On the Piecewise Continuous Control of Methane Fermentation Processes

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Abstract: The paper deals with a new approach for computer-based control of a methane fermentation process in stirred tank bioreactors. The process is described by a non-linear mathematical model based on one-stage reaction scheme. The process control is realized using a piecewise continuous regulator which enables tracking of a variable set point trajectory and overperforms the existing anaerobic digestion control schemes. Computer simulation examples illustrate the performance of the proposed approach.

Keywords: Anaerobic digestion, Non-linear systems, Piecewise continuous control

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1. Introduction

During the last two decades the anaerobic digestion (methane fermentation) has been widely used in the biological wastewater treatment and depolution of highly concentrated wastes from animal farms and agroindustries [1]. Anaerobic processes result in the production of a biogas, an important energy source that can replace fossil fuel sources and thus contribute to the greenhouse gas reduction.

Anaerobic digestion processes are very complex and their efficient control is still an open problem. Due to the very limited on-line information, the process control is usually reduced to the regulation of the biogas production rate (energy supply) or of the concentration of polluting organic matter (depolution control) at a desired value in presence of disturbances [2]. In this case, however, the classical linear regulators do not have good performances because of the strongly non-linear input-output process characteristics [3]. More sophisticated linearizing controllers have been proposed in [2, 4], but due to some implementation difficulties they haven't found practical application.

In this paper a new control scheme for anaerobic digestion processes is proposed, based on the piecewise continuous regulator developed in [6-8]. The proposed control scheme can be efficiently implemented and overperforms the existing anaerobic digestion controllers both in disturbance rejection and set point tracking. The proposed controller is particularly adapted in this context due to its original property which allows itself to be independent of the plant's model. Thus, it shows robustness against the non linear nature of processes. The paper gives the theory of piecewise control leading to the robust control method. Moreover, simulation examples are proposed so as to show the performance of the approach.

2. Mathematical Model of the Process and Formulation of the Control Problem

Consider the model of anaerobic digestion in a stirred tank bioreactor, based on a one-stage reaction scheme [2, 5]:

$$\frac{dX}{dt} = \mu X - DX$$
$$\frac{dS}{dt} = -k_1 \mu X + D(S_{0i} - S) \qquad (1)$$
$$Q = k_2 \mu X$$

with

$$\mu = \frac{\mu_{max} S}{k_S + S},$$

where X is the biomass concentration (g/l), μ - the specific growth-rate of the methane producing bacteria (day^{-1}), D - the dilution rate (day^{-1}), S - the concentration of the soluble organic (g/l), S_{0i} - influent organic pollutant concentration (g/l), Q - the biogas flow rate (litre biogas for litre of the medium per day), and k_1, k_2, k_S and $\mu_{\max}$ are coefficients.

To avoid the washout of microorganisms, the variations of D and S_{0i} are limited in some admissible ranges:

$$0 \le D \le D_{\max}, \; S_{0i}^{\min} \le S_{0i} \le S_{0i}^{\max}.$$

For a laboratory-scale bioreactor the following estimates of the model parameters k_1, k_2, k_S and $\mu_{\max}$ have been obtained [5]:

$$\hat{k}_1 = 6.7, \; \hat{k}_2 = 16.8, \; \hat{k}_S = 2.3, \; \hat{\mu}_{\max} = 0.35. \qquad (2)$$

The non linear mathematical model (1) together with the parameter values (2) are used in the computer simulation of the anaerobic digestion control systems.

The problem of optimal control of anaerobic digestion may be decomposed in three sub problems [4]:

a) static optimisation;
b) optimal start-up;
c) dynamic optimisation.

In turn, the dynamic optimisation problem is reduced to regulation of:

1) the biogas production rate Q (energy supply),

or

2) the organics concentration S (depolution control),

at a prescribed value (Q^* and S^* respectively) by acting upon the dilution rate D.

In this paper the regulation of both biogas production rate and organics concentration is considered for the anaerobic digestion process (1). New, efficient solutions for these regulation problems are obtained using the piecewise continuous control scheme presented in the next section.

2. Piecewise Continuous Control

In [6,7] a class of hybrid systems called Piecewise Continuous Systems (PCS) has been introduced. These systems, characterized by autonomous switchings and controlled impulses can be used as regulators: Piecewise Continuous Controllers (PCC). PCC are easily implemented on digital calculators and allow set point tracking by the plant's state. Though the standard PCC requires a linear model of the plant to be controlled, it is shown in [8] that an adaptation of the PCC gives rise to a particular regulator that allows control without knowledge of the plant's model, thus suitable for some time-varying or non linear plants.

2.1 Piecewise continuous controller

The behaviour of a PCC can be summarized as follows:

- The state $\lambda(t) \in \Sigma^{\hat{n}}$ of the PCC is switched to forced values at regular intervals of period t_e. The corresponding switching set is represented by $S = \{ kt_e, k = 0,1,2,... \}$.
- The equations of the controller are

$$\dot{\lambda}(t) = \alpha\lambda(t), \forall t \in \left] kt_e, (k+1)t_e \right], \tag{3a}$$

$$\lambda(kt_e^+) = \delta\psi(kt_e), \forall k = 0,1,2,..., \tag{3b}$$

$$w(t) = \gamma\lambda(t), \forall t \tag{3c}$$

Equation (3a) describes the continuous evolution of the controller's state $\lambda(t) \in \Sigma^{\hat{n}}$ upon $\left] kt_e, (k+1)t_e \right]$, $\alpha \in \Re^{\hat{n}\times\hat{n}}$ being the state matrix of the controller. The only parameter that defines the behaviour of the controller's state in this interval of time is α which can take an arbitrary value. Usually, it is fixed such that the PCC is stable between switching instants.

Equation (3b) defines the controller's state at switching instants, by means of a bounded discrete input $\psi(k.T_e) \in V^s$, and according to the linear relationship characterized by the matrix $\delta \in \Re^{\hat{n}\times s}$.

Equation (3c) is the output equation of the controller, characterized by the full rank matrix $\gamma \in \Re^{\hat{m}\times\hat{n}}$. The output $w(t) \in Y^{\hat{m}}$ constitutes the input command to be fed to the plant.

Figure 1a gives the realization diagram of a PCC and figure 1b shows its state's evolution.

Note that from now on, the discrete values of every function will be considered as being sampled at t_e period and to simplify the notations, any time function $f(t)$ at a given $k.t_e$ instant will be written as $f(kt_e) = f_k \quad \forall k = 0,1,2,...$. Moreover, if any signal $f(t)$ is discontinuous, we shall consider the right value at the discontinuity since the switchings at each kt_e imply consequences occurring at every kt_e^+. However, for simplification sake, the notation f_k will be used, instead of the strict one: $f_k^+ = f(kt_e^+)$.

2.2 Control strategy

In order to illustrate the functioning of the PCC in its standard form, we consider that the plant to be controlled is described by the usual linear state representation

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{4a}$$

$$y(t) = Cx(t), \tag{4b}$$

with $A \in \Re^{n \times n}$, $B \in \Re^{n \times r}$ and $C \in \Re^{m \times n}$ being real constant matrices, and $x(t) \in \Sigma^n$, $u(t) \in U^r$ and

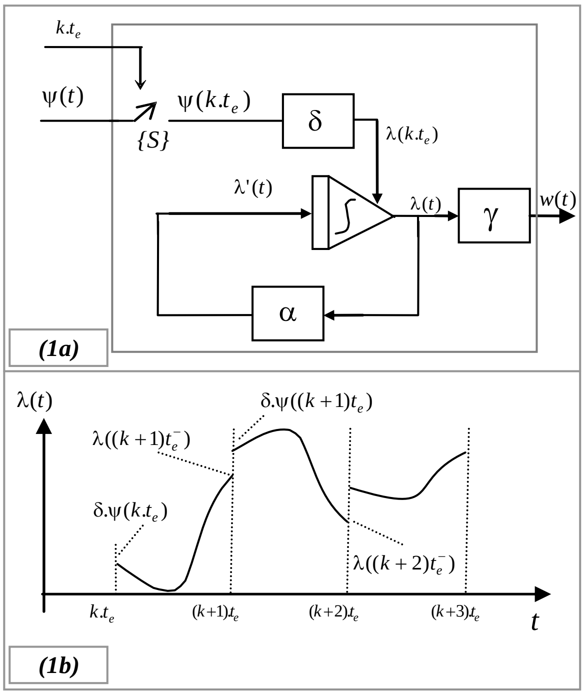

Fig.1a. Realisation diagram,
Fig.1b. State evolution of a PCC

$y(t) \in Y^m$ representing respectively the state, the input and the output of the plant.

The aim is to define the PCC parameters $\psi(t)$ and δ so as to achieve discrete tracking of a $c(t)$ state trajectory by the plant's state $x(t)$ at each switching instant and with one sampling period delay: $x((k+1)t_e) = c(kt_e)$, $\forall k = 0,1,2,...$.

To do so, we consider the closed loop system whose equations are

$$\dot{x}(t) = Ax(t) + Bu(t), \forall t, \tag{5a}$$

$$\dot{\lambda}(t) = \alpha\lambda(t), \forall t \in \left]\ kt_e, (k+1)t_e\right], \tag{5b}$$

$$u(t) = \gamma\lambda(t), \forall t, \tag{5c}$$

$$\lambda_k = \delta\psi_k, \forall k = 0,1,2,.... \tag{5d}$$

By integration, the first three equations allow us to write in a sampled format, the next step value x_{k+1} of the state as a function of its previous one x_k :

$$x_{k+1} = fx_k + M\lambda_k, \tag{6}$$

with $f = e^{At_e}$ and $M = f\int_0^{t_e} e^{-A\tau}B\gamma e^{\alpha\tau}d\tau$.

In order to realize the discrete tracking which is defined above, we only have to fix down the tracking condition which is $x_{k+1} = c_k$, where $c(t)$ is the desired state trajectory. Thus, from (6) we have

$$\lambda_k = M^{-1}\{c_k - fx_k\} \tag{7}$$

Equation (7) gives the switching value of the controller's state, under the condition that M^{-1} exists [6]. Hence, in this case, we are able to define the PCC with

$\delta = M^{-1}$ and $\psi(t) = c(t) - fx(t)$,

2.3 Adaptation of the PCC

According to [8], it is possible to enhance the performance of a PCC by enabling switching at high frequencies, i.e. $t_e \to 0^+$. The author shows that in this case, a PCC allows control of time varying plants or even non linear systems, using state or output reference trajectories for state and output feedback respectively.

To understand the effect of fast switching, let's rewrite the equation of the closed loop structure in the case of an output feedback:

$$y_{k+1} = Cfx_k + Cf \left\{ \int_0^{t_e} e^{-A\tau} B\gamma e^{\alpha\tau} d\tau \right\} \lambda_k \tag{8}$$

with $\dim(\lambda(t)) = m$, $\dim(\alpha) = m \times m$, $\dim(\gamma) = r \times m$.

By realizing $t_e \to 0^+$, it is possible to simplify (8) by

$$y_{k+1} = \Upsilon x_k + \Upsilon \left\{ \int_0^{t_e} (I_n - A\tau) B\gamma (I_n + \alpha t_e) d\tau \right\} \lambda_k \tag{9}$$

with $\Upsilon = C(I_n + At_e)$ and I_n being the *n*-th order identity matrix.

We can thus write (9) as

$$y_{k+1} = y_k + CAt_e x_k + (CB\gamma t_e + \varepsilon(t_e^{\,2}))\lambda_k, \tag{10}$$

$\varepsilon(t_e^{\,2})$ being negligible when $t_e \to 0^+$.

In order to evaluate the initial condition of the PCC state at each switching instant, we fix the tracking condition as $y_{k+1} = (c_o)_k$, where $c_o(t)$ is the output's desired trajectory, such that the closed loop structure becomes

$$(CB\gamma t_e + \varepsilon(t_e^{\,2}))\lambda_k = (c_o)_k - y_k - CAt_e x_k \tag{11}$$

In order to solve (11) numerically, we rewrite the latter as

$$\lambda_k - \lambda_k + (CB\gamma t_e + \varepsilon(t_e^{\,2}))\lambda_k = (c_o)_k - y_k - CAt_e x_k \tag{12}$$

With fast switching ($t_e \to 0^+$), equation (12) becomes

$$\lambda_k = I_m^- \lambda_k + (c_o)_k - y_k \tag{13}$$

Note that I_m^- is the *m*-th order diagonal matrix whose non zero terms are strictly less than 1 and tend to 1. Equation (13) can be interpreted algorithmically by an iterative evaluation of λ_k at each calculation step:

$$\lambda_k \leftarrow I_m^- \lambda_k + (c_o)_k - y_k \tag{14}$$

The calculation of the initial condition of the PCC state at each switching instant is thus highly simplified.

Moreover, the structure of the PCC is simplified by the fact that $t_e \to 0^+$. In fact, in this condition, the evolution of the controller's state is negligible, such that the integrator setup of figure 1a acts as a Zero Order Holder (ZOH). Furthermore, if we consider that switching occur at each calculation step of a digital calculator, the ZOH can be replaced by a short circuit.

Figure 2 illustrates the structure of such a regulator.

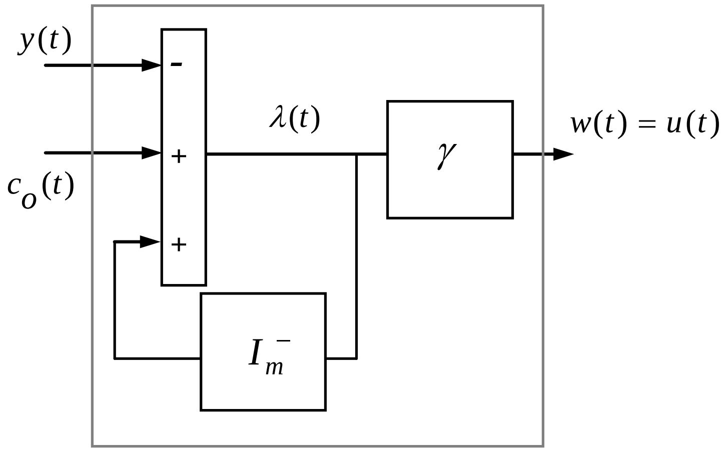

Fig.2. Adaptation of a PCC (fast switching)

The major advantage of this adaptation of the usual PCC is the fact that the model of the plant is unnecessary. It thus allows control of non linear systems.

3. Simulation Experiments

By means of its mathematical model, the anaerobic digestion process has been simulated in Matlab/Simulink® and controlled with the presented regulator.

Regulation of the output *Q* (energy supply)

The performances of the piecewise continuous control have been evaluated for step changes of:

- the set point Q^* in the interval from 0.1 to 0.5 (Fig. 3a),

- the disturbance S_{0i} in the interval from 3 to 4 (Fig. 3b).

Regulation of the output S (depolution control)

In the same way, the controller's performance has been tested with step changes of:

- the set point S^* in the interval from 1 to 0.5 (Fig. 4a),

- the disturbance S_{0i} in the interval from 3 to 4 (Fig. 4b).

According to [5], the input D is limited in the interval $0 \le D \le 0.35$ in all cases.

The simulation results show that the proposed piecewise continuous controller presents very good performances of set point tracking (Figs. 3a, 4a) of a discontinuous reference trajectory. It also ensures regulation (Figs. 3b, 4b) of the plant's output, rejecting disturbances in a large range.

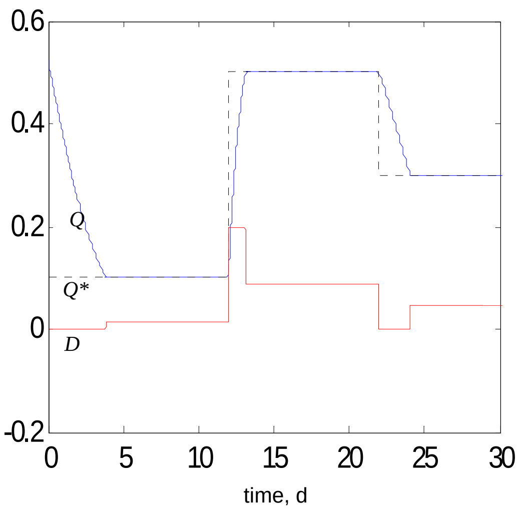

Fig .3a. Step changes of set point (Q^*)

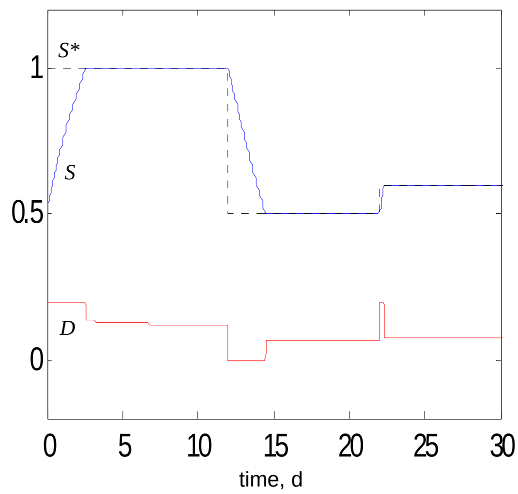

Fig. 4a. Step changes of set point (S^*)

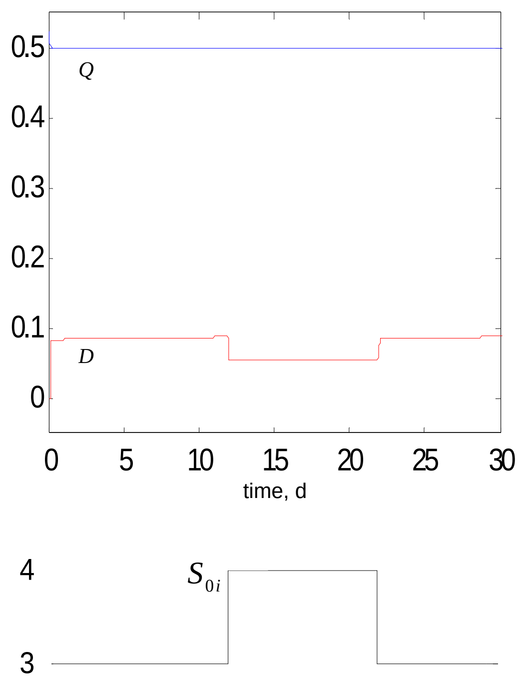

Fig. 3b. Step changes of influent organics concentration (S_{0i})

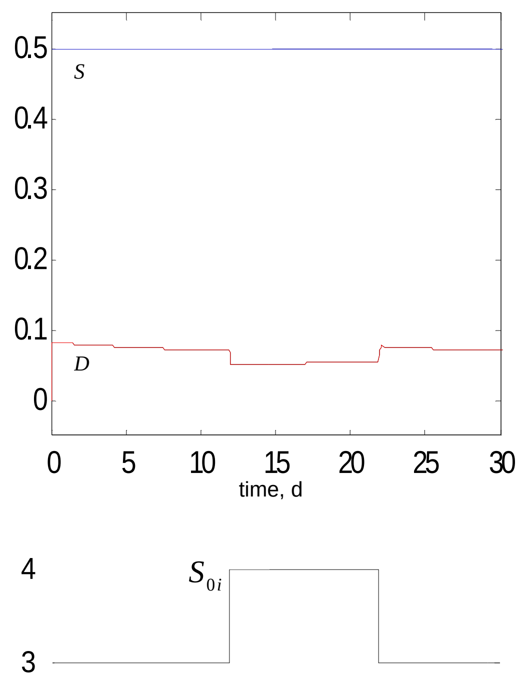

Fig. 4b. Step changes of influent organics concentration (S_{0i})

4. Conclusion and Future Works

Control of the anaerobic digestion process presents difficulties due to its non linear nature. However, the proposed method uses a controller, which is independent of the plant's model, thus overcoming the non linear effects in the present working conditions. Moreover, the regulator can be very easily implemented and requires few step calculations.

As a perspective of our study, works are presently being carried out to realize control of the anaerobic digestion process with a wider range of the set points and disturbances.

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REFERENCES

1. FORSTER, C.F., D.A. JOHN WASE, **Environmental Biotechnology**, Ellis Horwood Ltd, N.Y., 1990.
2. BASTIN, G., D. DOCHAIN, **On-line estimation and adaptive control of bioreactors**, Elsevier Science Publ., Amsterdam, 1991.
3. SIMEONOV, I., **Modelling and control of anaerobic digestion of organic waste**, Chem. Biochem. Eng. Q., **8**, 1994, pp. 45-52.
4. SIMEONOV, I., I. QUEINNEC, **Linearizing control of the anaerobic digestion with addition of acetate**, Control Engineering Practice, 14, 2006, pp. 799-810.
5. SIMEONOV, I. S., **Mathematical modeling and parameter estimation of anaerobic fermentation processes**, Bioprocess Engineering, **21**, 1999, pp. 377-381.
6. KONCAR, V., C. VASSEUR, **Systèmes à fonctionnement par morceaux et poursuite échantillonnée**, APII-JESA, **35**, 2001, pp. 665-689.
7. KONCAR, V., C. VASSEUR, Control of linear systems using piecewise continuous systems. *IEE Control Theory & Applications*, **150**, 2003, pp. 565-576.
8. CHAMROO, A., **Studies in Piecewise Continuous Systems: Application in Online Identification and Real Time Control Fields**. Ph.D. Thesis, Université des Sciences et Technologies de Lille (in French), 2006.