Outer Feedback Correction Loops in Particle Filtering-based Prognostic Algorithms: Statistical Performance Comparison

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Abstract: This paper presents, analyzes, and evaluates two different approaches for outer feedback correction loops (OFCL) in particle-filtering-based prognostic algorithms. These approaches incorporate information, from the short-term prediction error, back into the implementation of the estimation routine, to improve its performance in terms of both the resulting state and time-of-failure (ToF) pdf estimates. Three indicators are also proposed and used to measure the performance of the prognostic routines that result from the implementation of these OFCL in terms of precision, accuracy, and steadiness of the solution. Both approaches are tested using actual data from a seeded fault test in a critical component of rotorcraft transmission system.

Keywords: Particle filtering, failure prognosis, nonlinear state estimation, feedback loops.

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1. Introduction

Uncertainty management of prognostics holds the key for a successful penetration of health management strategies in industrial applications. While methods to estimate and handle uncertainty have received a reasonable amount of attention in the diagnostics domain, uncertainty management for prognostics is an area which awaits major advances.

In the field of failure prognosis, several approaches intend to solve the issue of uncertainty management. Probabilistic, soft computing methods and tools derived from evidential theory or Dempster-Shafer theory [1] have been explored for uncertainty representation in prediction. Probabilistic methods are mathematically rigorous assuming, of course, that a statistically sufficient database is available to estimate the required distributions. Possibility theory (fuzzy logic) offers an alternative when scarce data and even incomplete or contradictory data are available. Dempster's rule of combination and such concepts from evidential theory as belief on plausibility based on mass function calculations can support uncertainty representation tasks. Probabilistic reliability analysis tools employing an inner-outer loop Bayesian update scheme have also been used to "tune" model hyper-parameters given observations [2].

In this sense, particle filters (PF) have been established as the de facto state of the art in failure prognosis and uncertainty representation [3]. PF-based algorithms are capable of combining advantages of the rigors of Bayesian estimation to nonlinear prediction while also providing uncertainty estimates for a given solution. The outcome of these algorithms – an estimate of the probability density function (pdf) of the state – allows online computation of expectations, 95% confidence intervals, and other statistics of the time of failure (ToF). All the more, PF-based algorithms provide the framework to implement corrective schemes aimed at an online performance improvement; see Figure 1.

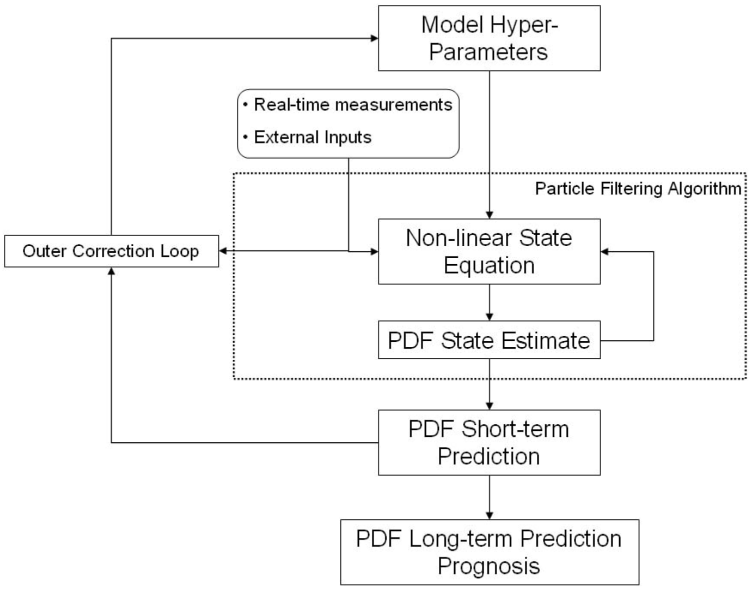

Figure 1. Particle Filtering-based prognostic algorithms and outer correction loops

This paper proposes, tests, and assesses a systematic method for the uncertainty management problem in failure prognosis consisting in two possible options for outer feedback correction loops. These loops incorporate information about the short-term prediction error to improve the performance of the overall prognostic framework. The structure of the paper is as follows. Section 2 summarizes the basic concepts associated to the usage of PF in the field of failure prognosis. Section 3 presents two approaches that can be used to implement outer correction loops, while Section 4 illustrates obtained results when using these loops in several realizations of PF-based prognostic algorithms that are fed with actual fault data. Section V presents an assessment of the results on the basis of three performance indicators that help to quantify the concepts of accuracy, precision, and steadiness of prognostic results. All proposed approaches are tested using actual data from a seeded fault test in a critical component of rotorcraft transmission system [4].

2. Particle Filtering in Failure Prognosis

Nonlinear filtering is the process of using noisy observation data to estimate at least the first two moments of a state vector governed by a dynamic nonlinear, non-Gaussian state-space model [4]. Moreover, from a Bayesian standpoint, any nonlinear filtering procedure generates an estimate of the posterior probability density function $p(x_k \mid y_{1:k})$ for the state x_k, based on the set of received measurements $\{y_{1:k}\}$. In this sense, Particle Filtering [3]-[15] (PF) is an algorithm that solves this estimation problem by efficiently selecting a set of N particles (samples drawn from the state domain) $\{x^{(i)}\}_{i=1\ldots N}$ and weights $\{w_k^{(i)}\}_{i=1\ldots N}$, such that the state pdf at time k may be approximated by

$$\tilde{\pi}_k^N(x_k)=\sum_{i=1}^{N} w_k^{(i)}\delta(x_k - x_k^{(i)}) \qquad (1)$$

Weights in (1) are updated in an online fashion, using the information collected from the measurements set $\{y_{1:k}\}$.

Failure prognostic, on the other hand, implies predicting the Remaining Useful Life (RUL) of the piece of equipment [3]-[5] via the generation of long-term predictions describing the evolution of a fault dimension over time. The problem of failure prognostics goes beyond the scope of filtering-only applications, since it involves future time horizons where no measurements are available for weight update purposes in a PF-based approximation of the predicted state pdf. It has been shown however [3], that a combination of PF-based algorithms, limited bandwidth kernels, and resampling schemes allows to represent the uncertainty associated with long-term predictions. Therefore, this combination enables the application of these algorithms in the field of failure prognosis, generating discrete approximations for the predicted state pdf $\hat{p}(x_{k+s})$ at future time s based on the recursion:

$$\hat{p}(x_{k+s} \mid \hat{x}_{1:k+s-1}) \approx \sum_{i=1}^{N} w_{k+s-1}^{(i)} K\left(x_{k+s} - E\left\{x_{k+s}^{(i)} \mid \hat{x}_{k+s-1}^{(i)}\right\}\right) \quad (2)$$

where $K(\cdot)$ is a rescaled version of the Epanechnikov kernel [3]-[4] and $E\{\bullet\}$ represents the expectation of a random variable.

Given the state pdf approximation (2), the problem of failure prognostic summarizes in computing the pdf of the RUL for the piece of equipment under analysis, since from that pdf estimate it is possible to obtain any necessary statistics describing the evolution of the fault dimension in time, either in the form of expectations or 95% confidence intervals. The procedure needed to obtain the RUL pdf from the predicted path of the state pdf is detailed and discussed in [3], [5] and it is now briefly described. Basically, the RUL pdf can be computed from the probability of failure at future time instants. This probability is computed through a procedure that combines long-term predictions for the state associated to the fault dimension and empirical knowledge about critical conditions for the system. This knowledge is usually included in the form of thresholds for main fault indicators, also referred to as the hazard zones.

The aforementioned procedure assumes the existence of a dynamic system that describes the evolution of the fault dimension in time. For failure prognostic purposes, this dynamic system is represented through the nonlinear state equation:

$$\begin{aligned} x_{k+1} &= x_k + \alpha_k \cdot F(x_k, \alpha_k) + \omega_k \\ \alpha_{k+1} &= L(\alpha_k, e_k^s) + \omega_k{}' \\ y_k &= x_k + \upsilon_k \end{aligned} \quad (3)$$

where x_k is the state associated to the fault dimension, α_k is a time-variant parameter related to the growth rate of the state x_k; $F(\bullet)$ is a nonlinear function representing a model based on first principles, a neural network, or even a fuzzy system; α_k is a time-variant parameter; $L(\bullet)$ is a nonlinear and discontinuous function defining the evolution of α_k (usually referred to as the transition function of α_k); $e_k^s = y_k - \hat{y}_k^s$ is the s-step-ahead prediction error (note that $\hat{y}_k^s = \hat{x}_k^s$ is computed at k-s); $\{\omega_k\}_{k\in\mathbb{N}}$ and $\{\omega_k'\}_{k\in\mathbb{N}}$ are independent noise sequences; $\{\upsilon_k\}_{k\in\mathbb{N}}$ is iid measurement noise sequence and y_k is the noisy observation at time k.

Model parameter estimate α_k has a direct effect in the accuracy and precision [3] of long-term predictions that could be generated using model (3) [3]. There should be a compromise between the accuracy of the current state estimate (usually checked via the usage of the likelihood of measurements) and the capability of the state model to describe the evolution of the fault condition in the long term. Moreover, biased estimation schemes lead to excessively aggressive (or conservative) approaches that may affect the operation performance of the system.

In this sense, an optimality criterion for prognosis should consider both the accuracy and the precision of the RUL estimates obtained from the state long-term predictions. To achieve an optimal prognosis procedure, in the aforementioned sense, [3] and [4] have suggested the usage of outer feedback correction loops that modify the estimate of the model parameter α_k, basically considering $L(\bullet)$ as a function of short-term prediction error. This paper presents two specific implementations for the outer correction loop and analyzes the effect of both changes in noise hyper-parameters [2]-[3] and transition function design, within a PF-based prognosis framework.

3. Outer Feedback Correction Loops in PF-based Prognosis Framework

Outer feedback correction loops (OFCL) play an important role in the assessment of on-line prognostic algorithms, ensuring both accuracy and precision of the resulting RUL estimates. These correction loops typically measure the prediction capability of the fault progression model [3]-[5], via the analysis of the short-term prediction error, and improve the algorithm performance either modifying the structure of the model or updating hyper-parameters that define process/measurement noise of model update equations [2]-[3]. A number of different approaches/strategies can be considered in the implementation of suitable OFCLs: heuristic rules, fuzzy expert systems, neural network control, and optimal control, among others. Particularly, this paper focuses its analysis on two OFCL that directly modify hyper-parameters associated with the process noise definition in the fault propagation model (3). The first proposed OFCL modifies the variance associated to the noise kernel in the equation describing the evolution of the system parameter α_k; i.e., the variance of $\{\omega_k'\}_{k\in\mathbb{N}}$ in (3), while the second updates the transition function $L(\bullet)$ for the aforementioned parameter.

3.1 First approach: Outer correction loop manipulating variance of noise kernel ω_k'

Let α be a fixed and unknown model parameter that is being estimated using the concept of "artificial evolution" [12]; i.e., let the transition function for α at time k be:

$$L(\alpha_k, e_k^s) = \alpha_k \tag{4}$$

where "s" is the time horizon considered for the computation of the short-term prediction error.

The first approach for an OFCL considers the short-term prediction error as a measure of the long-term accuracy of model (3), given the current PF-based state estimates x_k and α_k. The feedback loop then updates the variance of the noise kernel ω_k', associated to equation that describes the evolution in time of the unknown model parameter α, as follows:

$$var(\omega_k') := \begin{cases} p \cdot var(\omega_k') & \left|e_k^s\right| \le e^{th} \\ q \cdot var(\omega_k') & \left|e_k^s\right| > e^{th} \end{cases} \tag{5}$$

where e^{th} is an operator defined threshold, and $[p; q]$ are such that $0 < p < 1$ and $1 < q$.

The proposed correction for the variance of the noise kernel allows the particle-filtering-based algorithm to increase the probability of drawing samples from a broader subset of the domain for the second component of the state vector $[x_k \quad \alpha_k]^T$, thus facilitating a model parameter update that considers the measurement likelihood. On the other hand, whenever the sort-term prediction error is bounded the algorithm reduces the size of the domain where samples are drawn to obtain a PF-based estimate of the α_k, which is in accordance with the assumption $\alpha = \alpha_k$ (fixed model parameter) [12].

3.2 Second approach: Outer correction loop manipulating both the transition function of α_k and variance of noise ω_k'

Although the OFCL proposed in Section 3.1 facilitates the adjustment of unknown model parameter estimates, a task of paramount importance when observability issues are present, it does not provide the means to increase/decrease the correction energy according to the significance of the measured error. Therefore, and with the purpose of achieving more accurate RUL estimates, this paper introduces a second OFCL using a transition function $L(\bullet)$ to describe the evolution in time of the estimate of the unknown model parameter.

In this sense, let the transition function $L(\bullet)$ be defined as follows:

$$L(\alpha_k, e_k^s) := \begin{cases} \alpha_k & \left|e_k^s\right| \le e^{th} \\ \alpha_k + \eta e_k^s & \left|e_k^s\right| > e^{th} \end{cases} \tag{6}$$

And let the second approach for OFCL be defined by the following the update law for the variance of the process noise $\omega_k^{'}$:

$$var(\omega'_{k+1}) := \begin{cases} p \cdot var(\omega'_k) & \left|e_k^s\right| \le e^{th} \\ \sigma_0^2 & \left|e_k^s\right| > e^{th} \end{cases} \quad (7)$$

where η is the feedback gain for the correction of the unknown model parameter α and σ_0^2 is a constant value.

Similarly to the approach presented in Section 3.1, this OFCL defines a limit e^{th} for the s-step-ahead prediction error. If the magnitude of the short-term prediction error is under this value, then only the variance of the process noise $\omega_k^{'}$ is modified. On the other hand, if the absolute error is above this threshold, both the noise variance and the transition function $L(\bullet)$ are modified. It is assumed that short-term prediction error consistently small is an indicator that the estimate of the unknown model parameter α has reached a steady state value. In that case, the variance of the noise driving the "artificial evolution" of α can be gradually decreased within the implementation of the PF-based filtering algorithm.

On the other hand, in the event of large short-term predicted errors (*i*) the variance of the process noise associated to the model parameter noise variance is fixed to a constant value that facilitates the estimation task of the PF-based algorithm and (*ii*) a correction term dependent of the error is introduced in the transition function. In this approach, the inclusion of a PF-based algorithm is critical since the update stage of the filtering process [5]-[15] ensures that the one-step-ahead state estimate will consider the likelihood of measurements, regardless of the effect of the OFCL on the a priori estimate of the filter (to diminish the difference between $x_{k|k-1}$ to $\hat{x}_k^s$, for $\eta < 0$). As the variance of $\omega_k{'}$ is set to σ_0^2, the particle in the filtering algorithm can be drawn from a wider region of the state space, facilitating the correction of the model parameter estimate.

Given that in the event of significant errors in the short-term prediction the variance of $\omega_k{'}$ is set to σ_0^2 in this OFCL (value typically larger than the variance at the time of the update), both the state and RUL estimates that are generated immediately after the correction will exhibit an negative effect in the precision of the corresponding pdf's. Nevertheless, this method offers the possibility of strong and sudden adjustments in the value of the unknown model parameter that is being estimated on-line. Due to these capabilities, responses present a shorter transient time, and the s-step-ahead predicted error remains outside the interval $[-e^{th}\ ;\ e^{th}]$ for shorter periods of time, compared to the approach presented in Section 3.1.

Both of the aforementioned approaches for OFCL in prognostic algorithms have been implemented and tested with actual fault data to evaluate their respective performance in terms of the resulting estimate for the time-of-failure (ToF). Results are described and analysed in the next section.

4. Implementation of Outer Feedback Correction Loops: Analysis of Results

Consider the case of implementing the approaches described in Section 3 to help determining the ToF for a propagating fatigue crack on a critical component in a rotorcraft transmission system. The objective in this seeded fault test is to analyze how a cyclic load profile affects the growth of an axial crack. Although a physics-based model for a system of these characteristics may be complex, it is possible to represent the growth of the crack (fault dimension) using the much simpler population-growth-based model [3]:

$$\begin{cases} x_{k+1} = x_k + C \cdot \alpha_k \cdot (a - b \cdot k + k^2)^m + \omega_k \\ \alpha_{k+1} = \alpha_k + \omega'_k \end{cases} \quad (8)$$

where x_k is a state representing the fault dimension at the k^{th} cycle of operation, α_k is an unknown model parameter, C and m are constants associated with the fatigue properties of the material. The constants a and b depend on the maximum load and duration of the load cycle. The initial value for the model parameter α_0 is arbitrarily set to 0.5 and a PF-based algorithm is applied to generate a pdf estimate of the state $[x_k \ \alpha_k]^T$ considering a vibration-based feature (Sideband Ratio, SBR [4]-[5]) as online measurements.

The analysis described in the following paragraphs is based on statistical comparison of 40 realizations for each of the proposed OFCLs. The number of prediction steps considered to compute the short-term prediction error (at the k^{th} cycle) e_k^s, is set to $s = 5$. The authors would like to state that previous experimentation was conducted to define an appropriate value for this parameter. Future work will be focused in determining the optimal value of the short-term horizon.

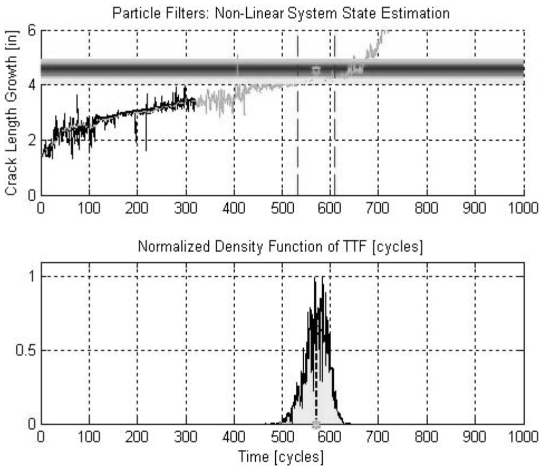

Figure 2. First OFCL approach: Evolution in time of crack length and ToF pdf estimates using $[p \ q]^T = [1.2 \ 0.95]^T$. Time axis is measured in cycles of operation.

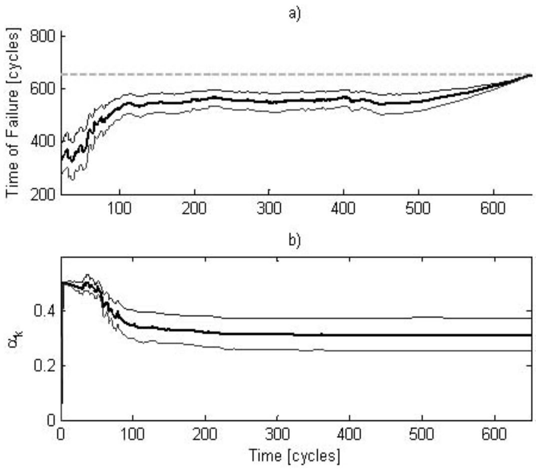

Figure 3. First OFCL approach: Average values for 95% confidence intervals for the ToF and estimates of the model parameter α_k. Black line indicates the averaged expectation of the variable under analysis, while light-dark lines are the upper and lower bound of the 95% confidence interval, respectively. The dotted horizontal line in Figure 3.a) corresponds to the ground truth failure time of the system

4.1 Analysis of results for the first OFCL proposed approach

The first OFCL approach, presented in Section 3.1, has been originally tested for two different choices for the algorithm parameter vector $[p \ q]^T$: $[1.05 \ 0.9]^T$ and $[1.2 \ 0.95]^T$. Although the first choice results in a shorter transient times whenever is needed to apply sudden updates in the value of the model parameter α_k, it does not provide the best results in terms of prognostic results. Indeed, the algorithm provides more accurate results in terms of accuracy in the RUL estimate when $[p \ q]^T = [1.2 \ 0.95]^T$. This is based on the fact that larger values for p and q imply higher values in the variance of the process noise ω_k' for more extended periods of time, thus providing better results from the PF-based estimation algorithms [5]. Results for one particular realization of the PF-based prognostic algorithm, using the proposed OFCL, are shown in Figure 2.

Figure 3 shows the average values for both the Time of Failure (ToF) and the estimate of αk using the $[p \ q]^T = [1.2 \ 0.95]^T$; computed over 40 realizations of the PF algorithm, and using the same measurement data for each realization. Results show that the resulting ToF estimate is biased during most part of the simulation, which is a key issue when evaluating the performance of the prognostic algorithm. It is important to mention, though, that the ToF estimate is always conservative, in such a manner that the actual failure time (defined by the instant when the crack reaches 4.5 inches) is always posterior to the upper bound of the 95% confidence interval (computed from the PF-based pdf estimate [3]-[5]).

4.2 Analysis of results for the second OFCL proposed approach

The second OFCL approach, presented in Section 3.2, has been tested using the same data set and using identical initial conditions for the unknown model parameter α_k. The algorithm implementation used η = 0.01 and p = 0.95. Figure 4 shows results for one of the 40 realizations tested using the aforementioned approach. It can be noticed that this procedure provides a more precise estimate of the ToF, compared to the results obtained when implementing the first OFCL.

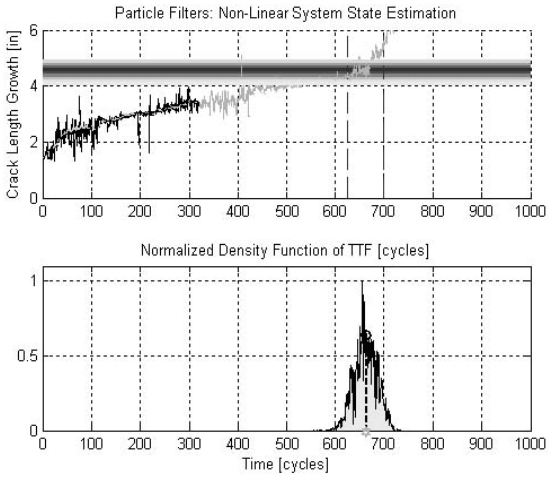

Figure 4. Second OFCL approach: Evolution in time of crack length and ToF pdf estimates using transition-function management. Time axis is measured in cycles of operation.

Figure 5 shows average values for both the expectation of the ToF and bounds of the 95% confidence interval, computed with results obtained in all 40 realizations of the PF-based prognostic algorithm when using the second proposed OFCL. It is important to note that all estimates exhibit a considerably smaller bias when compared to results from the first proposed approach. Also, the transient time associated to the estimation of the model parameter αk has decreased with respect to the one presented in Figure 3.

In this sense, the correction loop helps the prognostic routine in correcting the estimate of the unknown model parameter, thus obtaining more accurate results. In fact, changes in the operating condition of the seeded fault test 5 are reflected in updates in the model parameter estimate that are not detected by the PF-based prognostic algorithm when using the first OFCL. From a prognostic algorithm standpoint, these results are far more desirable. This approach, in summary, shows a more accurate and reliable tracking of the model parameter value, but at a price of having a noisier behavior in the resulting ToF expectation.

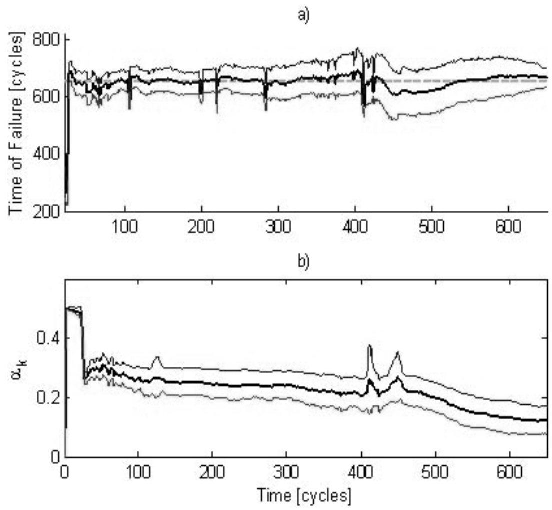

Figure 5. Second OFCL approach: Average values for 95% confidence intervals for the ToF and estimates of the model parameter α_k. Black line indicates the averaged expectation of the variable under analysis, while light-dark lines are the upper and lower bound of the 95% confidence interval, respectively. The dotted horizontal line in Figure 5.a) corresponds to the ground truth failure time of the system

5. Performance Indices for Prognostic Algorithms

Three performance indices have been developed to provide the means for an adequate quantitative comparison among the proposed outer feedback loops and their effects on prognostic algorithms. This section of the paper presents a mathematical definition and the results that are obtained when using these indices to evaluate the performance of the proposed outer correction loops in prognosis.

5.1 RUL Precision index

This index considers the relative width of the 95% RUL confidence interval (CIt), computed at time t, when compared to the expected RUL.

$$I_1(t) = e^{-\left(\frac{\sup(CI_t)-\inf(CI_t)}{E_t\{ToF\}-t}\right)} \qquad (9)$$
$$0 < I_1(t) \le 1, \forall t \in [1, E_t\{ToF\}), t \in \mathbb{N},$$

where E_t\{ToF\} is the expectation of the time-of-failure, conditional to measurements up to time t. It allows quantifying the concept “the more data the algorithm processes, the more precise should the prognostic results be”. Precise prognostic results are associated to values of I_1(t) ~ 1.

5.2 RUL accuracy-precision index

This index represents the amount of bias in ToF estimates, relative to the width of the corresponding 95% confidence interval CIt, and penalizes the fact that E_t\{ToF\} > *Ground Truth* \{ToF\} (actual failure happens before the ToF expectation, conditional to measurements up to time *t*).

$$I_2(t) = e^{-\left(\frac{Ground\,Truth\{ToF\}-E_t\{ToF\}}{\sup(CI_t)-\inf(CI_t)}\right)} \qquad (10)$$
$$0 < I_2(t), \forall t \in [1, E_t\{ToF\}), t \in \mathbb{N}$$

Accurate and useful prognostic results are associated to values of the index such that $0 \le 1 - I_2(t) \le \varepsilon$, where ε is a small positive constant.

5.3 RUL on-line steadiness index

This index represents a measure of the algorithm capability to provide consistent prognostic results in time. It considers the evolution in time of the conditional *ToF* expectation, E_t\{*ToF*\}, and quantifies the concept “the more data the algorithm processes, the more steady should the prognostic results be”.

$$I_3(t) = \sqrt{Var\left(E_t\{ToF\}\right)} \qquad (11)$$
$$I_3(t) \ge 0, \forall t \in \mathbb{N}$$

Steady results are associated to small values of this index.

As it has been mentioned in Section 4, statistical comparison between the two proposed prognostic methods has been performed on the base of 40 realizations of the stochastic process, for each one of the proposed OFCL in PF-based prognostic algorithms, and using the aforementioned indices.

Figure 6 shows the obtained results, where averaged precision, accuracy and steadiness indices are compared for both approaches. Particularly, the RUL precision index shows that that both schemes are similarly precise (width of the resulting 95% CI), even considering the fact that the system undergoes a change in the operating conditions around the 400th cycle (situation that is reflected in the resulting estimate of the unknown model parameter; see Figure 5).

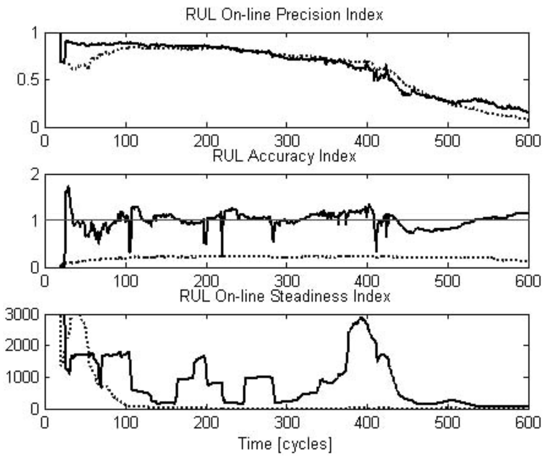

Figure 6. Performance indices for prognostic algorithms. First and second OFCL approaches for PF-based prognostic algorithms are shown in dotted and continuous lines, respectively

Regarding the other two indices, it can be noted that the second approach for OFCL is more accurate, although less steady, than the first one. This follows from the fact that, as it has been mentioned in Section 3.2, the second OFCL ensures high likelihood of estimates with respect to the current measurements, minimizing the one-step-ahead prediction error at the price of decreasing the steadiness of the *ToF* estimate.

6. Conclusions

This paper presents, analyzes, and evaluates two different approaches for outer feedback correction loops (OFCL) in PF-based prognostic algorithms. These approaches incorporate information, about the short-term prediction error, back into PF-based estimation routines to improve its performance in terms of both the resulting state and ToF pdf estimates. Statistical analysis of the proposed OFCL for prognostic algorithms is performed on the basis of three performance indicators: precision, accuracy, and steadiness indices. Results show that an OFCL modifying both the variance of the process noise and transition functions associated to unknown model parameters improves the precision and accuracy of the obtained ToF 95% confidence intervals. Both approaches are tested using actual data from a seeded fault test in a critical component of rotorcraft transmission system.

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