A Recognition Algorithm for a Class of Partitionable Graphs that Satisfies the Normal Graph Conjecture

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Abstract: A graph is normal if admits a clique cover and a stable set cover so that every clique may intersect every stable set. The Normal Graph Conjecture says that every $\{C_5, C_7, \overline{C}_7\}$-free graph is normal. In this paper we prove this conjecture for the class of O-graphs and we give a recognition algorithm for O-graphs.

Keywords: Normal graphs, O-graphs, Partitionable graphs, Partite graphs, Triangulated graphs.

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1. Introduction

Normal graphs form a superclass of perfect graphs and can be considered as closure of perfect graphs by means of co-normal products [9] and graph entropy [8]. Perfect graphs have been characterized as those without odd holes and antiholes as induced subgraphs (Strong Perfect Graph Theorem, [5]). Korner and de Simone [6] observed that $C_5, C_7, \overline{C}_7$ are minimal, not normal graphs. As a generalization of the Strong Perfect Graph Theorem, Korner and de Simone conjectured that every $\{C_5, C_7, \overline{C}_7\}$-free graph is normal (Normal Graph Conjecture, [11]). Wagler [15] proved the conjecture for the class of circulant graphs.

The entropy [10] of a graph is a functional depending both one the graph itself and on a probability distribution on its vertex set.

In [4] we find a recent survey of results on combinatorial optimization problems in which the objective function is the entropy of a discrete distribution.

In [8], the authors prove that a graphs is perfect if and only if it "splits graph entropy". Using this derive the following strengthening of the normality of perfect graphs:

Let G be a perfect graph. Then G contains a family $\mathcal{A}$ of independent sets and a family $\mathcal{B}$ of cliques with the following properties:

i) $|\mathcal{A}|+|\mathcal{B}|=k+1$;

ii) the sets in $\mathcal{A}$ ($\mathcal{B}$) *cover all vertices;*

iii) the incidence vectors of sets in $\mathcal{A}$ ($\mathcal{B}$) *are linearly independent;*

iv) every $A \in \mathcal{A}$ *intersects every* $B \in \mathcal{B}$.

The class of perfect graphs is important because many problems of interest in practice but intractable in general can be solved efficiently when restricted to the class of perfect graphs [6].

Partitionable graphs contain all the potential counterexemples to Berge's famous Strong Perfect Graph Conjecture ([3]) which was proved by Chudnovski, Robertson, Seymour, and Thomas in [5]. Partitionable graphs ([7])

are one of the central objects in the theory of perfect graphs due to the following theorem of Lovasz: *A graph is perfect if and only if* $\alpha(H)\omega(H)\geq n(H)$ *for every induced subgraph* H *of* G.

In this paper we find a class of partitionable graphs that are not perfect, but are normal. We prove Normal Graph Conjecture for the class of O-graphs and give a recognition algorithm for O-graphs.

Throughout this paper, $G=(V,E)$ is a simple (i.e. finite, undirected, without loops and multiple edges) graph [2]. Let $\overline{G}$ denote the complement graph of G. For $U\subseteq V$ let $G(U)$ denote the subgraph of G induced by U. By G-X we mean the graph $G(V$-$X)$, whenever $X\subseteq V$, but we often denote it simply by G-v $(\forall v\in V)$, when there is no ambiguity. If $v\in V$ is a vertex in G, the neighborhood $N_G(v)$ denotes the vertices of G-v that are adjacent to v. We write $N(v)$ when the graph G appears clearly from the context. The neighborhood of the vertex v in the complement of the graph G is denoted by $\overline{N}(v)$. For any subset S of vertices in G, the neighborhood of S is $N(S)=\cup_{v\in S}N(v)$-S and $N[S]=S\cup N(S)$. A clique is a subset of V with the property that all the vertices are pairwise adjacent. The *clique number* (*density*) of G, denoted by $\omega(G)$ is the cardinal of the maximum clique. A clique cover is a partition of the vertices set such that each part is a clique. $\theta(G)$ is the cardinal of a smallest possible clique cover of G; it is called the *clique cover number* of G. The *stability number* of G is $\alpha(G)=w(\overline{G})$; the *chromatic number* of G is $\chi(G)=w(\overline{G})$.

By P_n, C_n, K_n we mean a chordless path on $n\geq3$ vertices, the chordless cycle on $n\geq3$ vertices, and the complete graph on $n\geq1$ vertices. If $e=xy\in E$, we also write $x\sim y$; we also write $x\nsim y$ whenever x, y are not adjacent in G. A set A is totally adjacent (non adjacent) with a set B of vertices ($A\cap B=\phi$) if ab is (is not) edge, for any a vertex in A and any b vertex in B; we note with $A\sim B$ ($A\nsim B$). A graph G is F-free if none of its induced subgraphs is in F.

A graph G is called α-*partitionable* if

$\alpha(G)=\theta(G)$ holds.

A graph G is *perfect* if $\alpha(H)=\theta(H)$ (or, equivalent, $\chi(H)=\omega(H)$) holds for every induced subgraph H of G, i.e. every induced subgraph is α-partitionable.

We call *matching* a set $F\subset E$ such that the edges in F are not adjacent.

A $[p,q,r]$-*partite* graph is a graph whose set of vertices is partitioned in p stable sets $S_1,S_2,...,S_p$, each of them consisting of exactly q vertices and every subgraph induced by $S_i\cup S_j$ consists of exactly r independent edges, for $1\leq i<j\leq p$.

A *circulant* $C^{k,n}$ is a graph with nodes $1,...,n$ where ij is an edge if i and j differ by at most $k(mod\ n)$ and $i\neq j$.

The subset $A\subset V$ is called a *cutset* if G-A is not connected. If, in addition, some $v\in A$ is adjacent to every vertex in A-$\{v\}$, then A is called a star cutset and v is called the center of A.

The paper is organized as follows. In Section 2 we give sufficient conditions for a graph to be normal. In Section 3 we give a recognition algorithm for O-graphs.

2. The Normal Graph Conjecture is True for *O*-graphs

Definition 1. *A graph G is called normal if G admits a clique cover C and a stable set cover S such that every clique in C intersects every stable set in S.*

In this section we address the problem of finding another class of α-partitionable graphs that are not perfect, but are normal.

Definition 2. *A graph G is partitionable if* $\theta(G)=\alpha(G)$ *and* $\chi(G)=\omega(G)$.

Definition 3. [13] *A graph G is O-graph if there exists a coloring of G and a coloring of* $\overline{G}$*, the complement of G, such that any class of colors of G intersects any class of colors of* $\overline{G}$.

Sufficient conditions for a graph to be normal are set by the following result. The equivalent conditions, (i) with (ii) and (i) with (iii), are stated and [12].

Theorem 1. *Let G by a graph with n vertices, m edges, stability number* α *and density* ω*. Then the following conditions are equivalent:*

(i) G is O-graph;

(ii) G is partitionable and $n=\alpha\omega$;

(iii) V can be partitioned in $\omega\alpha$-stable set and $\alpha\omega$-cliques;

(iv) G is $[\omega,\alpha,\alpha]$-partite.

Proof. Let G an O-graph. We show that G fulfills condition (ii). Let $\chi(G)=p$, $\theta(G)=\chi(\overline{G})=q$, $S=(S_1,\ldots,S_p)$ a p-coloring of G and $Q=(Q_1,\ldots,Q_p)$ a q-coloring of $\overline{G}$ such that $S_i\cap Q_j\neq\phi$, for $i=1,...,p$ and $j=1,...,q$ hold. Then

$$\begin{aligned}\alpha(G) &\geq |S_i| = \left|\cup_{j=1}^{q}(S_i\cap Q_j)\right| \\ &= \sum_{j=1}^{q}\left|S_i\cap Q_j\right| \\ &= q = \theta(G) \geq \alpha(G)\end{aligned}$$

for all $i=1,...,p$. Therefore:

$\alpha(G)=\theta(G)(\chi(G)=\theta(\overline{G})=\alpha(\overline{G})=\omega(G))$;

$|S_i|=\alpha(G)$, $i=1,...,p$; $q=\alpha(G)$;

$|Q_j|=\omega(G)$, $j=1,...,q$; $p=\omega(G)$;

$n=\sum_{i=1}^{p}|S_i|=\alpha(G)\omega(G)$.

Suppose that G satisfies condition (ii) and we show (i). Because $\chi(G)=\omega(G)$ $(=\omega)$ and $n=\alpha(G)\omega(G)$, there exists an optimal colouring of G with ω stable sets $S_1,\ldots,S_\omega$ with $|S_i|=\alpha$ $(\alpha(G))$, $i=1,...,\omega$. Similarly, there exists an optimal colouring $Q_1,\ldots,Q_\alpha$ of $\overline{G}$ with $|Q_j|=\omega$, $j=1,...,\alpha$. Obviously $S_i\cap Q_j\neq\phi$ for all $i=1,...,\omega$ and $j=1,...,\alpha$, which means that G is O-graph.

It is clear that (ii) is equivalent to (iii).

Let G be O-graph. We show that (iv) holds. Let $\{S_1,\ldots,S_\omega\}$ a partition of G in $\omega\alpha$-stable sets, and $\{Q_1,\ldots,Q_\alpha\}$ a partition in $\alpha\omega$-cliques with $S_i\cap Q_k\neq\phi$ for all $i=1,...,\omega$ and $k=1,...,\alpha$. The subgraph induced by $S_i\cup S_j$ $(i,j=1,...,\omega, i\neq j)$ admits a matching with α elements, which obviously is maximal. Indeed, let $\{x_k^i\}=S_i\cap Q_k$. For $k\neq1$ we have $x_k^i\neq x_l^i$ because $Q_k\cap Q_l=\phi$. So $S_i=\{x_1^i,\ldots,x_\alpha^i\}$, , $i=1,...,\omega$. Because $\{x_k^i, x_k^j\}\subseteq Q_k$ it follows that $x_k^i x_k^j\in E(G)$, for $k=1,...,\alpha$, $i,j=1,...,\omega$ with $i\neq j$. Consequently, the set of edges $\{x_k^i, x_k^j \mid k=1,\ldots,\alpha\}$ is a matching in $G(S_i\cup S_j)$ for $i,j=1,...,\omega$ with $i\neq j$. Because $\overline{G}$ is an O-graph with $\alpha(\overline{G})=\omega(G)$ and $\omega(\overline{G})=\alpha(G)$ it follows that $\overline{G}$ is $[\alpha,\omega,\omega]$-partite.

Suppose that G satisfies (iv). We prove that (i) holds.

As G is $[\omega,\alpha,\alpha]$-partite, it follows that there exists a partition of V in $S=\{S_1,\ldots,S_\omega\}$ α-stable sets and, as $\overline{G}$ is $[\alpha,\omega,\omega]$-partite it follows that there exists a partition of V in $C=\{Q_1,\ldots,Q_\alpha\}$ with Q_i cliques and $|Q_i|=\omega(1\leq i\leq\alpha)$, which means that G is an O-graph and this completes the proof.

Corollary 1. *If G is a graph that satisfies one of the conditions (i), (ii), (iii) or (iv) in the above theorem, then G is a normal graph.*

Remark 1 ([12]). *For an O-graph G with n vertices, stability number α and density ω, the number of edges, m, verifies the following:* $\alpha\omega(\omega=1)/2\leq m\leq\alpha^2\omega(\omega-1)/2$.

Proof. Because the disjoint reunion of $\alpha\omega$-cliques is O-graph of minimal length it follows that $\alpha\omega(\omega=1)/2\leq m$. Because for an ω-colouring $\{S_1,\ldots,S_\omega\}$ with α-stable sets of the O-graph G and because for two non-adjacent vertices $x\in S_i$ and $y\in S_j$ $(i\neq j)$ the graph $G'=G+xy$ is an O-graph with the same n, α, ω it follows that the ω-partite, complete graph $K_{\alpha,\ldots,\alpha}$ is an O-graph of maximal length, that is $m\leq\alpha^2\omega(\omega-1)/2$.

3. An Algorithm for *O*-graph Recognition

At first, we recall the notion of weakly decomposition.

Definition 4. ([13], [14]) *A set $A\subset V(G)$ is called a weakly set of the graph G if $N_G(A)\neq V(G)-A$ and $G(A)$ is connected. If A is a weakly set, maximal with respect to set inclusion, then $G(A)$ is called a weakly component. For simplicity, the weakly component $G(A)$ will be denoted with A.*

Definition 5. ([13], [14]) *Let $G=(V,E)$ be a connected and non-complete graph. If A is a weakly set, then the partition $\{A,N(A),V\text{-}A\cup N(A)\}$ is called a weakly decomposition of G with respect to A.*

The name of "*weakly component*" is justified by the following result.

Theorem 2. ([13], [14]) *Every connected and non-complete graph* $G=(V,E)$ *admits a weakly component* A *such that* $G(V-A)=G(N(A)+G(\overline{N}(A))$.

Theorem 3. ([13], [14]) *Let* $G=(V,E)$ *be a connected and non-complete graph and* $A\subset V$. *Then* A *is a weakly component of* G *if and only if* $G(A)$ *is connected and* $N(A)\sim\overline{N}(A)$.

The next result, based on Theorem 2, ensures the existence of a weakly decomposition in a connected and non-complete graph.

Corollary 2. *If* $G=(V,E)$ *is a connected and non-complete graph, then* V *admits a weakly decomposition* (A,B,C), *such that* $G(A)$ *is a weakly component and* $G(V\text{-}A)=G(B)+G(C)$.

Theorem 3 provides an $O(n+m)$ algorithm for building a weakly decomposition for a non-complete and connected graph.

Algorithm for the weakly decomposition of a graph ([13])

Input: A connected graph with at least two nonadjacent vertices, $G=(V,E)$.

Output: A partition $V=(A,N,R)$ such that $G(A)$ is connected, $N=N(A)$, $A\nsim R=\overline{N}(A)$.

```
begin
  A:= any set of vertices such that
        A∪N(A)≠V
  N:=N(A)
  R:=V-A ∪ N(A)
  while (∃n∈N, ∃r∈R such that nr∉E ) do
    begin
      A:=A ∪ {n}
      N:=(N-{n})∪ (N(n) ∩ R)
      R:=R-(N(n) ∩ R)
    end
  end
```

In [13] some applications of weakly decomposition have been depicted. In the following proposition we present two of those applications.

Proposition 1. *Let* $G=(V,E)$ *be connected, non-complete graph and* (A,N,R) *a weakly decomposition, with* A *the weakly component. The following hold:*

a) G *is* P_4*-free iff* $A\sim N\sim R$ *and* $G(A)$, $G(N)$ *and* $G(R)$ *are* P_4*-free;*

b) G *is* $K_{1,3}$-free *iff* R *and* $N(n)\cap A$ *are cliques,* $\forall n\in N$ *and* $G\text{-}A$ *and* $G\text{-}R$ *are* $K_{1,3}$-free.

Each of the above results lead to recognition algorithms for the corresponding graphs.

Proposition 2. *Let* $G=(V,E)$ *be connected, non-complete graph and* (A,N,R) *a weakly decomposition, with* $G(A)$ *the weakly component.*

G is triangulated iff:

1) N is a clique and

2) R and G-R are triangulated.

Proof. We note the fact that N is a minimal cutset, because N is the set of neighbours of A and N~R. Then there exists a P_3:*anr*, for every $n\in N$, with $a\in A$ and $r\in R$. Because G is triagulated, N is a clique. Graphs $G(R)$ and $G\text{-}R$ are triangulated. Conversly, let G a graph that satisfies conditions 1) and 2). Let C_k ($k\geq 4$) a cycle induced in G. As N is a clique, it follows that $|N\cap V(C_k)|\leq 2$. If $N\cap V(C_k)=\phi$ then $C_k\subseteq G\text{-}N$, contradicting 2) or $A\nsim R$ does not hold. If $|N\cap V(C_k)|=1$ then $C_k\not\subset(N\cup R)$ and $C_k\not\subset(N\cup R)$ and $C_k\not\subset G(A\cup N)$. So $V(C_k)\cap A\neq\phi$. Furthermore, $|V(C_k)\cap R|=1$. Because $A\nsim R$, we obtain a contradiction. If $|V(C_k)\cap R|=2$, as N~R, it follows that $C_k\subseteq G\text{-}R$, contradicting 2).

Remark 2. *Let* $G=(V,E)$ *be connected, non-complete graph and* (A,N,R) *a weakly decomposition, with* $G(A)$ *the weakly component. Then we have:* $\alpha(G)=\max\{\alpha(G(A))+\alpha(G(R)), \alpha(G(N[A]))$.

Proof. Any stable set of maximum cardinal either intersects R and so it has the cardinal

$\alpha(G(A))+\alpha(G(R))$ or it does not intersects R and so it has cardinal $\alpha(G(N[A]))$.

Proposition 3. *Let* $G=(V,E)$ *be connected, non-complete graph and* (A,N,R) *a weakly decomposition, with* $G(A)$ *the weakly component. If* G *is triangulated then we have:* $\alpha(G)=\alpha(G(A))+\alpha(G(R))$.

Proof. We will use the formula given in Remark 2. Let $T\subset A\cup N=N[A]$ such that T is stable and $|T|=\alpha(G(N[A]))$. As $N=N(A)$ is a clique it follows that $|T\cap N(A)|\leq 1$. If $T\cap N(A)=\phi$ then $T\cup\{r\}$ is a stable set in $G(A\cup R)$. If $T\cap N(A)=\{n_0\}$ then $(T\text{-}\{n_0\})\cup\{r\}$

is stable in $G(A \cup R)$ ($r \in R$). It follows that, in the expression of $\alpha(G)$, the maximum is always obtained by the first component.

Every non-empty graph has a triangulated induced subgraph since every graph on three or fewer vertices is triangulated.

Balas and Yu ([1]) developed a polynomial-time algorithm to find a vertex-maximal triangulated induced subgraph of a given graph, and devised a branching strategy that has been used in many subsequent research efforts (see also [16]).

We give a recognition algorithm for O-graphs, based on the branching strategy by Balas and Yu, but using Proposition 2 and Proposition 3 to determine a stable set of maximum cardinal for a triangulated graph.

Procedure Stable-*O*-graph(*G*)

Input: A connected graph with at least two nonadjacent vertices, $G=(V,E)$.

Output: An answer to the question: is G an O-graph ?

begin

0. *While* $G \neq \theta$ *do*
1. find a maximal induced subgraph $H=G(T)$ such that H is triangulated
2. build in H a stable set S of maximum cardinal
3. build in H the disjoint cliques $K_1,\ldots,K_{|S|}$
4. let $U=\cup_{i=1}^{|S|} K_i$
5. let $V\text{-}U=\{x_1,\ldots,x_k\}$
6. for every i from 1 to k: V_i= $V_i = \overline{N}_G(x_i) - \{x_j \mid j < i\}$ determine a stable set S_i in $G(V_i)$ using Stable-O-graph ($G(V_i)$)
7. let S_0 a maximal stable set (one of S or $S_1 \cup \{x_1\}$ or $\ldots S_k \cup \{x_k\}$)
8. $G \leftarrow G\text{-}S_0$
9. If all sets S_0 have the same cardinal then G is an O-graph.

$\alpha(G)=|S_0|$; $\omega(G)=n/\alpha(G)$;

$\theta(G)=\alpha(G)$; $\chi(G)=\omega(G)$

end

In what follows, we give some remarks on the algorithm.

Step 1 is $O(m+n)$, according to [1].

Step 2 is the following:

begin
  $S \leftarrow \varphi$
  $L \leftarrow \{H\}$ // L is a list of graphs
 while ($L \neq \phi$) *do*
  begin
   extract an element F from L
   if (F is complete) *then*
    Return: $S \leftarrow S \cup \{v\}, \forall\, v \in V(F)$
   else
    begin
     determine a weakly decomposition (A,N,R) for F
     put in L the subgraph induced by A and the connected component of the subgraph induced by R
    end
  end
end

where:

$S=\{s_1,\ldots s_{|S|}\}$

The test "F is complete" is done as follows: if there exists a vertex v in F whose neighbor list (or the degree of v) is not $V(F)\text{-}\{v\}$ (corresponding to $|V(F)|$-1) then F is not complete.

Step 2 is $O(nm)$.

Step 3 is the following:

begin
 for $i \leftarrow 1$ to $|S|$ *do*
  begin
   $K_i \leftarrow \{s_i\}$
   for every v in $T\text{-}S$ *do*
    if $\{v\} \sim K_i$ then
     $K_i \leftarrow K_i \cup \{v\}$
   $T \leftarrow T - K_i$
  end
end

Step 3 is $O(n^3)$.

In Step 4 we have:

$K_1,\ldots,K_{|S|}$ is a covering with cliques of $G(U)$ and $\alpha(G(U))=\theta(G(U))=|S|$

In Step 5 we have an arbitrary order of the vertices in $V\text{-}U$.

In Step 6 we use Stable-O-graph for $G(V_i)$.

In Step 7, either S or $S_1 \cup \{x_1\},\ldots,S_k \cup \{x_k\}$ is a maximum stable set for G ([16]).

It follows that the complexity of the recognition algorithm for O-graphs is $O(n^3)$.

4. Conclusions and Future Work

In this paper we have proved that the normal graph conjecture is true for O-graphs and we have presented a recognition algorithm for these graphs.

Our future work will verify the conjecture on classes of graphs characterized by means of forbidden subgraphs.

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