

A Portfolio Theory Approach to Fishery Management

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Abstract: Several portfolio selection models for fishery management are presented. The financial risk is measured by the first lower partial moment of the return. The purpose of the models is to obtain optimal fishing plans that minimize the financial risk or maximize the expected return. The ranges of variation for the parameters of the minimum risk model are determined. A numerical example for a fishery from the Galati county, Romania is analyzed.

Keywords: portfolio selection, fishery management, lower partial moment, risk, maximum sustainable yield, risk.

1. Introduction

In the last decades fisheries management has been shifting toward an ecosystem-based fisheries management, also called an ecosystem approach to fisheries. The ecosystem approach to fishery management is a significant step towards sustainable use of the natural resources. It combines harmoniously economic objectives with environmental and social objectives. In this paper, we present several portfolio selection models for fishery management. Our fishery models consider multiple fish species and take into account both economic and environmental constraints. They are inspired from Sanchirico's paper [29] and take into account the ecosystem approach. Note that the approach based on financial portfolio theory to fish management is compatible with the ecosystem-based approach. The differences between our model and the Sanchirico portfolio selection model from Sanchirico [29] are the following:

- in our models an additional constraint connected to the budget for fish harvesting or with the input costs is introduced;
- we determine the range of variation for the parameters of our models. This facilitates the integration of our models in a decision support system;

- we use the first lower partial moment as a measure for the financial risk. This approach implies the introduction of a target parameter for the return of the fishing plans. The financial risk occurs if the return of the fishing plan is below this target parameter.

An approach for managing discrete assets based on the first lower partial moment can be found in Radulescu [24].

2. The Portfolio Approach to Multi-species Management of Fish Stocks

According Edwards et al. [9] and [10], a portfolio framework for fishery management is a technical methodology that systematically combines fish stocks that are joined by ecology (e.g., predation, competition) and unspecialized fishing technologies (e.g., mixed-species trawls) into a portfolio which balances expected aggregate returns against the risks associated with stock-attribute and other uncertainties.

The portfolio approach to multi-species management of fish stocks supposes that:

- fish stocks are real assets capable of generating a flow of returns indefinitely;

- society's objectives and constraints are clearly defined and capable of being evaluated;
- the combinations of stock sizes and other attributes for each species are evaluated for their effects on aggregate returns;
- there exists a tradeoff between the expected aggregate returns and risk (i.e., variation in returns).

In general, portfolios are used to find the most desirable combination of assets given their individual value and risk properties. Financial securities and real capital portfolios can be designed to manage risk by balancing the mix of assets. In contrast to selecting financial securities from independent companies or governments, fish stocks which have co-evolved over many years and are harvested, in many cases, by unspecialized technologies in mixed species. Fisheries should be managed jointly due to spillover interactions between them in order to optimize their overall value. That is, harvesting one species generally has important implications for other species.

Fish populations are real assets belonging to society. They are capital stocks that can potentially yield benefits indefinitely to the owners of stock rights (governments, commons, or private individuals) and the public. From an economics standpoint, the value of a renewable asset, such as a fish stock, is the present value of income (revenues minus costs) from future harvests, appropriately discounted and balanced against the income from current harvest (a dividend).

The economic value of a fish stock is a function of current and future prices and extraction costs, and is subject to production technology and the resource growth rate Fisher [12].

Prices are derived from people's demand (preferences) for seafood and other commodities and services. Fish market prices are influenced by product attributes, such as species and fish size. Extraction costs are determined by market prices for inputs, such as fuel and insurance, and by extraction technology and stock attributes. Other costs are associated with administration, management, and enforcement (i.e., transaction costs).

Ecological and technological interactions are sufficient reasons to manage multi-species fish stocks for their aggregate benefits because stocks affect each others' dynamics. Harvesting one species can affect the stock sizes and capitalized values of many other species in the community. There are three distinct classes of risk that affect fishery returns: availability of fish, market price, and institutions Pontecorvo [20].

3. Application of Financial Portfolio Theory to Fishery Management

An important mathematical instrument which was successfully applied to modelling the problems from fisheries management is portfolio theory. The above mentioned theory was developed as a result of the research in the domain of financial management. Its aim is the elaboration of a quantitative analysis of how investors can diversify their portfolio in order to minimize risk and maximize returns. The theory was introduced in 1952 by University of Chicago economics student Harry Markowitz, who published his doctoral thesis, "Portfolio Selection" in the Journal of Finance, Markowitz [18]. There exist many applications of portfolio theory to domains that do not imply finance such as agriculture Blandon [7], Blank [8], Romero [27], [28], Radulescu [22], sire selection Schneeberger [30], biodiversity, Figge [11], Koellner and Schmitz [16], energy Bazilian and Roques [5], sustainable production planning Radulescu [23] and [25] etc. For supplementary references regarding applications of portfolio theory to non-financial areas see Radulescu [21]. The application of portfolio theory for finding an optimal harvesting portfolio is popular in the literature. Managing fish stocks in terms of a portfolio of economic assets is likely to significantly increase benefits for society relative to single-species approaches. Fishery portfolio selection models for fishery management were formulated and analyzed in Hatch and Atwood [14], Sanchirico [29], Sethi [31]. In Sanchirico [29] the authors adapt financial portfolio theory as a method for ecosystem-based fishery management that accounts for species interdependencies, uncertainty, and sustainability constraints.

They illustrate their method with routinely collected data available from the Chesapeake Bay and demonstrate the gains from taking into account variances and covariances of gross fishing revenues in setting species total allowable catches.

In Larkin [17] the authors embed a portfolio decision framework into a multi-period bio-economic model in order to quantify the risk-benefit tradeoffs of alternative strategies. They develop alternative sets of processed seafood products for managing the risks that occur as a result of harvests from commercial fish stocks. The model is used to generate an efficient portfolio frontier to estimate possible rent dissipation from status quo management. Frontiers are also generated for seafood processors and brokers. The authors discuss implications for the different industry agents.

Drawing the analogy between managing risky assets and managing multispecies fisheries is a relatively new idea, even though the foundation for this idea is neither new to ecology nor economics. In ecology, Walters [32] derives a mean-variance frontier for single-species management, while Real [26] uses portfolio theory to describe animal behaviour.

Portfolio management of fisheries can be a means of allocating catch across life history stages. Baldursson and Magnusson [4], Arnason [2] alludes to multispecies portfolio management in a deterministic bioeconomic model by suggesting that managers choose a vector of Total Allowable Catches (TACs), while Hanna [13] explicitly discusses the idea of selecting "species portfolios" as a means to match management objectives with ecosystem structure. Hilborn et al. provides in [15] a justification for portfolio management at the regional level by pointing out that total productivity aggregated across species is subject to less variability than the productivity of individual species. Edwards et al. formally develop in [9] the analogy within the context of standard bio-economic models and provide a stylized simulation of a three-species system. More recently, Perusso et al. apply portfolio theory in [19] to individual fishermen targeting decisions in the U.S. Atlantic and Gulf of Mexico pelagic longline fishery. An interesting survey on the application of portfolio theory to fisheries management may be found in Yang [33].

Several references on applications of operations research to fish management can be found in Arnason [3] and Bjørndal [6].

4. Portfolio Selection Models for Fisheries Management

In this section we shall present a multi-objective programming model for fisheries management. Starting from it we define several single objective programming problems: a minimum risk problem, a maximum expected return problem and an optimal trade-off problem. In order to facilitate the use of these models in a decision support system we gave procedures for finding the ranges of variation for the parameters.

Suppose that we have n fish species S_1, S_2, \dots, S_n . Denote by ξ_i the market price of one kg of fish of species S_i . Of course all ξ_i are random variables. Let $\xi = (\xi_1, \xi_2, \dots, \xi_n)$ denote the random vector of returns. Denote by μ_i the mean of ξ_i . Let $\mu = (\mu_1, \mu_2, \dots, \mu_n)$. Denote by b_i the cost associated to fishing one kg of fish (or the cost of inputs for producing one kg of fish) from species S_i and with a_i the maximum sustainable yield for the fish of species S_i . Let $\mathbf{a} = (a_1, a_2, \dots, a_n)$, $\mathbf{b} = (b_1, b_2, \dots, b_n)$. Let x_i be the quantity harvested from fish of species S_i . Of course $x_i \geq 0$ for every $i=1, 2, \dots, n$. The vector $\mathbf{x} = (x_1, x_2, \dots, x_n)$ will be called a portfolio. The cost of harvesting a portfolio \mathbf{x} is equal to $\sum_{i=1}^n b_i x_i = \mathbf{b}^T \mathbf{x}$. The return (resp. the expected return) from harvesting a portfolio \mathbf{x} is equal to $\sum_{i=1}^n \xi_i x_i = \xi^T \mathbf{x}$ (resp. to $\sum_{i=1}^n \mu_i x_i = \mu^T \mathbf{x}$).

In reality the vectors $\xi = (\xi_1, \xi_2, \dots, \xi_n)$ and $\mu = (\mu_1, \mu_2, \dots, \mu_n)$ depend on time. Fishing and pollution are important factors that make that the above mentioned vectors vary along the time. The financial risk of the portfolio \mathbf{x} may be defined in several ways.

One traditional way is to define the risk as the variance of the return $f_1(\mathbf{x}) = \text{Var}(\xi^T \mathbf{x}) = \mathbf{x}^T \mathbf{C} \mathbf{x}$.

If t is a real number we shall denote by t_+ the positive part of t , that is:

$$t_+ = \max(t, 0) = \frac{t + |t|}{2}$$

Alternative ways to define the risk are $f_2(\mathbf{x}) = E[(\tau - \xi^T \mathbf{x})_+]$, $f_3(\mathbf{x}) = P(\xi^T \mathbf{x} < \tau)$.

A general problem in the fisheries management is the following multi-objective problem

$$\begin{cases} \text{minimize the risk of portfolio } \mathbf{x} \\ \text{maximize the expected return of portfolio } \mathbf{x} \\ \text{subject to} \\ \mathbf{b}^T \mathbf{x} = M \\ \mathbf{0} \leq \mathbf{x} \leq \mathbf{a} \end{cases}$$

Starting from the above multi-objective problem one can formulate the following single objective problems: the minimum risk problem, the maximum expected return problem, the optimal trade-off problem.

In this paper we shall use as a measure of financial risk the map $f_2(\mathbf{x}) = E[(\tau - \xi^T \mathbf{x})_+]$. This risk measure is called the first partial lower moment of the return.

The minimum risk problem

In the frame of this problem the decision maker looks for a portfolio $\mathbf{x} = (x_1, x_2, \dots, x_n)$ that minimizes the financial risk and satisfies the following constraints: the expected return is greater than a given limit W and the sum invested in the portfolio of fish is equal to M . One can easily see that the risk occurs when the return is smaller than the target τ .

$$(P_1) \begin{cases} \min E[(\tau - \xi^T \mathbf{x})_+] \\ \mu^T \mathbf{x} \geq W \\ \mathbf{b}^T \mathbf{x} = M \\ \mathbf{0} \leq \mathbf{x} \leq \mathbf{a} \end{cases}$$

From the last two constraints one can easily see that $\mathbf{0} \leq M \leq \mathbf{b}^T \mathbf{a}$.

Denote by r_{it} the market price of one kg of fish of species S_i at moment t . Suppose that there are m moments of time, that is $t \in \{1, 2, \dots, m\}$. Consider the vectors $\mathbf{r}_t = (r_{t1}, r_{t2}, \dots, r_{tn})$, $t \in \{1, 2, \dots, m\}$ and the matrix $\mathbf{R} = (r_{it})$. For every $i \in \{1, 2, \dots, n\}$ denote

$v_i = \frac{1}{m} \sum_{t=1}^m r_{it}$ and $\mathbf{v} = (v_1, v_2, \dots, v_n)$. Note that

$\frac{1}{m} \sum_{t=1}^m (\tau - \mathbf{r}_t^T \mathbf{x})_+$ is an estimation of the risk

function $E[(\tau - \xi^T \mathbf{x})_+]$ and $\mathbf{v} = (v_1, v_2, \dots, v_n)$ is an estimation of the vector $\boldsymbol{\mu} = (\mu_1, \mu_2, \dots, \mu_n)$.

Consider the problem

$$(P'_1) \begin{cases} \min \left[\frac{1}{m} \sum_{t=1}^m (\tau - \mathbf{r}_t^T \mathbf{x})_+ \right] \\ \mathbf{v}^T \mathbf{x} \geq W \\ \mathbf{b}^T \mathbf{x} = M \\ \mathbf{0} \leq \mathbf{x} \leq \mathbf{a} \end{cases}$$

One can easily see that the problem (P'_1) is the problem (P_1) where the risk function and the expected return are replaced with estimation of them. From now on we shall focus our attention on the problem (P'_1) .

An important problem is to find the range of the parameter W . In order to find the limits of this range we shall solve the following problems

$$(W_{\min}) \min \{ \mathbf{v}^T \mathbf{x} : \mathbf{b}^T \mathbf{x} = M, \mathbf{0} \leq \mathbf{x} \leq \mathbf{a} \}$$

$$(W_{\max}) \max \{ \mathbf{v}^T \mathbf{x} : \mathbf{b}^T \mathbf{x} = M, \mathbf{0} \leq \mathbf{x} \leq \mathbf{a} \}$$

Denote the optimal value of the problem (W_{\min}) with W_1 and the optimal value of the problem (W_{\max}) with W_2 . The decision maker may choose the parameter W in the range $[W_1, W_2]$. Let $W \in [W_1, W_2]$. For every $t \in \{1, 2, \dots, m\}$ define the numbers

$$s'_t = \min \{ \mathbf{r}_t^T \mathbf{x} : \mathbf{v}^T \mathbf{x} \geq W, \mathbf{b}^T \mathbf{x} = M, \mathbf{0} \leq \mathbf{x} \leq \mathbf{a} \}$$

$$s''_t = \max \{ \mathbf{r}_t^T \mathbf{x} : \mathbf{v}^T \mathbf{x} \geq W, \mathbf{b}^T \mathbf{x} = M, \mathbf{0} \leq \mathbf{x} \leq \mathbf{a} \}$$

The range of variation for the parameter τ is $[\tau_1, \tau_2]$ where $\tau_1 = \min \{ s'_t : t \in \{1, 2, \dots, m\} \}$ and $\tau_2 = \max \{ s''_t : t \in \{1, 2, \dots, m\} \}$.

By introducing m additional variables y_1, y_2, \dots, y_m one can build a linear programming problem (P''_1) which is equivalent with the problem (P'_1) .

$$(P_1'') \begin{cases} \min \left[\sum_{t=1}^m y_t \right] \\ \tau - \mathbf{r}_t^T \mathbf{x} \leq y_t, t=1,2,\dots,m \\ \mathbf{v}^T \mathbf{x} \geq W \\ \mathbf{b}^T \mathbf{x} = M \\ \mathbf{0} \leq \mathbf{x} \leq \mathbf{a} \\ y_t \geq 0, t=1,2,\dots,m \end{cases}$$

Consider the m dimensional vectors $\mathbf{e}=(1,1,\dots,1)$ and $\mathbf{y}=(y_1, y_2, \dots, y_m)$. Then the problem (P_1'') may be written in the vectorial form as follows:

$$(P_1''') \begin{cases} \min [\mathbf{e}^T \mathbf{y}] \\ \tau \mathbf{e} - \mathbf{R} \mathbf{x} \leq \mathbf{y} \\ \mathbf{v}^T \mathbf{x} \geq W \\ \mathbf{b}^T \mathbf{x} = M \\ \mathbf{0} \leq \mathbf{x} \leq \mathbf{a}, \mathbf{y} \geq \mathbf{0} \end{cases}$$

The maximum expected return problem

In the frame of this problem the decision maker looks for a portfolio $\mathbf{x}=(x_1, x_2, \dots, x_n)$ that maximizes the expected return and satisfies the following constraints: the risk is smaller than a given limit ρ and the sum invested in the portfolio of fish is equal to M .

$$(P_2) \begin{cases} \max \mu^T \mathbf{x} \\ E[(\tau - \xi^T \mathbf{x})_+] \leq \rho \\ \mathbf{b}^T \mathbf{x} = M \\ \mathbf{0} \leq \mathbf{x} \leq \mathbf{a} \end{cases}$$

As in the minimum risk problem one can define a new problem (P_2') where the risk function and the expected return are replaced with some of their estimations.

$$(P_2') \begin{cases} \max [\mathbf{v}^T \mathbf{x}] \\ \frac{1}{m} \sum_{t=1}^m (\tau - \mathbf{r}_t^T \mathbf{x})_+ \leq \rho \\ \mathbf{b}^T \mathbf{x} = M \\ \mathbf{0} \leq \mathbf{x} \leq \mathbf{a} \end{cases}$$

For every $t \in \{1,2,\dots,m\}$ define the numbers

$$s_t' = \min \{ \mathbf{r}_t^T \mathbf{x} : \mathbf{b}^T \mathbf{x} = M, \mathbf{0} \leq \mathbf{x} \leq \mathbf{a} \}$$

$$s_t'' = \max \{ \mathbf{r}_t^T \mathbf{x} : \mathbf{b}^T \mathbf{x} = M, \mathbf{0} \leq \mathbf{x} \leq \mathbf{a} \}$$

The range of variation for the parameter τ is $[\tau_1, \tau_2]$ where $\tau_1 = \min \{ s_t' : t \in \{1,2,\dots,m\} \}$ and $\tau_2 = \max \{ s_t'' : t \in \{1,2,\dots,m\} \}$.

An important problem is to find the range of the parameter ρ . In order to find the limits of this range we shall solve the following problems

$$(\rho \min) \min \left\{ \frac{1}{m} \sum_{t=1}^m (\tau - \mathbf{r}_t^T \mathbf{x})_+ : \mathbf{x} \in X \right\}$$

$$(\rho \max) \max \left\{ \frac{1}{m} \sum_{t=1}^m (\tau - \mathbf{r}_t^T \mathbf{x})_+ : \mathbf{x} \in X \right\}$$

Here we denoted $X = \{ \mathbf{b}^T \mathbf{x} = M, \mathbf{0} \leq \mathbf{x} \leq \mathbf{a} \}$

Denote the optimal value of the problem $(\rho \min)$ with ρ_1 and the optimal value of the problem $(\rho \max)$ with ρ_2 . The decision maker may choose the parameter ρ in the range $[\rho_1, \rho_2]$.

In the case the risk map $R(\mathbf{x})$ is equal to $f_3(\mathbf{x})$ then the maximum expected return problem is called the chance constrained problem.

The optimal trade-off problem

For every $\alpha \in [0,1]$ consider the problem

$$P(\alpha): \min \{ (1-\alpha) E[(\tau - \xi^T \mathbf{x})_+] - \alpha \mu^T \mathbf{x} : \mathbf{x} \in X \}$$

Here we denoted $X = \{ \mathbf{b}^T \mathbf{x} = M, \mathbf{0} \leq \mathbf{x} \leq \mathbf{a} \}$

As in the minimum risk problem one can define a new problem $(P'(\alpha))$ where the risk function and the expected return are replaced with some of their estimations.

$$P'(\alpha): \min \left\{ (1-\alpha) \frac{1}{m} \sum_{t=1}^m (\tau - \mathbf{r}_t^T \mathbf{x})_+ - \alpha \mathbf{v}^T \mathbf{x} : \mathbf{x} \in X \right\}$$

Every optimal portfolio of the above problem is called an efficient portfolio.

5. Numerical Results

Recall the input data, the user parameters and the decision variables for the minimum risk model:

Input data:

- n is the number of fish species
- m is the number of moments for which are gathered historical data

- The matrix $\mathbf{R}=(r_{ii})$. r_{ii} is the market price of one kg of fish of species S_i at moment t .
- The vector $\mathbf{a}=(a_1, a_2, \dots, a_n)$ of maximum sustainable yield for the fish of species
- The vector $\mathbf{b}=(b_1, b_2, \dots, b_n)$ of the fishing costs (or input costs) of fish from species

User parameters

- The sum M invested in the fishing plan. The range of M is the interval $[0, \mathbf{b}^T \mathbf{a}]$.
- The lower limit for the expected return of the fishing plan W . The range of W is the interval $[W_1, W_2]$.
- The return target τ . The range of variation for τ is the interval $[\tau_1, \tau_2]$.

Decision variables

- The fishing plan or the portfolio $\mathbf{x}=(x_1, x_2, \dots, x_n)$

In this section we shall study an application of the minimum risk model for a fish farm, called Malina, located near the villages Sendreni and Smardan, Galati county, Romania. The fishery area is 131 ha and 127 ha of it are covered by water. We study 6 fish species: Carp, Sanger, Novac, Perch, Catfish, Crucian. The historical data on the fish market prices are from the period 2000-2008. They are displayed in Table 2. The sum invested in the harvest plan is $M=5000$ RON. As a result of computation we find $W_1 = 4141.66$ RON, $W_2 = 4394.2$ RON.

Table 1. The maximum sustainable yield (MSY) and the harvesting cost for the fish species

Fish Type	MSY in kg	Harvesting cost in RON/Kg
Carp	200	8.455
Sanger	300	3.680
Novac	400	3.680
Perch	100	7.500
Catfish	100	8.000
Crucian	500	2.680

In the second (resp. third) column from Table 1 is displayed the vector \mathbf{a} (resp. the vector \mathbf{b}).

In Table 2 are displayed the historical data on the market prices for the fish species regarding the period 2000-2008. The prices are measured in RON/kg. Table 2 can be considered as the transpose of the matrix $\mathbf{R}=(r_{ii})$ of historical market prices.

In Table 3 (resp. in Table 4) are displayed on the columns the optimal portfolios of harvested fish for a fixed value of W , $W=4141.66$, (resp. $W=4192.17$) and various values of parameter τ .

Table 2. Historical data on the market prices for the fish species (in Ron/Kg)

Fish Type	2000	2001	2002	2003	2004	2005	2006	2007	2008
Carp	6.0	6.0	6.5	7.0	7.2	7.5	8	8.5	9.0
Sanger	2.0	2.25	2.5	3.0	3.2	3.2	3.5	3.8	4.0
Novac	2.2	2.5	2.8	3.2	3.2	3.5	3.5	3.8	4.0
Perch	4.5	5.0	5.5	6.0	6.5	7.0	7.5	7.8	8.0
Catfish	6.5	7.0	7.0	7.5	7.8	8.0	8.0	9.0	9.5
Crucian	1.5	1.6	1.8	2.0	2.0	2.25	2.5	3.0	3.2

Table 3. The fish harvesting portfolios for $W=4141.66$ and various values of the parameter τ

	$W=4141.66$			
Carp	103.4	103.4	200.0	200.0
Sanger	300.0	300.0	77.9	77.9
Novac	400.0	400.0	400.0	400.0
Perch	100.0	100.0	100.0	100.0
Catfish	100.0	100.0	100.0	100.0
Crucian	0.0	0.0	0.0	0.0
$\tau =$	2853.2	3120.8	3388.5	3656.2

Table 4. The fish harvesting portfolios for $W=4192.17$ and various values of the parameter τ

	$W=4192.17$			
Carp	200.0	200.0	200.0	200.0
Sanger	295.4	249.0	77.9	77.9
Novac	400.0	400.0	400.0	400.0
Perch	100.0	100.0	100.0	100.0
Catfish	0.0	21.3	100.0	100.0
Crucian	0.0	0.0	0.0	0.0
$\tau =$	2904.1	3166.7	3429.3	3691.9

In Figure 1 (resp. Figure 2) is displayed the graph of the efficient frontier map associated to the minimum risk problem for $W=4141.66$, (resp. $W=4192.17$). That is to every value of

the parameter τ corresponds the minimum value of the objective map (the lower partial moment of the return).

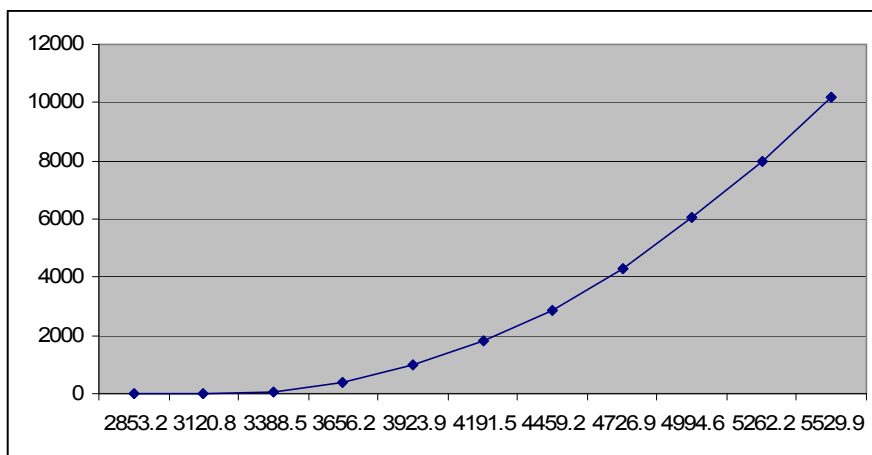


Figure 1. The efficient frontier map associated to the minimum risk problem for $W=4141.66$

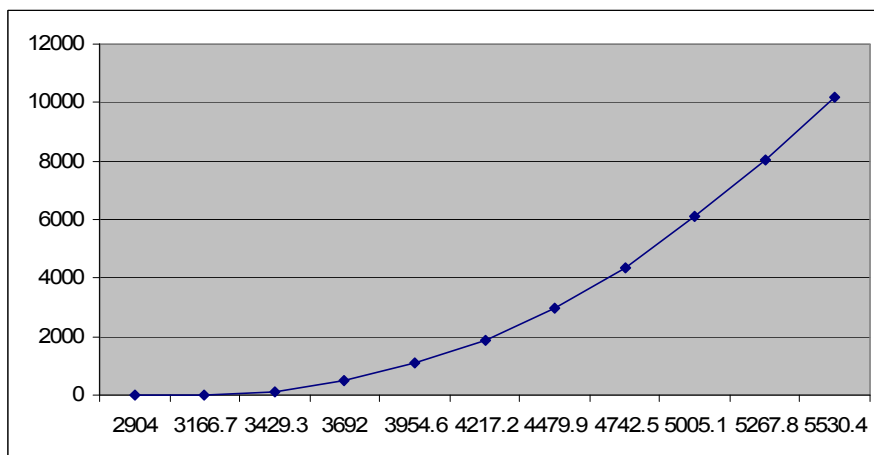


Figure 2. The efficient frontier map associated to the minimum risk problem for $W=4192.17$

In table 5 for each value of the parameters W and τ is displayed Fob, that is, the minimum value of the objective map. In Figure 3 is displayed the 3-D efficient frontier of the minimum risk problem. That is to every value of W and τ corresponds the minimum value of the objective map (the lower partial moment of the return). The minimum risk problem was solved for various values of the parameters W and τ with the MINOS solver,

type NLP, from GAMS. The limits of the range of variation of the parameter W were obtained with the DICOPT solver, type MINLP from GAMS. Alternative solutions of the minimum risk problem may be found following the approach from Andrei [1].

In Figure 3 along the x-axis varies the parameter W , along the y-axis varies parameter τ and along the vertical z-axis varies the optimal value of the objective map.

Table 5. The values of the risk map versus the values of W and τ

$W=4141.66$		$W=4192.17$		$W=4242.68$		$W=4293.19$		$W=4343.7$		$W=4394.2$	
τ	Fob	τ	Fob	τ	Fob	τ	Fob	τ	Fob	τ	Fob
2853.6	0	2904	0	2974.9	0	3049	0	3150.4	0	3323.7	0
3120.8	0	3166.7	0	3230.5	0	3294.3	0	3382.7	46.7	3535.6	199.6
3388.5	52.5	3429.3	93.3	3486	150.0	3539.5	203.5	3614.9	318.4	3747.5	583.5
3656.2	400.9	3691.9	472.5	3741.5	571.6	3784.8	658	3847.2	782.9	3959.3	1101.6
3923.9	995.1	3954.6	1087.3	3997.1	1214.8	4030	1313.6	4079.5	1461.9	4171.2	1737.2
4191.5	1798.1	4217.2	1875.2	4252.6	1981.4	4275.3	2060.6	4311.7	2206.5	4383	2491.9
4459.2	2856	4479.9	2959.3	4252.6	3049.3	4520.5	3162.6	4543.9	3279.9	4594.9	3534.8
4726.9	4271.7	4742.5	4365.5	4763.7	4492.4	4765.8	4504.9	4776.2	4567.8	4806.8	4751.4
4994.6	6049.3	5005.1	6123.4	5019.2	6221.9	5011	6164.5	5008.5	6146.9	5018.7	6218.4
5262.2	7988.9	5267.8	8033.3	5274.7	8088.9	5256.2	7941.1	5240.7	7817	5230.57	7735.7
5529.9	10198.3	5530.4	10202.8	5530.3	10201.5	5501.5	9942.6	5473	9686.1	5442.4	9430.7

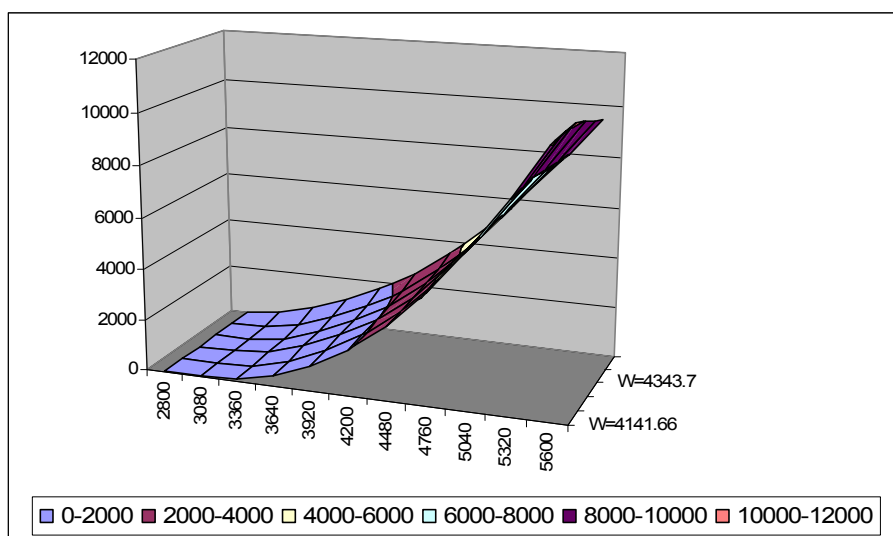


Figure 3. The 3-D efficient frontier of the minimum risk problem

6. Conclusions

We presented a multi-objective portfolio selection model for fisheries management. Starting from it we defined several single objective programming problems: a minimum risk problem, a maximum expected return problem and an optimal trade-off problem. In order to facilitate the use of these models in a decision support system we gave procedures for finding the ranges of variation for the parameters. We applied the theoretical results obtained for the minimum risk model for a fish farm from Galati county. The numerical results confirm that the minimum model is a very useful tool for the management of fish farms. Several extensions of the model may be done. For example one can consider several ponds in the fish farm or one can consider the environmental impact of the fish farm and look for management solutions that meet at the same time the economic and the environmental objectives.

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