Identification and Supervised Equalization of a MIMO Non-linear Communication Channel

Nabiha Saidi , Anouar Ben Amor, Hassani Messaoud

Unité de recherche ATSI

Ecole Nationale d'Ingénieurs de Monastir

Rue Ibn ELJazzar, 5019 Monastir – Tunisia

saidinabiha @ yahoo.fr , anouar.benamor@yahoo.fr, hassani.messaoud@enim.rnu.tn

Abstract: Due to their general non linear structural and their linearity with respect to their parameters, Volterra models are widely used to describe the behaviour of non linear process. In this paper we are concerned by the modelling and the supervised equalization of a Multi Input Multi Output non linear communication channel using Volterra models. To overcome the burden induced by the parameter number increasing, we develop the Volterra kernel on the General Orthogonal Basis GOB to provide a reduced complexity model known as GOB-Volterra model.

Keywords: MIMO Volterra model, identification, modelling, supervised equalization, non linear communication channel.

Saidi Nabiha was born in 1981. She received her Electrical Engineering diploma in 2005 from the School of Engineers in Monastir, Tunisia. She is currently preparing her doctorate in the School of Engineers in Monastir about Modelling, identification and equalization of numerical communication channel.

Ben Amor Anouar was born in 1981. He received his Electrical Engineering diploma in 2005 from the School of Engineers in Gabes, Tunisia. He is currently preparing his doctorate in the School of Engineers in Monastir about Modelling, identification of non linear systems.

Hassani Messaoud is a Professor at the School of Engineers in Monastir, Tunisia. He was graduated from the University of Tunis. He is majoring in process identification and control. Actually his main interest is numerical communication channel equalization using reduced Volterra models.

1. Introduction

Till while ago Volterra models still the most usual and popular way to describe non linear system behaviour as it provides a model linear with respect to its parameters [3]. Truncated Volterra filters constitute a class of non recursive polynomial models without output feedback which guarantees their stability. Such models can approximate any time invariant nonlinear system with fading memory [3], [28]. These models have been successfully applied to a wide variety of engineering problems such as modelling a nonlinear communication channels, biological systems and acoustic noise cancellation [15]. In communication systems, Volterra models have been used for modelling communication channels exhibiting nonlinear behaviours [10] and [14] that is the case of those including amplifiers and optical fiber. Indeed, high power amplifiers, currently used in mobile radio and satellite communication channels, have to operate near their nonlinear region for maximizing the utilization of the available power. However, the main drawback of Volterra models is their complexity due to the high parameter number. To eliminate this disadvantage, two ways can be followed to consider simplified models like Hammerstein or Wiener models or to reduce the number of parameters associated with a Volterra series.

During the last decade, the issue of Volterra model complexity reduction has been addressed in two main different ways. The first is based on the use of Singular Value Decomposition (SVD) of the second kernel and PARAFAC tensor decompositions [7] and [17] of the higher kernels as these latter can be described by tensors. The resulting reduced model known as SVD PARAFAC Volterra model [19], [20], [21]. The second is based on expanding the Volterra kernels on orthonormal basis (OB) such as the Laguerre functions basis [4], [5] and [6] or the Generalized Orthonormal Basis (GOB) [16], [22] and [23]. The complexity reduction depends on the choice of the basis parameter structure such as poles and truncating order. Recently, many approaches are given to identify Multiple-Input- Multiple-Output (MIMO) Volterra models to describe wireless communication channel [11][12][13]. The provided model can be used to design equalizers to restitute the transmitted signals. Channel identification and equalization consist in the retrieval of unknown information about the transmission channel and source signals, respectively. In order to reach a desired quality of service, broadband wireless communication systems classically perform channel identification and/or equalization using pilot symbols, i.e. training sequences composed of a priori known signals.

In this paper we are interested to the identification and supervised equalization of a MIMO non linear communication channel described by a reduced MIMO Volterra model. This reduction is ensured by developing Volterra kernels on Generalized Orthogonal Bases (GOB). The resulting model is used to synthesise the supervised equalizer.

2. Volterra Model

Volterra models have several important properties that make them very useful for the modelling and analysis of non linear systems [3]. These models which are linear with respect to their parameters, the kernel coefficients, suffer from the huge increasing of the parameter number depending on non linearity hardness.

2.1 SISO Volterra model

The model output is written as:

$$y(k)=\sum_{i=1}^{\infty}\left\{\sum_{m_1=0}^{\infty}\cdots\sum_{m_i=0}^{\infty} h_i(m_1,\cdots,m_i)\prod_{n=1}^{i}u(k-m_n)\right\} \quad (1)$$

Where u and y are the input and the output of the process respectively and $h_i(m_1,\cdots,m_i)$ is the i[th] Volterra kernel. For causal and stable system, the infinite sums in (1) can be truncated to a finite one as:

$$y(k)=h_0+\sum_{i=1}^{P}\sum_{m_1=0}^{M-1}\sum_{m_2=0}^{M-1}\ldots\sum_{m_i=0}^{M-1}h_i(m_1,\ldots,m_i)\,u(k-m_1)\ldots u(k-m_i) \quad (2)$$

Where P is the process non linearity degree, M is the memory and h_0 is the statistical characteristic.

The Volterra model can be seen as a natural extension of the linear system impulse response to non linear systems. Although it is linear with respect to its parameter such model suffers from the increasing of its parameter number and any attempt for its using in real time application may fail if a reduction operation doesn't precede such attempt. The parameter number of the Volterra model given by (2) is:

$$n_p=1+\sum_{i=1}^{P}M^{i} \quad (3)$$

To reduce this number we use generally the triangular form of the Volterra model, given as:

$$y(k)=h_0+\sum_{i=1}^{P}\sum_{m_1=0}^{M-1}\sum_{m_2=m_1}^{M-1}\ldots\sum_{m_i=m_{i-1}}^{M-1}h_i(m_1,m_2,\ldots,m_i)\times\prod_{j=1}^{i}u(k-m_j) \quad (4)$$

And the relevant parameter number of such model is:

$$n_{tri}=1+\sum_{i=1}^{P}\frac{(M-1+i)!}{(M-1)!\,i!} \quad (5)$$

2.2 MISO Volterra model [29], [30]

For multiple inputs; the output of the Volterra model in its triangular form is:

$$y(k) = h_0 + \sum_{i=1}^{P}\sum_{j_1=1}^{n}\cdots\sum_{j_i=1}^{n}\sum_{m_1=0}^{M-1}\cdots\sum_{m_i=m_{i-1}}^{M-1} h_{j_1,j_2,\ldots,j_i}(m_1,\ldots,m_i) \times \prod_{e=1}^{i} u_{j_e}(k-m_e) \quad (6)$$

Where $u(k) = [u_1(k)\ u_2(k)\ \cdots\ u_n(k)]^T$ and $y(k)$ are the process input vector and output respectively and $h_{j_1,j_2,\ldots,j_i}(m_1,\ldots,m_i)$ is the Volterra kernel. P is the non linearity degree and M is the memory. The corresponding parameter number is:

$$n_{MISO} = 1 + \sum_{i=1}^{P} n^i \frac{(M-1+i)!}{(M-1)!\,i!} \quad (7)$$

2.3 MIMO Volterra model

The MIMO system can be considered as a collection of Multi Input Single Output (MISO) sub systems. Thus the modelling of the MIMO System is equivalent to the modelling of its sub systems. Let a MIMO system with n inputs and S outputs, each subsystem output $y_s(k)$ can be developed on Volterra series as:

$$y_s(k) = h_0^s + \sum_{i=1}^{P}\sum_{j_1=1}^{n}\cdots\sum_{j_i=1}^{n}\sum_{m_1=0}^{M-1}\cdots\sum_{m_i=m_{i-1}}^{M-1} h^s_{j_1,j_2,\ldots,j_i}(m_1,\ldots,m_i) \times \prod_{e=1}^{i} u_{j_e}(k-m_e),\ s=1,\ldots,S \quad (8)$$

$h^s_{j_1,j_2,\ldots,j_i}(m_1,\ldots,m_i)$ is the Volterra Kernel of i[th] order corresponding to the sub system the output of which is $y_s(k)$ and h_0^s : is the statistical characteristic corresponding to $y_s(k)$.

The parameter number is:

$$n_{MIMO} = \left(1 + \sum_{i=1}^{P} n^i \frac{(M-1+i)!}{(M-1)!\,i!}\right) * S \quad (9)$$

Reduced Volterra model

3.1 Generalised Orthogonal Basis (GOB)

When developing a linear transfer function on an orthogonal basis it can be written as a linear combination of the basis functions and the coefficient of such combination [24], known as the Fourrier coefficients, are the yielded model parameters.

$$G(z) = \sum_{n=0}^{\infty} g_n\, B_n(z) \quad (10)$$

Where $B_n(z)$ are the z-transform of the basis functions and g_n are the Fourrier coefficients.

$$B_n(z) = \frac{\sqrt{1-|\xi_n|^2}}{z-\xi_n} \prod_{k=0}^{n-1}\left(\frac{1-\xi_k^* z}{z-\xi_k}\right) \quad (11)$$

ξ_k is the pole of order k, $|\xi_k| < 1$

If the considered system is stable, the infinite sum in (10) can be truncated to a finite value called truncating order.

The GOB model can be described by a state space representation

$$\begin{cases} X(k+1) = A X(k) + B u(k) \\ y(k) = C X(k) \end{cases} \quad (12)$$

With

$$X(k) = \begin{bmatrix} x_0(k) \\ x_1(k) \\ x_2(k) \\ \vdots \\ x_n(k) \end{bmatrix};\ B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n+1} \end{bmatrix};\ C^T = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ \vdots \\ g_n \end{bmatrix};$$

$$A = \begin{bmatrix} a_{1,1} & 0 & 0 & \cdots & 0 \\ a_{2,1} & a_{2,2} & 0 & \cdots & 0 \\ a_{3,1} & a_{3,2} & a_{3,3} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n+1,1} & a_{n+1,2} & a_{n+1,3} & \cdots & a_{n+1,n+1} \end{bmatrix}$$

$a_{ij} = \xi_{i-1}$, for $i = j$

$a_{ij} = (1-\xi^*_{j-1}\,\xi_{j-1})\,A_j$, for $i - j = 1$

$$a_{ij} = (-1)^{i-j+1}\left(1 - \xi^*_{j-1}\,\xi_{j-1}\right) \prod_{k=j}^{i-1} A_k \prod_{k=j+1}^{i-1} \xi^*_{k-1}$$

for $i - j > 1$

$$b_1 = \sqrt{1-|\xi_0|^2}$$

$$b_i = (-1)^{i+1}\sqrt{1-|\xi_0|^2}\left(\prod_{k=1}^{i-1} A_k\ \xi_{k-1}^{*}\right),$$

for $i > 1$

3.2 GOB MIMO Volterra model

In the following we will be only interested to the decomposition of MIMO system on GOB [1]. As mentioned in the introduction, because of the linearity of Volterra model with respect to its parameters, we proceed in this paragraph to the decomposition of Volterra kernels of the Volterra model on the Generalised Orthogonal Base GOB. This decomposition will be done on the same GOB basis. The output of the model $y_s(k)$ is then written as:

$$y_s(k)=h_0^s+\sum_{i=1}^{P}\sum_{j_1=1}^{n}\cdots\sum_{j_i=1}^{n}\sum_{k_{j_1}=0}^{\infty}\sum_{k_{j_2}=0}^{\infty}\cdots\sum_{k_{j_i}=0}^{\infty} gs_{k_{j_1},k_{j_2},\ldots,k_{j_i}}^{j_1,j_2,\cdots,j_i} \times \prod_{e=1}^{i} xs_{k_{j_e}}^{j_e}(k) \quad (13)$$

With $xs_{k_l}^{l}(k)$ is the filtered input given by:

$$xs_{k_l}^{l}(k)=\sum_{m=0}^{M-1} bs_{k_l}(i)\ u_l(k-m) \quad l=1\ldots n \quad (14)$$

And $gs_{k_{j_1},k_{j_2},\ldots,k_{j_i}}^{j_1,j_2,\cdots,j_i}$ are the coefficients of the kernels on the GOB base.

Taking into account the truncating order K of the GOB base, relation (13) can be written in triangular form as:

$$y_s(k)=h_0^s+\sum_{i=1}^{P}\sum_{j_1=1}^{n}\cdots\sum_{j_i=1}^{n}\sum_{k_{j_1}=0}^{K-1}\sum_{k_{j_2}=k_{j_1}}^{K-1}\cdots\sum_{k_{j_i}=k_{j_{i-1}}}^{K-1} gs_{k_{j_1},k_{j_2},\ldots,k_{j_i}}^{j_1,j_2,\cdots,j_i} \times \prod_{e=1}^{i} x_{k_{j_e}}^{j_e}(k) \quad (15)$$

The total number of parameters is:

$$\left[1+\sum_{i=1}^{P} n^i \frac{(K-1+i)!}{(K-1)!\,i!}\right] S \quad (16)$$

In Table 1, for a double output double input system we vary the truncating order K of a reduced quadratic Volterra model with memory M = 5 and we note the parameter number.

Table 1. Parameter number of the reduced and classical Volterra models

Reduced Volterra Model		Volterra Model
Truncating order	Parameter number	Parameter number
K = 1	14	142
K = 2	34	
K = 3	62	
K = 4	98	

It resorts that the decomposition of Volterra kernels on GOB reduces efficiently the model parameter number if the truncating order is low.

Pole optimisation:

The complexity reduction is then very depending on the choice of poles that characterize the GOB basis. Poles can be optimized by means of a gradient-type algorithm [16] and [24] or to use the experimental method which selects poles that minimise the Normalized Mean Square Error (NMSE) between the real output $y_s(k)$ and the output of the model $\hat{y}_s(k)$. In this paper we adopt the latter to optimise the GOB poles for a MIMO Volterra model.

$$\xi^{s}_{i,opt} = Arg\min_{p_j} NMSE(s)\left(p_j\right);\ j=1,\ldots,P \quad (17)$$

Where $NMSE(s)$ is the Normalised Mean Square Error relative to the subsystem s.

$$NMSE(s)\left(p_j\right)=\frac{\sum_{k=0}^{N_m}[y_s(k)-\hat{y}_s(k)]^2}{\sum_{k=0}^{N_m}[y_s(k)]^2} \quad (18)$$

N_m is the observation number

4. Supervised Equalization of a Non Linear Communication Channel

The received signal depends on the number of the channel inputs and outputs i.e. sources and antennas. In this section the case multiple inputs multiple outputs non linear communication channels will be addressed so that the users number corresponds to the channel input number and the antenna number is related to the output number.

4.1 Transmission channel (SISO case)

The numerical transmission systems carry the information from the source to the destination using a transmission channel composed [18], as shown in **Figure 1**, of a modulator, a physical support which serves as a transmission medium and a demodulator.

noise or attenuates some frequencies because of its frequency selectivity. To avoid these imperfections and generate the source signal, many techniques have been proposed in literature such as supervised equalisation.

4.2. Supervised equalization of numerical communication channels

The equalization is a processing operation carried out in the receiver of a communication system to reduce the inter symbol interferences. The supervised equalization is based on the emission of a set

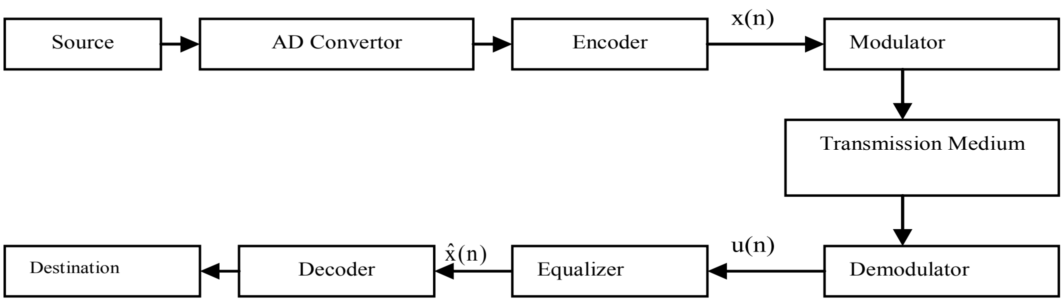

Figure 1. Numerical SISO Communication system

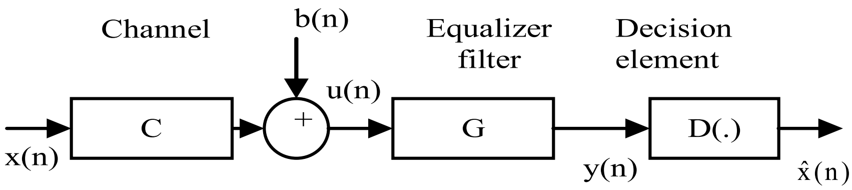

Figure 2. Outline of a communication system

The source generates the analogue signal to be transmitted which is converted by the AD converter to yield a numerical word. This word is converted, using an encoder, in a coded word, then to an amplitude modulated signal the frequency of which depends on the transmission medium. The channel output is that signal after being demodulated. The equaliser, composed of the decision element and a filter, has to resituate the coded symbols sent to the channel. The resulting signal is then decoded before being sent to its destination. However from the source to the destination the signal may have many perturbations which influence its quality. In fact, the channel often introduces an additive

known by the destination and called learning sequence to identify the channel and to initialise the equalizer [18], [2]. **Figure 2** gives an outline of a communication system.

u(k) represents the discrete sequence of data transmitted time instant k ; b(k) are the additive noise samples to the channel output, $y_C(k)$ the disturbed samples at the channel output, y(k) the discrete sequence at the equaliser filter output and $\hat{u}(k)$ the estimated sequence at the decision element output. The equaliser filter and the decision element form together the reconstruction bloc which accommodates the signal $y_C(k)$ by minimising the distortions effects on x(k) so that the provided sequence û(k) be as close as possible to the channel input u(k).

task can be outlined by **Figure 3.**

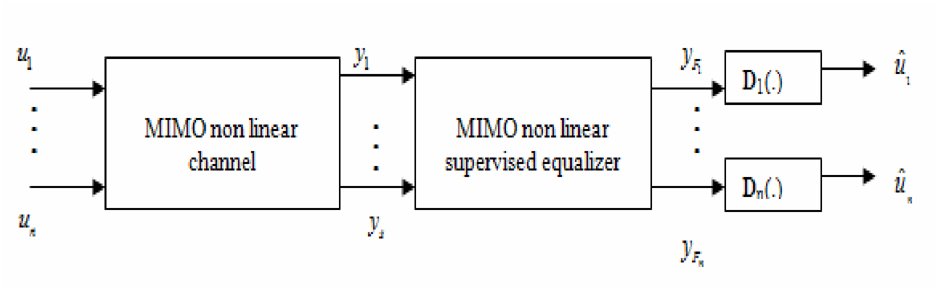

Figure 3. MIMO channel equalization

The decision element is generally a non linear function which transforms the filter equalizer output into symbols to yield û(k), example the sign function

$$D(y) = sign(y) = \begin{cases} +1 & \text{if } y \geq 0 \\ -1 & \text{if } y < 0 \end{cases} \quad (19)$$

The supervised equalization of a linear communication channel using Laguerre function has been proposed in [18] and [2] this work has been tackled using of Generalized Orthogonal Basis in [27].

4.3 MIMO channel

Consider a MIMO non linear channel characterised by n sources (users) and S the received antenna. This channel can be modelled by a MIMO Volterra given by:

$$y_s(k) = h_0^s + \sum_{i=1}^{P} \sum_{j_1=1}^{n} \cdots \sum_{j_i=1}^{n} \sum_{m_1=0}^{M-1} \cdots \sum_{m_i=m_{i-1}}^{M-1} h^s_{j_1, j_2, \cdots, j_i}(m_1, \cdots, m_i) \times \prod_{e=1}^{i} u_{j_e}(k - m_e) + v_s(k)$$

(20)

Where y_s (s = 1, …, S) is the signal received by the s[th] antenna at time instant k, P is the non linearity order of the channel and M is the channel memory.

$h^s_{j_1, j_2, \cdots, j_i}(m_1, \cdots, m_i)$ are the kernel coefficients of the s[th] subchannel and $v_s(k)$ is the additive white Gaussian noise to the s[th] antenna, it is assumed that the noise components are zero mean.

The MIMO non linear channel equalization

As the considered communication system is non linear its description will be more accurate once it uses a non linear model. Therefore we adopt the MIMO GOB-Volterra models to synthesise a supervised equalizer of a non linear multiple input multiple output communication channel. In fact, in a transmission system, the receiver antennas which collect the information signal collect perturbations as well which may cause the Inter Symbol Interferences: ISI that is responsible of transmission errors. To improve the performances of transmission systems, the receiver has to identify the distortions introduced by the channel.

The equalizer is composed of a MIMO filter G described by a reduced (GOB) quadratic Volterra model. Using its input output observations, the filter coefficient identification is ensured by the RLS (Recursive Least Square) algorithm and the basis pole optimisation is achieved by the minimisation of the Normalised Mean Square Error *NMSE*. The decision error for the i[th] input of the channel (i[th] output of the equalizer) is defined as:

$$e_i(k) = \begin{cases} 1 & if \quad u_i(k) \neq \hat{u}_i(k) \\ 0 & if \quad u_i(k) = \hat{u}_i(k) \end{cases} ; i=1,\ldots,n \quad (21)$$

The performance criteria used to evaluate the equalization quality is the *NMSE* between the filter output $y_{Fi}(k)$ and the information signal $u_i(k)$.

$$NMSE(i) = \frac{\sum_{k=1}^{N_m} (u_i(k) - y_{Fi}(k))^2}{\sum_{k=1}^{N_m} (u_i(k))^2} \quad (22)$$

This *NSME* is due to the Inter Symbol Interferences and the additive noise evaluated by the signal to noise ratio SNR(s) for the s^{th} output of the channel.

$$SNR(s) = \frac{\sum_{k=0}^{N_m} (y_S(k) - \overline{y_S})^2}{\sum_{k=0}^{N_m} (v_S(k) - \overline{v_S})^2} \quad (23)$$

With N_m the observation number, $\bar{y}_S$ and $\bar{v}_S$ are the mean values of the s^{th} channel output $y_s(k)$ of and the s^{th} noise value $v_s(k)$ respectively.

4.4. Simulation results

Consider the non linear Multiple Input Multiple Output MIMO Volterra channel [31] described by:

$$\begin{cases} y_1(k) = 2\,u_1(k) + 0.4\,u_1(k-1) + 0.08\,u_1(k-2) \\ \quad 2\,u_1(k)\,u_1(k-1) + 0.2\,u_1(k)\,u_2(k-1) + \\ \quad 0.2\,u_2(k)\,u_1(k-1) + 0.1\,u_1^2(k-1) + v_1(k) \\ \\ y_2(k) = u_2(k) + 0.3\,u_2(k-1) + 0.09\,u_2(k-2) \\ \quad 0.01\,u_1^2(k-1) + 0.01\,u_2^2(k-1) + v_2(k) \end{cases} \quad (24)$$

Where $u_1 \in \{-1,\ 1\}$ and $u_2 \in \{-2,\ 2\}$ are the channel inputs, y_1 and y_2 are its outputs and v_1 and v_2 are additive white noises.

4.4.1 Identification of the Channel:

This MIMO non linear channel can be modelled by a GOB MIMO Volterra model with P = 2 and a truncating order K = 2.

The total parameter number of the reduced model is 34 and the number of poles is 4.

- *First output of the channel:*

The optimal poles for the first subsystem optimised by the minimisation of the NMSE, are $\xi_{01} = -0.1$ and $\xi_{11} = 0.1$. We plot in **Figure 4** the validation of the first output of the channel and the output of the model; we note the concordance between both outputs.

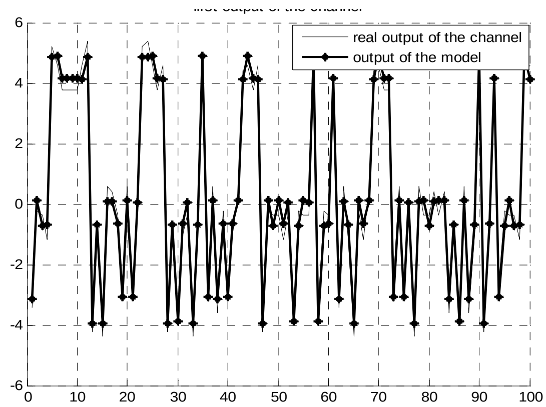

Figure 4. Validation of the first output of the channel

The NMSE for the first subsystem is 2.08%.

- *Second output of the channel:*

The optimal poles for the second subsystem are $\xi_{02} = 0.3$ and $\xi_{12} = -0.3$. In **Figure 5** we plot the second output of the channel and the output of the model we note the concordance between both outputs. The NMSE is 0.03%.

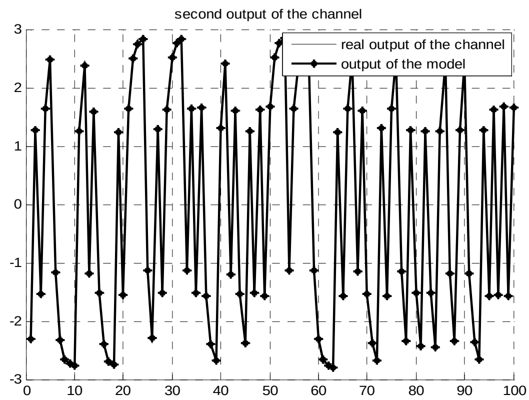

Figure 5. Validation of the second output of the channel

4.4.2 Supervised equalization of the Channel

The equalizer can be modelled by a GOB MIMO Volterra model with two inputs and two outputs, a non linearity degree P = 2 and a truncating order K = 2. The total parameter number of the equalizer model is 34 and the number of GOB poles is 4. First we assume that the equalizer inputs are noise free and we proceed to the equalization of both subsystems describe the MIMO channel.

- *First input equalization:*

The poles of the first subsystem of the equalizer are $\xi_{01} = -0.2$ and $\xi_{11} = -0.1$ and the NMSE is 3.36%. In **Figure 6** we plot the first source of the channel and the first output of the equalizer we note the similarity between both signals.

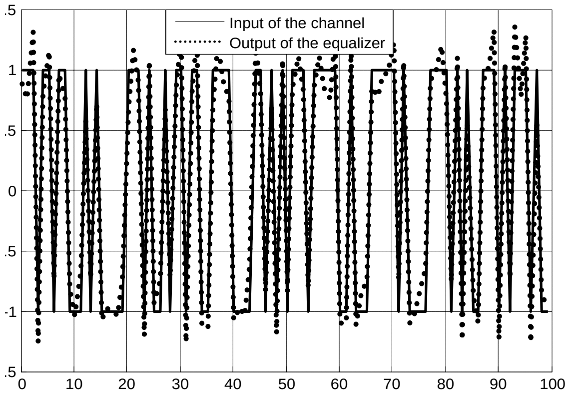

Figure 6. Validation of the first equalizer output

- *Second input Equalization:*

The poles of the second equalizer subsystem are $\xi_{02} = 0$ and $\xi_{12} = -0.3$, the NMSE is equal to 0.11%. **Figure 7** plots the second channel input and the second output equalizer and both signal fit each other.

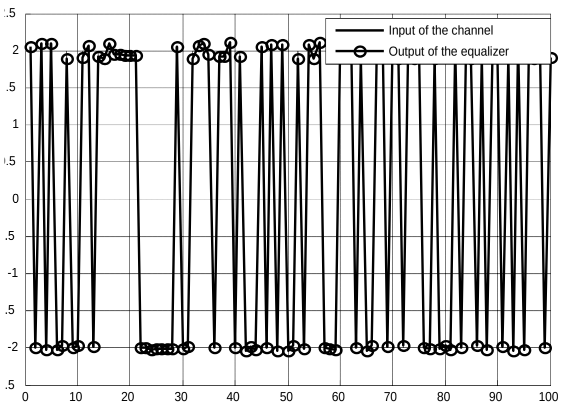

Figure 7. Validation of the second equalizer output

To test the effect of additive noise to the equalizer input on the equalizer behaviour we assume that the channel outputs are composed of an additive noise and a signal with SNR(s)=20.

For the first subsystem, the GOB poles are $\xi_{01} = 0.1$ and $\xi_{11} = -0.1$. The NMSE is 14.62%, **Figure 8** draws the first channel input and the first equalizer output.

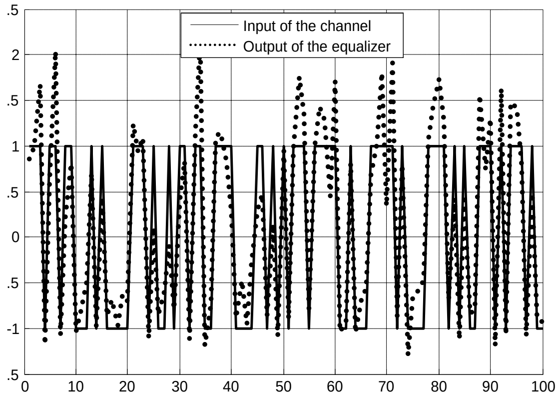

Figure 8. Validation of the first equalizer output

The second subsystem poles $\xi_{02} = 0.1$ and $\xi_{12} = -0.3$ and the corresponding NMSE is 1.88%. **Figure 9** displays the second channel input and the second equalizer output.

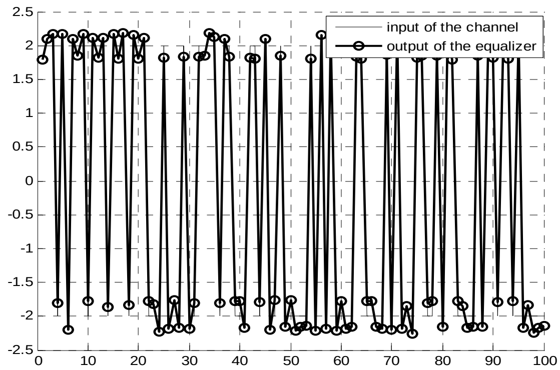

Figure 9. Validation of the second equalizer output

5. Conclusion

In this paper we have proposed a MIMO supervised equalizer based on GOB MIMO Volterra model for a Multi Input Multi Output non linear communication channel. The equalizer synthesised is tested in simulation on numerical example to resituate inputs of two input two output non linear communication channel and results are satisfactory. Simulations are carried out to evaluate the equalizer performances and the influence of an additive noise on these performances.

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