A Fuzzy Analysis Approach to Part Family Formation in Cellular Manufacturing Systems

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Abstract: Part family formation using fuzzy cluster analysis in Group Technology (GT) and feature recognition for tool access direction using relationship matrix in Computer Aided Process Planning (CAPP) have been presented in this paper. In the process of identifying part-families and machine groups, Fuzzy Cluster Analysis to form machine-part fuzzy relative matrix which converted into a zero-one conventional matrix. A new similarity coefficient, which involves all the entries of the machine-part fuzzy relative matrix resulting in a more realistic part family formation was suggested. The feature extraction system presented in this paper is designed to extract features from a STEP file in Boundary Representation (B-Rep) and Associated Adjacency matrix. The system is able to automatically extract alternate tool axis directions (TADs). There is a potential applications in set-up change cost optimisation and fixture selection.

Keywords: CAD/CAM, GT, CAPP, part family formation, feature recognition.

1. Introduction

Researches have been done on part family formation based on the usage of binary classification logic (zero-one data) (Chandrasekaran and Rajagopalan, 1986); (Chandrasekaran and Rajagopalan, 1987); and a modified form of grouping heuristic in GT (Islam and Sarker, 2000).

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method are suggested in this paper. The other researches have been done on feature recognition and process planning in CAPP in defining the tool access directions using a hierarchical structure (Gindy, 1989), generating the tool approach directions for grouping machines for set-ups (Ozturk, Kaya, Alanks, and Sevinc, 1996), determining the cutter sweep directions using a floor-based algorithm (Han and Requicha, 1998); (Han, Han, Lee and Juneho, 2001) introducing the concept of attributed adjacency graph (AAG) for the recognition of machining features from a B-rep of a solid model (Han, Kang, and Choi, 2001). A feature recognition system in Java language to automatically capture the candidate TADs is developed and presented in this paper.

2. Methods

2.1 Part family formation

In a cellular manufacturing system, the similarity between parts should be related to similarity between machines for the creation of cells. The similarity coefficients have been suggested to apply for the entries which are either 0 or 1. Hence the similarity coefficient needs to be changed suitably.

\[
S_{ij} = \sqrt{\sum_{k=1}^{N} (x_{ik} - x_{jk})^2}
\]  

(1)

Where, \( S_{ij} \) is Similarity coefficient for \( i^{th} \) machine and \( j^{th} \) part

\( x_{ij} \) is Fuzzy machine-part incident matrix value corresponding to \( i^{th} \) machine and \( j^{th} \) part

\( N \) is Number of parts

Using this, similar machines are grouped.

The corresponding similarity coefficient for parts groupings will then be

\[
S_{ij} = \sqrt{\sum_{k=1}^{M} (x_{ki} - x_{kj})^2}
\]  

(2)

Where, \( M \) is Number of machines

The above two equations are used in grouping the machines and parts.

The proposed methodology applies fuzzy relative matrix, new similarity coefficient, cell formation heuristic and new grouping efficiency formula.

(i) Consider \( M \) machines and \( N \) parts, find the \( M \) by \( N \) clustering matrix \( (x_{ij}) \) where the values of \( x_{ij} \) lie between 0 and 1 which is the degree of relationship between the machine \( i \) and the part \( j \).

(ii) Find the lower triangular machine similarity matrix using the equation (1).

(iii) Apply a heuristic method of grouping of machines by selecting the \((i,j)\) cells of the similarity triangular matrix \( \{S_{ij}\}_{i>j} \) having least values of \( S_{ij} \)'s. The number of least values selected depends upon the number of groupings of machines required.

(iv) Now obtain the optimal grouping of machines by Linear Programming technique using TORA software.

(v) Repeat the above steps to group parts using equation (2).

(vi) Rearrange the rows and columns of the original fuzzy machine-part matrix according to the machine groupings and parts groupings.

(vii) Finally the grouping efficiency \( GE \) is found as follows:

\[ GE = Pa + (1-P) b \]  

(3)
Where, GE is Grouping Efficiency

P is A real number between 0 and 1 used in grouping efficiency equation

A is Number of entries \( \geq \beta \) in the diagonal blocks/Number of entries in the diagonal blocks

B is Number of entries \( < \beta \) in the off-diagonal blocks/Number of entries in the off-diagonal blocks.

The grouping problem minimizes the sum of differences (i.e. new similarity coefficient) and can be expressed as the objective to

Minimize \( Z = \sum \sum S_{ij} X_{ij} \) for \( i>j \)

where \( X_{ij} = \begin{cases} 1 \text{ if } S_{ij}<\frac{a+b}{2}, & \text{where } a=\max \{ S_{ij}:i>j \} \text{ & } b=\min \{ S_{ij}:i>j \} \\ 0 \text{ if } S_{ij}\geq\frac{a+b}{2} \end{cases} \)

subject to the constraints \( \sum \sum X_{ij} = K \) and \( i\geq j \), \( K=\text{number of } S_{ij}<\frac{a+b}{2} \)

\( U + \sum \sum X_{ij} \geq 0 \) for \( i, j=1,2,\ldots M \) and \( i\geq j \) and \( X_{ij} = 0 \) or \( 1 \) for \( i, j=0,2,\ldots M \)

Where, \( \beta \) is \( \min \{ 1/U, \frac{M}{Mc/M} \} \)

\( U \) is A value greater than or equal to \( [M/Mc] \).

2.2 Feature recognition for tool access direction

In a CAPP system, feature extraction is the initial step of obtaining process plans. A feature recognition system is developed to capture alternate interpretations of TAD in each feature. The tool axis direction is vital for reducing the number of set-ups thus reducing set-up change cost.

Consider a part model shown in fig.2.2.1(a) containing a corner pocket (notch). This feature can be machined in \(-x, -y\) and \(-z\) direction as shown in fig2.2.1(b). Hence, the alternate interpretations have to be evaluated to form candidate TADs.

Whenever a part needs machining on more than one face, an additional cost is incurred while changing the set-ups. For optimising the number of set-ups or minimising the set-up change cost, the knowledge of TAD is essential. This paper deals with the problem of determination of TADs in the manufacturing features like slots, holes and step features. Fig.2.2.2 shows the possible candidates for TADs for some sample features.

In this methodology, the first step is the creation of the part model using Pro/Engineer and then converted into a STEP file. The input to the system is the B-rep data, which is obtained by processing the entities contained in the STEP file. The B-rep data includes all information of the prismatic part. Using this data, the system then determines the orientation of each face. The relationships between adjacent faces of the part are found, and these relationships are stored in a relationship matrix. The features are extracted first and compared with feature codes in the
database to identify the type of feature. In extracting the features, boundary faces of the feature are determined and the root faces associated with each feature on the object are identified and labelled. The major and minor dimensions of the feature are determined. The orthogonal view direction of the boundary faces is then evaluated and is stored in feature database.

Figure 2.2.2. Tool access directions for some predefined features

3. Results

3.1 Part family formation

The fuzzy part-machine incident matrix with 10 machines and 11 parts used in the previous work [8] is taken for the numerical illustration and comparison of efficiency.

\[
\begin{bmatrix}
0.26 & 0.02 & 0.20 & 0.00 & 0.01 & 0.04 & 0.02 & 0.02 & 0.02 & 0.30 & 0.03 \\
0.01 & 0.03 & 0.03 & 0.01 & 0.21 & 0.01 & 0.03 & 0.23 & 0.01 & 0.02 & 0.29 \\
0.01 & 0.30 & 0.01 & 0.25 & 0.03 & 0.21 & 0.31 & 0.03 & 0.05 & 0.06 & 0.01 \\
0.0101 & 0.02 & 0.02 & 0.21 & 0.02 & 0.02 & 0.27 & 0.03 & 0.04 & 0.26 \\
0.21 & 0.01 & 0.20 & 0.01 & 0.04 & 0.01 & 0.01 & 0.23 & 0.23 & 0.23 & 0.04 \\
0.23 & 0.02 & 0.20 & 0.03 & 0.02 & 0.03 & 0.04 & 0.01 & 0.24 & 0.02 & 0.02 \\
0.02 & 0.29 & 0.01 & 0.30 & 0.06 & 0.32 & 0.43 & 0.05 & 0.06 & 0.03 & 0.01 \\
0.20 & 0.03 & 0.21 & 0.04 & 0.02 & 0.03 & 0.04 & 0.03 & 0.26 & 0.28 & 0.03 \\
0.01 & 0.28 & 0.00 & 0.01 & 0.37 & 0.03 & 0.05 & 0.25 & 0.09 & 0.02 & 0.25 \\
0.01 & 0.01 & 0.02 & 0.02 & 0.03 & 0.30 & 0.05 & 0.07 & 0.01 & 0.00 & 0.06 \\
\end{bmatrix}
\]

Applying the equation (1), the similarity triangular matrix for machines is obtained which is shown below.

\[
\begin{bmatrix}
0.000 \\
0.567 & 0.000 \\
0.637 & 0.635 & 0.000 \\
0.556 & 0.607 & 0.640 & 0.000 \\
0.232 & 0.538 & 0.637 & 0.520 & 0.000 \\
0.359 & 0.537 & 0.592 & 0.528 & 0.219 & 0.000 \\
0.762 & 0.737 & 0.170 & 0.741 & 0.759 & 0.705 & 0.000 \\
0.253 & 0.586 & 0.622 & 0.568 & 0.082 & 0.264 & 0.740 & 0.000 \\
0.696 & 0.313 & 0.618 & 0.323 & 0.644 & 0.641 & 0.712 & 0.677 & 0.000 \\
0.507 & 0.443 & 0.496 & 0.384 & 0.510 & 0.461 & 0.555 & 0.538 & 0.580 & 0.000 \\
\end{bmatrix}
\]

The machine groupings are done by a heuristic method by which it is natural to choose the least values in the similarity triangular matrix2. The corresponding cells and machine groupings are
marked in table 3.1.1.

<table>
<thead>
<tr>
<th>Minimum values</th>
<th>Cells</th>
<th>Machine groupings</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.067</td>
<td>(4,2)-I</td>
<td>Group1 = {4,2,9,10}</td>
</tr>
<tr>
<td>0.082</td>
<td>(8,5)-II</td>
<td>Group2 = {8,5,6,1}</td>
</tr>
<tr>
<td>0.17</td>
<td>(7,3)-III</td>
<td>Group3 = {7,3}</td>
</tr>
<tr>
<td>0.219</td>
<td>(6,5)-II</td>
<td></td>
</tr>
<tr>
<td>0.232</td>
<td>(5,1)-II</td>
<td></td>
</tr>
<tr>
<td>0.253</td>
<td>(8,1)-II</td>
<td></td>
</tr>
<tr>
<td>0.264</td>
<td>(8,6)-II</td>
<td></td>
</tr>
<tr>
<td>0.313</td>
<td>(9,2)-I</td>
<td></td>
</tr>
<tr>
<td>0.323</td>
<td>(9,4)-I</td>
<td></td>
</tr>
<tr>
<td>0.359</td>
<td>(6,1)-II</td>
<td></td>
</tr>
<tr>
<td>0.384</td>
<td>(10,4)-I</td>
<td></td>
</tr>
</tbody>
</table>

By applying the machines lower triangular similarity matrix into the linear programming model, the following results are obtained.

\[ X_{4,2} = X_{5,1} = X_{6,1} = X_{8,1} = X_{8,5} = X_{9,6} = X_{9,2} = X_{9,4} = X_{10,7} = 1 \]

The above result indicate that the optimal groupings of rows are \{4,2,9\}, \{5,1,6,8\} and \{10,3,7\}.

Similarly, the above steps are carried out for the parts grouping and the final grouping are \{4,2,6,7\}, \{8,5,11\} and \{9,1,3,10\}.

<table>
<thead>
<tr>
<th></th>
<th>3</th>
<th>1</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>8</th>
<th>5</th>
<th>7</th>
<th>4</th>
<th>2</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.21</td>
<td>0.20</td>
<td>0.26</td>
<td>0.28</td>
<td>0.03</td>
<td>0.03</td>
<td>0.02</td>
<td>0.04</td>
<td>0.04</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>5</td>
<td>0.20</td>
<td>0.21</td>
<td>0.23</td>
<td>0.23</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
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<tr>
<td>6</td>
<td>0.20</td>
<td>0.23</td>
<td>0.24</td>
<td>0.02</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
<td>0.04</td>
<td>0.03</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>1</td>
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<td>0.02</td>
<td>0.30</td>
<td>0.03</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
<td>0.04</td>
</tr>
<tr>
<td>4</td>
<td>0.02</td>
<td>0.03</td>
<td>0.03</td>
<td>0.04</td>
<td>0.26</td>
<td>0.27</td>
<td>0.21</td>
<td>0.02</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>2</td>
<td>0.03</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
<td>0.29</td>
<td>0.23</td>
<td>0.20</td>
<td>0.03</td>
<td>0.01</td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td>9</td>
<td>0.00</td>
<td>0.01</td>
<td>0.09</td>
<td>0.02</td>
<td>0.25</td>
<td>0.25</td>
<td>0.37</td>
<td>0.05</td>
<td>0.01</td>
<td>0.28</td>
<td>0.03</td>
</tr>
<tr>
<td>10</td>
<td>0.02</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00</td>
<td>0.06</td>
<td>0.07</td>
<td>0.03</td>
<td>0.05</td>
<td>0.01</td>
<td>0.28</td>
<td>0.03</td>
</tr>
<tr>
<td>7</td>
<td>0.01</td>
<td>0.02</td>
<td>0.06</td>
<td>0.03</td>
<td>0.01</td>
<td>0.05</td>
<td>0.06</td>
<td>0.43</td>
<td>0.30</td>
<td>0.29</td>
<td>0.32</td>
</tr>
<tr>
<td>3</td>
<td>0.01</td>
<td>0.02</td>
<td>0.05</td>
<td>0.06</td>
<td>0.01</td>
<td>0.03</td>
<td>0.03</td>
<td>0.31</td>
<td>0.25</td>
<td>0.30</td>
<td>0.21</td>
</tr>
</tbody>
</table>

From the above shown rearranged matrix, \( a = 30/36 = 0.833 \) & \( b = 72/74 = 0.972 \), the grouping efficiency is found to be 93.03% using the equation (3) by substituting the values \( M = 10 \), \( M_c = 3 \) and \( P = 0.3 \). Grouping Efficiency = 0.3*0.833 + (1-0.3)*0.972 = 0.9303 = 93.03%. The grouping efficiency value is found to be higher than that of other methods.

### 3.2 Feature recognition for tool access direction

A example part is created in Pro/Engineer, as shown in Fig.3.2.1. This part model is converted into a STEP file and given as the input to the feature recognition module. The feature recognition parses the STEP file for the entities. The faces and edges are identified and they are numbered. The faces and their surface normals are also extracted from STEP file as shown in Fig 3.2.2. The B-rep database is used to obtain the adjacency relationship between the faces. The left bottom vertices of the faces are always used as the origin for clock-wise loops. The type and orientations of each face using the B-rep database is modified accordingly. Geometric attributes of faces on the object and relational topology, which is basically concerned with the adjacency relationship between faces, are then determined. The attributes 1, -1 and 2 represent the convex, concave and non-adjacent surfaces respectively.
These attributes and relationships between all faces of the object are converted into a relationship matrix. The relationship matrix is derived from the part and this relationship matrix contains all the information for recognizing the feature. The feature extraction cycle is performed by scanning the matrix starting from the first row as shown in the Table 3.2.1.

**Table 3.2.1. Relationship matrix**

<table>
<thead>
<tr>
<th>$F_i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
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<td>0</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

---

*Figure 3.2.1. Example part*  
*Figure 3.2.2. Example part with faces identified*
The root and boundary face is evaluated to extract features. In Fig. 2.2.2, the boundary faces for the step feature are denoted by $F_1$ to $F_4$. If a negative entry is found (-1) in an off-diagonal cell $(i, j)$ of the relationship matrix, the faces $F_i$ and $F_j$ are defined as the root faces of the current feature as shown in shaded cells of Table 1. The column for the face having an off-diagonal negative (-1) entry in the $i^{th}$ row is also flagged as root faces. The column for the faces having a positive (1) entry in the $i^{th}$ row of root faces forms the boundary faces. The cycle is finished when no new root face is found. All the root and boundary faces of the features are successfully determined at the end of the feature extraction cycle. In the extraction cycle, the corresponding edge attributes are also saved for the future use in the feature identification stage. The number of boundary, root and adjacent faces are used in the extraction of features. The number of boundary faces gives the total number of tool approach directions for a given feature. The view directions for each boundary faces are established. Each view direction defines a TAD for the feature. The results are tabulated in Table 3.2.2.

<table>
<thead>
<tr>
<th>Feature</th>
<th>TADs</th>
<th>Number of boundary faces</th>
</tr>
</thead>
<tbody>
<tr>
<td>Through Step</td>
<td>$-x, +y$</td>
<td>4</td>
</tr>
<tr>
<td>Through Slot</td>
<td>$+x, -y$</td>
<td>3</td>
</tr>
<tr>
<td>Through Hole</td>
<td>$-z$</td>
<td>2</td>
</tr>
<tr>
<td>Blind Pocket</td>
<td>$z$</td>
<td>1</td>
</tr>
</tbody>
</table>

For each TAD, the dimensions are extracted separately and stored in a database. The TADs determined should be validated with respect to various constraints like tool accessibility, locating surfaces and datum requirements. The features having same TAD are grouped in a single set-up without violating the precedence constraints to minimise the set-up change cost.
4. Conclusions

4.1 Part family formation

A realistic approach to part family formation in cellular manufacturing systems using a new fuzzy approach in which the fuzzy relative values are used throughout the methodology instead of using them only in the first step is provided, which makes it possible to get a more realistic solution compared to earlier methods. A new formula for similarity coefficient and new grouping heuristic are used. Numerical example shows improvement of the grouping efficiency obtained for the proposed methodology is achieved.

4.2 Feature recognition for tool access direction

A feature recognition system that is developed to automatically capture the candidate TADs of a feature for a prismatic part. The system is developed in Java language to make it portable. Moreover, the system uses standard technologies like STEP, which is suitable for concurrent processes to enable interoperability. This work can be extended to integrate the feature recognition process with cost optimisation so that manufacturability is guaranteed during the extraction process itself.

REFERENCES


