Determination of the Switching Times of the Bang-bang Control for a Linear Stepping Motor by Lyapunov Functions

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Abstract: The linear tubular switched reluctance stepping motor is widely used in applications requiring high-precision linear positioning operations. But, the position evolution of this kind of stepping motors is oscillatory in the case of an open-loop control. In order to damp the position response of such motor, the bang-bang control is adopted. This classical control is simple to implement in practice. However, its successful application is dependent on the suitable determination of the switching instants for which there are no systematic methods allowing their determination. In this paper, a study allowing the determination of the switching instants of the bang-bang control is presented. This study is based on a stability analysis performed by Lyapunov's direct method which is suitable for the studied nonlinear system. The validity of the proposed method is proven by numerical simulations and in practice by the use of a test bench.

Keywords: Linear Tubular Switched Reluctance Stepping Motor, overshoot, Lyapunov fuction, Bang-Bang control

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1. Introduction

The Linear Stepping Motor divides linear distances into discrete incremental movements called steps. The major advantage of this kind of motor, in comparison with a rotary stepping motor, is that the huge part of the supplied mechanical power is converted directly to linear motion. Indeed, the rotary stepping motor needs the use of rotary to linear conversion devices such as leadscrews which introduce additive mechanical loses [1-3]. The predominant inertia is the leadscrew rather than the load to be moved. Thus, most of the motor torque goes to accelerate the leadscrew. So, the linear stepping motor is considered to be the most economical linear motor positioning solution [1-3].

The studied actuator is a Linear Tubular Switched Reluctance Stepping Motor (LTSRSM) characterized by four phases. Such motor represents the linear counterpart of the rotary variable-reluctance stepper motor. Its cylindrical mechanical geometry allows simplicity of construction. The LTSRSM is simple to use and it is characterized by its precision of positioning in open-loop operation. Moreover, it offers solutions to a variety of applications requiring high linear positioning precision. Nowadays, they are widely used in automobile industry, machine tools, robotics domains and electronic industry [4].

As all stepping motors, the step response of the LTSRSM is generally very oscillatory. In application requiring frequent accurate positioning this poorly damped response can be a great disadvantage [5-6]. For example, if the studied actuator is used to insert pins in plastic packages of some electronic components, the oscillations can induce the destruction of the pins or a bad insertion.

There are several solutions allowing the elimination of such oscillations. We focused in this work on the bang-bang control solution which is an open-loop control technique. Such control is easy to implement. However, the determination of the switching instants constitutes the major difficulty in its application. Generally, the switching instants are determined experimentally. Such an approach is very difficult to perform.

In this paper, a method allowing a theoretical determination of such switching instants is presented. It is based on a stability study of the LTSRSM by application of Lyapunov's direct method. The verification of the proposed approach is carried by numerical simulations and experimentally.

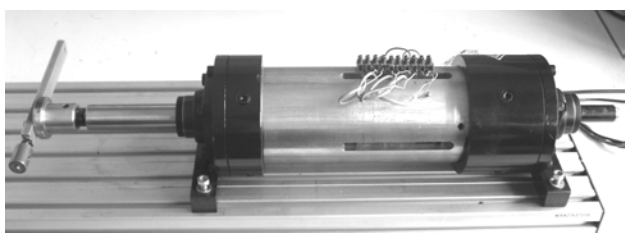
This paper is organized as follows: in section 2 the studied stepping motor structure and mathematical model are presented. The problem statement is introduced in section 3. Section 4 presents the proposed method for the motor oscillations suppression. Finally, the simulation results and the experimental results are given in section 5.

2. Studied Machine Description

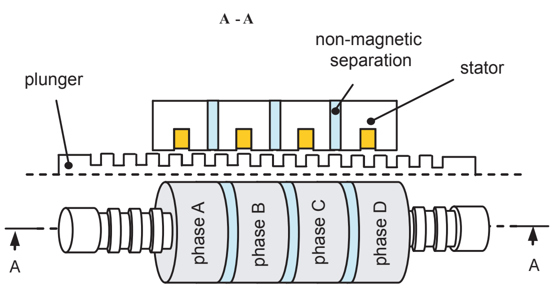
2.1 Studied machine structure

The studied LTSRSM has four electrically identical phases, denoted by A, B, C and D. The linear motor's moving element is called forcer or plunger, and the stationary one is called stator, Figure 1. The stator as well as the plunger are equally teethed. A linear switched reluctance stepping motor operates on the same electromagnetic principles as a rotary variable reluctance stepping machine. When a current passes through one phase winding the plunger tooth tends to align the stator tooth; that is it produces a force that tends to move the plunger to a minimum reluctance position.

The studied motor has 0,2 mm air gap. The elementary step is about 2,54 mm for a total course length of 100 mm.



a- Appearance of the motor



b- LTSRSM structure

Figure 1. Studied LTSRSM

2.2 The mathematical model of the LTSRSM

The approach for the elaboration of the dynamical consider the LTSRSM as a rotary switched reluctance stepping motor by cutting, along the shaft over its radius, both the stator and rotor and then rolling them out.

The phase j voltage of a linear switched reluctance stepping motor can be expressed by equation 1 [7-9]:

$$U_j = R_j i_j + \frac{d\Phi_j}{dt} \tag{1}$$

the subscript j indicates the phase A, B, C or D.

The LTSRSM is supposed to be non saturated. So, the fluxes Φ_j are expressed, in this case, as follows:

$$\Phi_j = \sum_{p=1}^{k} L_{jp}\, i_p \tag{2}$$

Substituting the fluxes expression, equation 2, into equation1, it becomes:

$$U_j = R_j i_j + \sum_{p=1}^{k} (L_{jp}\frac{di_p}{dt} + i_p\frac{dL_{jp}}{dt}) \tag{3}$$

with:

$$\frac{dL_{jp}}{dt} = \frac{\partial L_{jp}}{\partial x}\frac{dx}{dt} + \sum_{q=1}^{k}\frac{\partial L_{jp}}{\partial i_q}\frac{di_q}{dt} \tag{4}$$

Replacing $\frac{dL_{jp}}{dt}$ by its expression in equation 1 by the equation 4, the phase voltage is given by:

$$U_j = R_j i_j + \sum_{p=1}^{k} (L_{jp}\frac{di_p}{dt} + i_p\frac{\partial L_{jp}}{\partial x}\frac{dx}{dt} + \sum_{q=1}^{k} i_p\frac{\partial L_{jp}}{\partial i_q}\frac{di_q}{dt}) \tag{5}$$

Using equation (5), the following parameters can be identified:

- the saturation back EMF voltage: $\sum_{p=1}^{k}\sum_{q=1}^{k} i_p\frac{\partial L_{jp}}{\partial i_q}\frac{di_q}{dt}$,

- the movement back EMF voltage: $\sum_{p=1}^{k} i_p\frac{\partial L_{jp}}{\partial x}\frac{dx}{dt}$.

For the non saturated linear stepping motor, the saturation back EMF voltage can be neglected. Moreover, because of the non-magnetic separation, the statoric phases are supposed to be magnetically decoupled. So, the mutual inductances L_{jp}, for j ≠ p, are very small and they can be also neglected. Thus, the phase j voltage expression, is simplified into:

$$U_j = R_j i_j + L_{jj}\frac{di_j}{dt} + \frac{\partial L_{jj}}{\partial x}\frac{dx}{dt} i_j \tag{6}$$

The linear switched reluctance stepping motor inductances are supposed to be sinusoidal. For the case of a four phases LTSRSM, the inductance characteristics are $\frac{\pi}{2}$ off phased. Hence, the studied machine inductances are given by the following four relations:

$$L_A(x) = L_0 + L_1 cos(\frac{2\pi}{\lambda}x) \tag{7}$$

$$L_B(x) = L_0 + L_1 cos(\frac{2\pi}{\lambda}x - \frac{\pi}{2}) \quad (8)$$

$$L_C(x) = L_0 + L_1 cos(\frac{2\pi}{\lambda}x - \pi) \quad (9)$$

$$L_D(x) = L_0 + L_1 cos(\frac{2\pi}{\lambda}x - \frac{3\pi}{2}) \quad (10)$$

The mechanical equation of the studied machine is described by the following second order differential equation [3]-[6]:

$$m\frac{dx^2}{dt^2} = F(x) - \xi\frac{dx}{dt} - F_0 sign(\frac{dx}{dt}) - F_C \quad (11)$$

where $F(x)$ is the resultant of the forces developed by the four machine phases:

$$F(x) = F_A(x) + F_B(x) + F_C(x) + F_D(x) \quad (12)$$

In case of a non saturated machine, the phase j statoric force is determined by the following expression:

$$F_j(x) = -\frac{1}{2}i_j^2\frac{\partial L_j}{\partial x} \quad (13)$$

So, the theoretical force characteristics, for each machine phase, are sinusoidal and can be expressed by the following relations:

$$F_A(x) = -\frac{\pi L_1}{\lambda}i_A^2 sin(\frac{2\pi x}{\lambda}) \quad (14)$$

$$F_B(x) = -\frac{\pi L_1}{\lambda}i_B^2 sin(\frac{2\pi x}{\lambda} - \frac{\pi}{2}) \quad (15)$$

$$F_C(x) = -\frac{\pi L_1}{\lambda}i_C^2 sin(\frac{2\pi x}{\lambda} - \pi) \quad (16)$$

$$F_D(x) = -\frac{\pi L_1}{\lambda}i_D^2 sin(\frac{2\pi x}{\lambda} - \frac{3\pi}{2}) \quad (17)$$

By considering the expressions of the inductances and the forces, the mathematical model of the LTSRSM is given by the following system composed by five nonlinear differential equations:

$$U_A = Ri_A + \left[L_0 + L_1 cos(\frac{2\pi x}{\lambda})\right]\frac{di_A}{dt} + \frac{2\pi}{\lambda}L_1 sin(\frac{2\pi x}{\lambda})vi_A \quad (18)$$

$$U_B = Ri_B + \left[L_0 + L_1 cos(\frac{2\pi x}{\lambda} - \frac{\pi}{2})\right]\frac{di_B}{dt} + \frac{2\pi}{\lambda}L_1 sin(\frac{2\pi x}{\lambda} - \frac{\pi}{2})vi_B \quad (19)$$

$$U_C = Ri_C + \left[L_0 + L_1 cos(\frac{2\pi x}{\lambda} - \pi)\right]\frac{di_C}{dt} + \frac{2\pi}{\lambda}L_1 sin(\frac{2\pi x}{\lambda} - \pi)vi_C \quad (20)$$

$$U_D = Ri_D + \left[L_0 + L_1 cos(\frac{2\pi x}{\lambda} - \frac{3\pi}{2})\right]\frac{di_D}{dt} + \frac{2\pi}{\lambda}L_1 sin(\frac{2\pi x}{\lambda} - \frac{3\pi}{2})vi_D \quad (21)$$

$$\frac{d^2x}{dt^2}=-\frac{\pi L_1}{m\lambda}\left[i_A^2 sin(\frac{2\pi x}{\lambda})+i_B^2 sin(\frac{2\pi x}{\lambda}-\frac{\pi}{2})+i_C^2 sin(\frac{2\pi x}{\lambda}-\pi)+i_D^2 sin(\frac{2\pi x}{\lambda}-\frac{3\pi}{2})\right]$$
$$-\frac{1}{m}F_0 sign(\frac{dx}{dt})-\frac{\xi}{m}\frac{dx}{dt}-\frac{F_C}{m} \quad (22)$$

The studied machine has the following electrical and mechanical parameters:

$m = 5$ Kg ; $\lambda = 10,16$ mm ; $\xi = 65$ Nm/s ; $F_0 = 0.1$ N ; $L_0 = 225$ mH ; $L_1 = 50$ mH ; $R = 18\,\Omega$; $U_n = 18$ V .

3. Problem Statement

At low stepping rates the open-loop single step position response of a linear variable reluctance stepping motor presents some oscillations, overshoot and a long settling time. Moreover, if the inertia of the motor increases the overshoot and the oscillations increases also, figure 2. This case corresponds to an inertial load applied on the motor plunger.

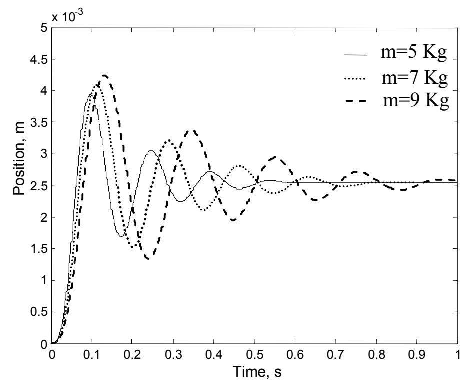


Figure 2. One step position evolutions of the LTSRSM for various plunger weight values

These characteristics disturb the normal operation of this kind of motors. At some speeds, the magnitude of oscillatory response increases with time. As a result, the motor can loose synchronism inducing dynamic unstability and erratic working, figure 3. In applications requiring frequent accurate positioning this poorly damped response is a great disadvantage.

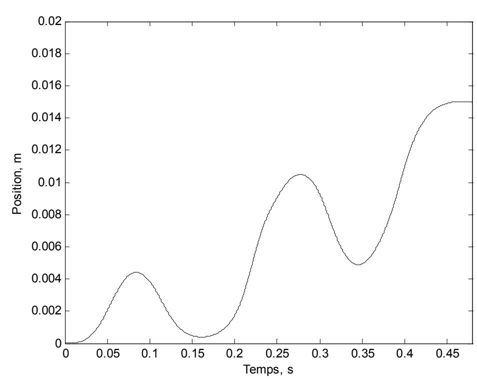


Figure 3. Four steps position evolution for the case of an erratic working

There are several solutions allowing the elimination of the oscillations of the linear stepping motors. These solutions can be classified into mechanical solutions and control solutions.

The mechanical solutions, consists of:

- connecting a mechanical reductor to the motor which is often costly,
- increasing friction at the expense of machine output,
- using a method of electric braking which obviously needs regular maintenance.

These mechanical solutions are cumbersome and they reduce the linear stepping motor nominal force.

The control solutions, more flexible than the mechanical ones, are classified into open loop and closed loop control solutions which require some sensors.

A stepping motor is a machine designed to be controlled in an open loop mode. The open loop control techniques have the merits of simplicity and consequent low cost. Thus, we propose to apply the bang-bang control technique allowing the elimination of the oscillations and the overshoot. It consists of the successive excitation of two of the motor phases. The first one (i+1) is used as a pull winding dragging the plunger to the equilibrium position. The second phase (i) is the braking phase. It is used to absorb the kinetic energy developed when the pull winding is excited. The excitation sequences of the Bang-Bang control are as follows: first phase (i+1) is excited until an instant t_1. At t_1 the excitation is switched from the phase (i+1) to the phase (i). The phase (i) remains excited until an instant t_2. Finally at t_2 the excitation is switched from the phase (i) to the phase (i+1) again to maintain the plunger on the equilibrium position. The bang-bang control voltages applied to the two motor phases are represented in figure 4.

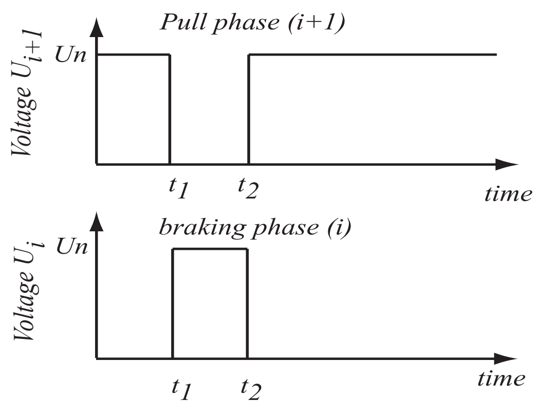


Figure 4. Sequence of the control voltages of the bang-bang control

The major difficulty for the application of the bang-Bang control solution is the determination of the switching instants t_1 and t_2.

Thus, we propose in the following a solution, based on stability analysis of the LTSRSM, allowing the determination of the bang-bang switching instants.

4. Determination of the Bang-Bang Switching Instants

It is known that Lyapunov's direct method allows checking stability of an equilibrium point of a system without explicitly integrating its differential equations. Thus, such method is suitable for studying stability of nonlinear systems or linear systems with uncertainty. The basic idea of the method of Lyapunov is the study of the rate of change of the energy of the system to ascertain stability. However, it is not necessary to have knowledge of the true physical energy. It is sufficient to find a Lyapunov candidate function which proves stability of equilibrium point. However, finding a Lyapunov candidate function remains the main difficulty in applying Lyapunov's direct method [10, 11].

Let us consider the mechanical dynamic equations of the LTSRSM expressed by the following system:

$$\begin{cases} m\dfrac{dv}{dt} = F(x) - Fc - \xi v - F_0 sign(v) \\ \dfrac{dx}{dt} = v \end{cases} \tag{23}$$

The four motor phases are supposed to be identical. In order to simplify the mathematical formulation of the problem, only two successive motor phases (i+1) and (i) are considered in what follows. Indeed, the electrical cycle of the LTSRSM consists of four steps or four dials. For all the control modes of the LTSRSM at most two successive phases can be excited simultaneously. For example, if the excitation of the phase B allows a full step positioning, the simultaneous excitation of the phases A and B gives an equilibrium position smaller than the full step called microstep.

Table 1. Definition of the pull phase and the braking phase for one electrical cycle

Step order	1	2	3	4
Phase (i+1)	B	C	D	A
Phase (i)	A	B	C	D

If we neglect F_0 and consider the change of variable $x_1 = x - x_0$, where x_0 is the equilibrium position relative to the phase (i+1), the dynamical model is expressed as follows:

$$\begin{cases} \dfrac{dx_2}{dt} = \dfrac{F_{i+1}(x_1)}{m} - \dfrac{F_c}{m} - \dfrac{\xi}{m} x_2 \\ \dfrac{dx_1}{dt} = x_2 \end{cases} \tag{24}$$

where $F_{i+1}(x) = -F_{max}\, sin(\dfrac{2\pi x_1}{\lambda})$ (25)

and $F_{max} = \dfrac{\pi L_1}{\lambda} I_n^2$ (26)

In this work, we will consider the case of an inertial load which increases the plunger weight m, so the force F_c is considered null.

Let us consider the function defined by:

$$V(x_1, x_2) = \frac{1}{2}x_2^2 + \frac{\lambda F_{\max}}{2\pi m}\left(1 - \cos(\frac{2\pi x_1}{\lambda})\right) \tag{27}$$

For $x_1 \in \left]-\lambda \;\; \lambda\right[$, corresponding to the domain of the plunger displacement, such function is positive definite.

The time derivative of the equation (27) is given by:

$$\dot{V}(x_1, x_2) = -\frac{\xi}{m}x_2^2 \tag{28}$$

So $\dot{V}(x_1, x_2)$ is a negative definite function and $V(x_1, x_2)$ is a Lyapunov candidate function. The figure 5 represents in the plunger position and speed phase plane the simulated motor trajectory obtained by excitation of the phase (i+1). The obtained trajectory is inside an equilibrium boundary which corresponding to the function $V(x_1, x_2) = V_0$. If the initial point of the trajectory is inside the stable boundary the system is stable. However, if the initial point of the trajectory exists outside the stable boundary, the system loses stability.

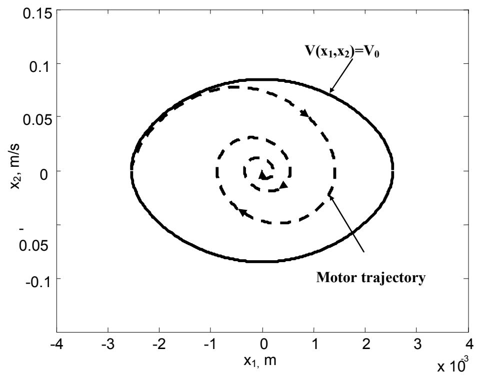
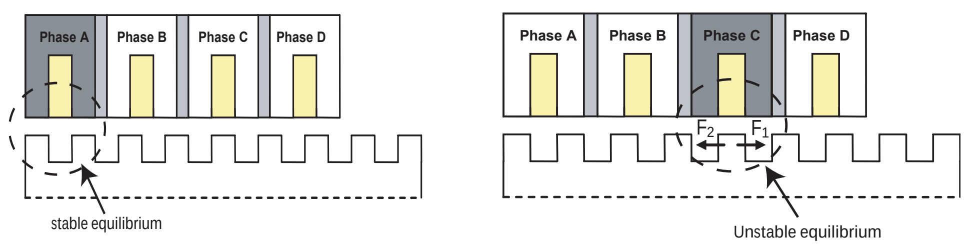


Figure 5. Simulated motor trajectory and stable boundary

To illustrate the notion of stable and unstable equilibrium position in practice, let us consider the stable equilibrium position obtained by the excitation of the phase A. In this case, there is a coincidence between the stator and the plunger teeth. In fact, after a disturbance, the plunger return to this stable equilibrium position where the resultant force in the plunger tooth is null, figure 6-a.

Let us suppose now that initially the plunger is located in the same position obtained by excitation of the phase A. In theory by excitation of the phase C the plunger does not move. In this case the plunger tooth is located between two stator teeth which generate two opposite vector forces $\vec{F_1}$ and $\vec{F_2}$ having in theoretically the same norm and applied to the plunger tooth. This second case corresponds to an initial point located outside the stable boundary corresponding to the phase C, figure 6-b. This position is unstable because for any disturbance the plunger moves away from this equilibrium position to an unknown position where the plunger and the stator tooth are in coincidence. For the normal operation of the studied motor and for the initial condition considered for this case the phase B or D have to be excited to perform one step evolution and to obtain a stable equilibrium.



(a) stable equilibrium position

(b) unstable equilibrium position

Figure 6. Stable and unstable equilibrium positions

The exploitation of the energy levels defined by the Lyapunov function is a method which was proposed by Senju and al., 1994 for the rotary stepping motors [12]. In this work such method is followed for the LTSRSM.

Let us consider the two energy levels relative to the phases (i) and (i+1) defined respectively by the functions $V_i(x_1,x_2)$ and $V_{i+1}(x_1,x_2)$:

$$\begin{cases} V_i(x_1,x_2) = V_e \\ V_{i+1}(x_1,x_2) = V_e \end{cases} \tag{29}$$

V_e is the energy level chosen so that the energy level of the phase (i) passes through the equilibrium position obtained by excitation of the phase (i+1) and the energy level of the phase (i+1) passes through the initial equilibrium position.

These two energy levels have the same shape because the two phases are supposed to be electrically identical. However, they are $\frac{\lambda}{4}$ shifted. This distance corresponds to the shift between two successive equilibrium positions, figure 8. By excitation of the phase (i+1), the plunger follows the corresponding energy level. On the switching point, defined by the intersection of the two energy levels, the phase (i+1) is switched off and the phase (i) is switched on. Thus, the plunger follows the energy level defined by the phase (i) until it reaches the equilibrium position. Finally, the phase (i+1) is switched on again and the phase (i) is switched off to maintain the plunger in this final equilibrium position.

t_1 Corresponds to the instant of time for which the plunger reaches the position $\frac{\lambda}{8}$. It can be determined experimentally or by simulation. The second time instant t_2 can be determined by numerical resolution of the equation $V_i(x_1, x_2) = V_e$ defining the energy level of the phase (i). In the case of the studied LTSRSM, the commutation instants are $t_1 = 0.043\text{s}$ and $t_2 = 0.080\text{s}$. These instants will be used for all the electrical cycle of the LTSRSM.

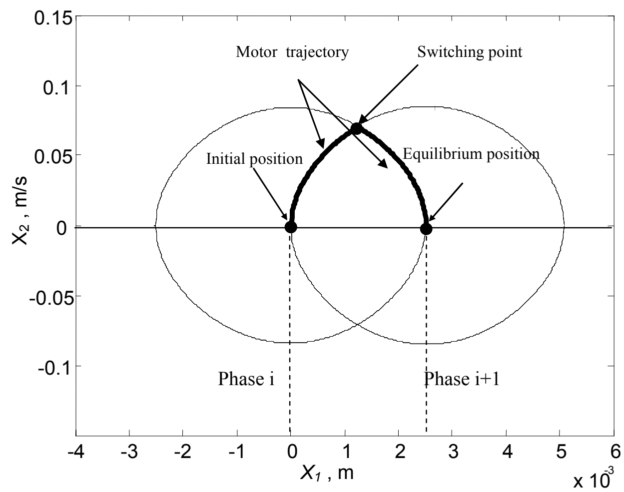


Figure 7. Definition of the switching point of the bang-bang control

5. Tests results

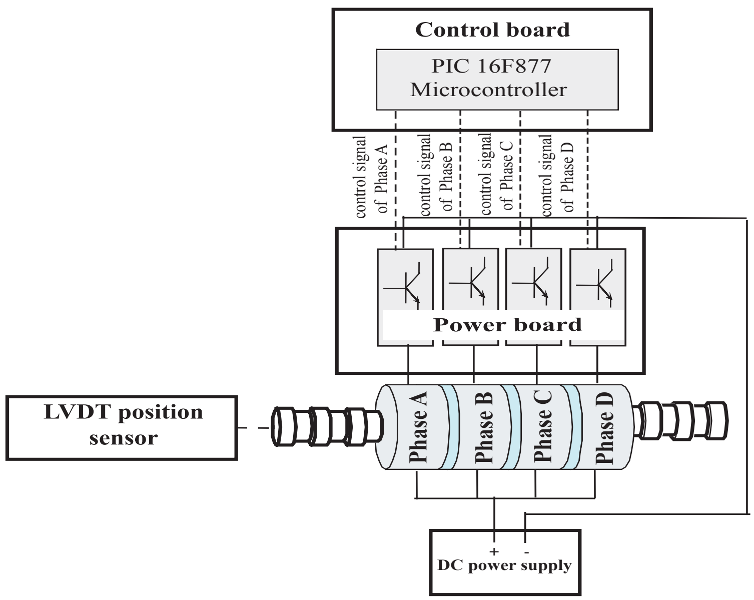
In order to validate the proposed approach some simulation and experimental tests are performed.

The test bench has been built with, figure 8:

- a control board with a 16F876 microcontroller,
- a power board with four legs converter connected to the four motor phases.
- the studied LTSRSM,
- a LVDT position sensor,
- a DC power supply.

The power board is composed of four identical converter legs corresponding to the four motor phases. One converter leg is composed of: an optocoupler allowing the electrical insulation between the control board and the power board, a free wheeling diode, and a switching aid device, figure 9.

(a) Test bench photo



(b) Test bench organization

Figure 8. Test bench description

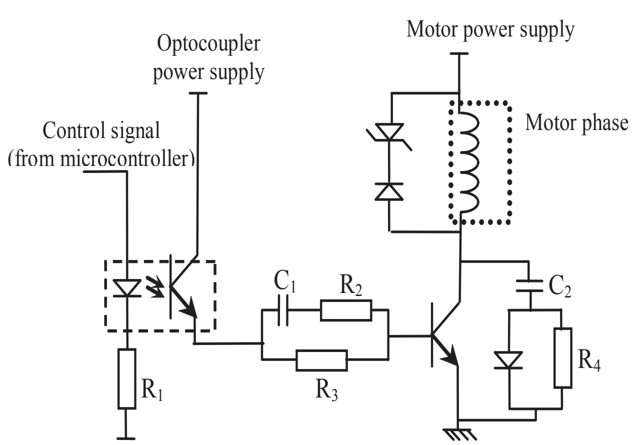


Figure 9. Structure of one converter leg

All the simulations are performed by the resolution of the LTSRSM mathematical model, presented in section 2, by the Runge- Kutta method.

Figure 9 shows in the phase plane the simulated trajectory of the motor. The obtained trajectory follows, with a small gap, the desired trajectory defined by the energy levels of the phases A and B. The small difference can be explained by the fact that only the mechanical part was considered in the proposed method and some frictions were neglected.

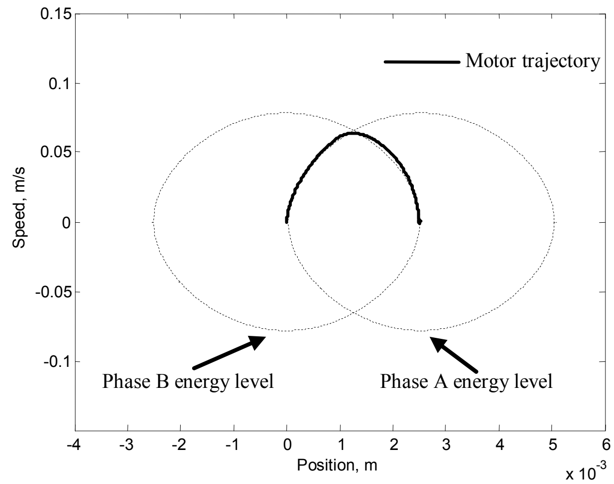
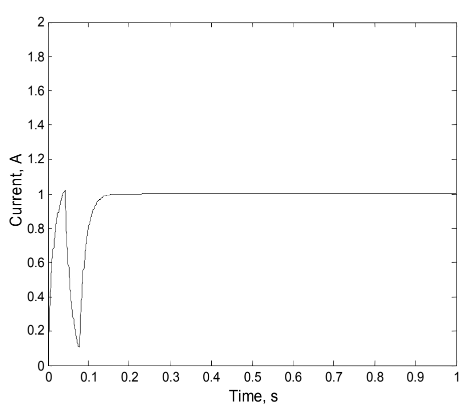


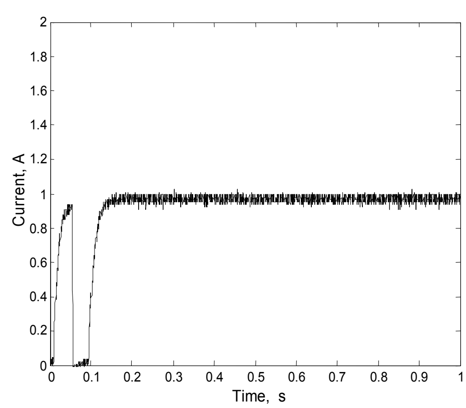
Figure 9. Simulated motor trajectory by application of the bang-bang control

The figure 10 shows the simulated and the experimental positions and currents responses where A is the braking phase and B the pulling phase. We can notice a similarity between the experimental and the simulated results. The obtained results prove the efficiency of the proposed method for the determination of the switching instants of the Bang-Bang control approach. In order to generalize the obtained result, the Bang-Bang control solution is applied to four steps position evolution. We can notice that the overshoots obtained for the normal four steps position evolution are reduced. The difference between the four damped position responses can be explained by the fact that in reality the four phases are not identical. In reality, a linear stepping motor presents some specificities in comparison with its rotary counterpart such as [7,9]:

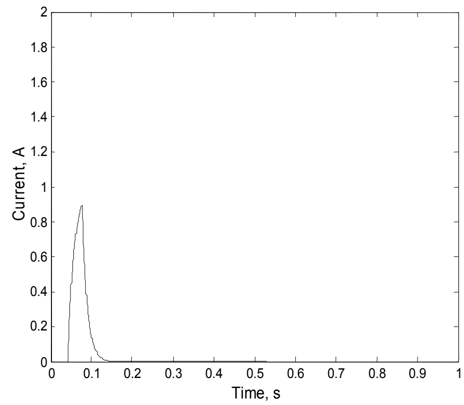
- the edge effects,
- the mechanical geometry.



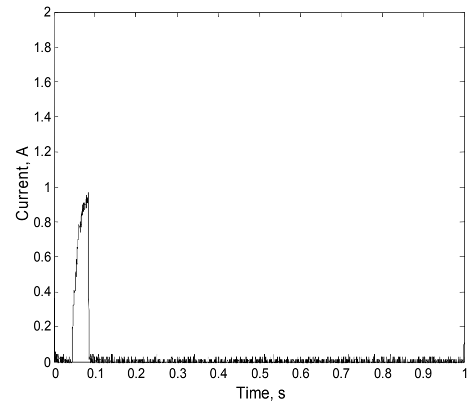
(a) Simulated phase B current



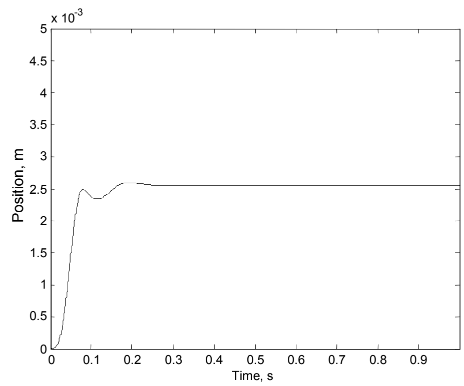
(b) Experimental phase B current



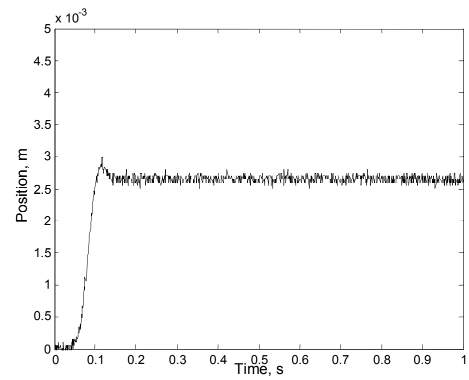
(c) Simulated phase A current



(d) Experimental phase A current



(e) Simulated position



(f) Experimental position

Figure 10. Simulated and experimental results obtained by application of the bang-bang control strategy

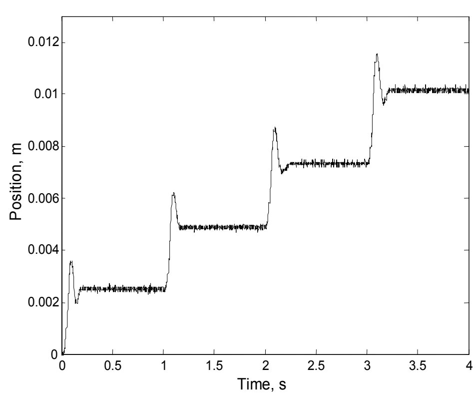


Figure 11. Four steps position evolution

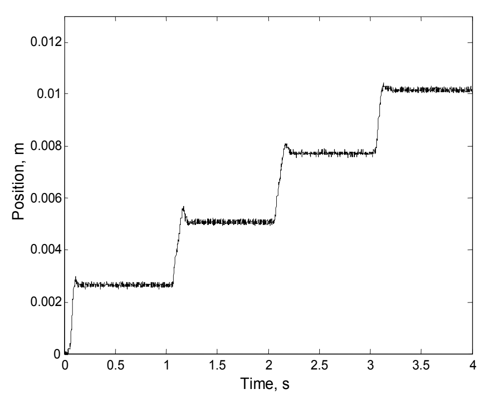


Figure 12. Four steps position evolution by application of the bang-bang control

6. Conclusion

The LTSRSM position evolutions present some oscillations which harm some application requiring high positioning quality and resolution. In this paper a method allowing the determination of the switching instants of the bang-bang control is presented. The bang-bang control is classical open-loop control solution allowing the attenuation of the stepping motors oscillations. It is difficult to determine analytically the two bang-bang switching instants because of the complexity of the nonlinear dynamical model of the studied LTSRSM. Also, the experimental determination of these two parameters is a tedious approach. In this work the Lyapunov's direct method is introduced to study stability of the LTSRSM. So, a Lyapunov candidate function is proposed to estimate the stability. The proposed method exploits the energy interpretation of Lyapunov's direct method to approximate the switching instants. The simulations and the experimental results prove the efficiency of the proposed approach. The experimental tests are performed on a test bench. Thanks to the good approximation of the switching instants, the experimental implementation of the bang-bang control is very easy.

Nomenclature

F_m : electromagnetic force
F_0 : dry friction force
F_C : resistant force
v : linear speed
x : plunger position
m : plunger weight
ξ : dynamic viscosity coefficient
R : statoric resistance
U_n : nominal voltage
U_i : voltage applied to phases i=A,B,C and D
i_j : **CURRENT OF THE PHASES j=A,B,C AND D**
I_n : nominal current
L_0 : average phase inductance
L_1 : amplitude of the inductance variation
Φ : fluxes density
λ : four step length

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