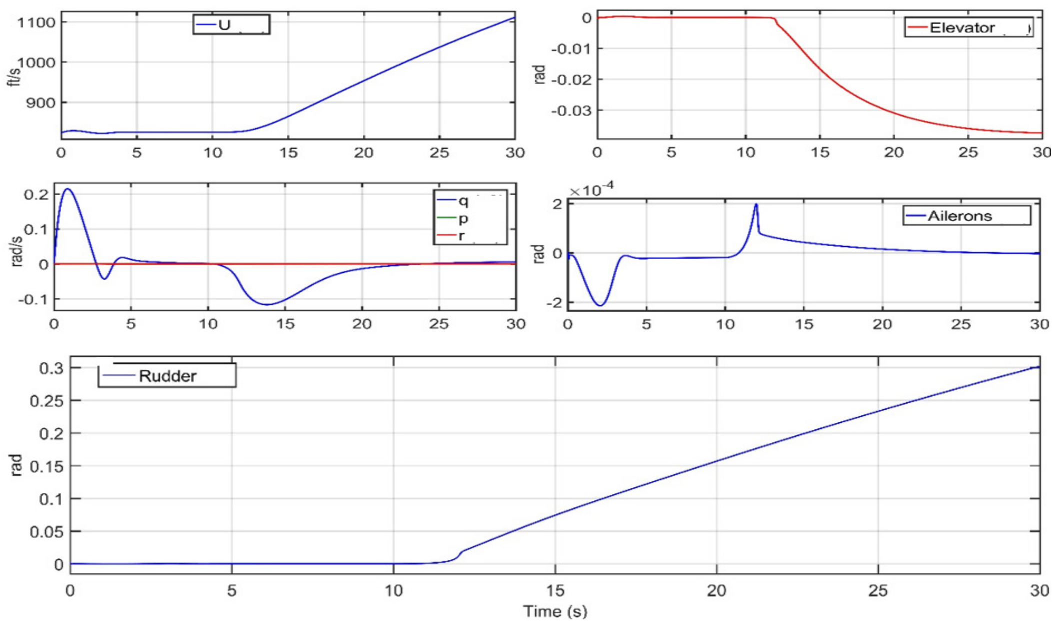

Figure 2. Output responses and actuators positions of the faulty system

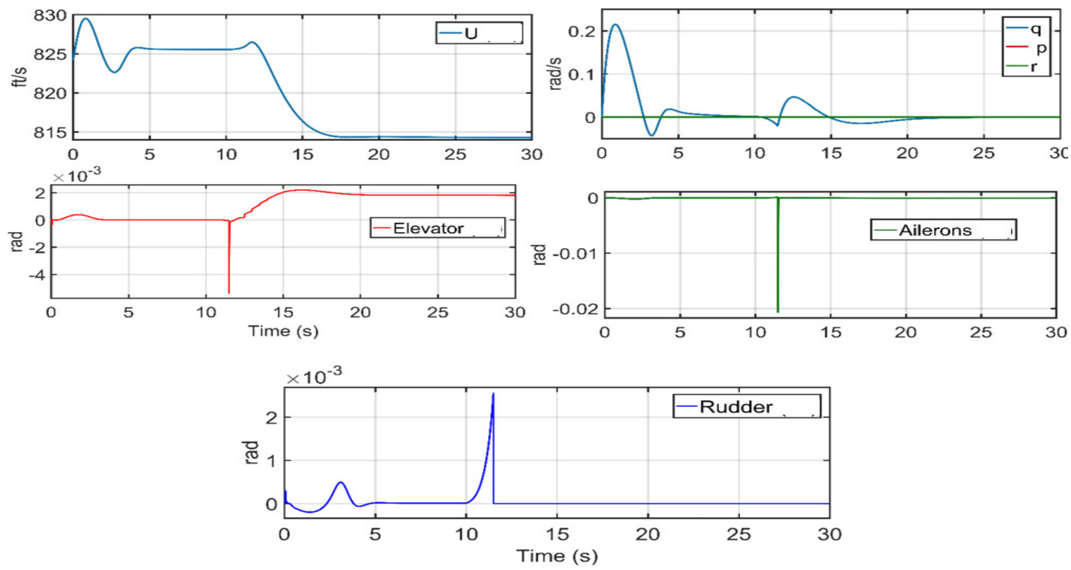

Figure 3. Output responses and actuators positions using the NPA

6. Conclusion

The aim of this study was to prove the efficiency of the NPA, dealing with a non-classical stabilization problem and a different model of an NL system, affected by a complete loss of an actuator. A NL model of a DC-8 aircraft where longitudinal and lateral channels are coupled was built, which made the model more realistic. Then, a modified NPA strategy was proposed by combining the SDRE control and a feed-forward compensator, derived from the FPRE. The proposed methodology accounts for the tracking problem of the aircraft, as well as the mismatch due to the state-independent terms in the aircraft dynamics. By applying the proposed strategy, and with only two controllable actuators, the system could achieve its control objectives. Despite some tracking error, the aircraft continued operating with a recovered stability. That is, the NPA can be considered as a good alternative to the use of redundant actuators, especially for aeronautical systems. Despite the advantages of the NPA strategy, some limitations can be encountered when selecting the SDC matrices that yield optimal control. Another limitation of the NPA is that it requires a very small sampling time which makes simulation time last longer, especially for high order systems.

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