

Sampled Tracking for Delayed Systems Using Two-Time-Scale Sampled-data Controllers

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Abstract: This article deals with the trajectory tracking of linear plants with sampled and delayed outputs. A class of sampled-data controllers with two time-scales is proposed which allows sampled tracking with a delay twice as much as that of the plant output. Numerical simulation results are presented to demonstrate the effectiveness of the proposed approach.

Keywords: Sampled tracking, sampled-data controllers, two time-scale systems, optimal control, Luenberger discrete-time observer.

1. Introduction

During the last two decades, two-time-scale sampled-data control of continuous-time plants has been studied in several papers dealing with both its theory and applications [1]-[5]. For instance, a sampled-data control scheme which control actions are scheduled at two different sampling rates (slow and fast) is suggested in [1]. This sampled-data control is in composite form and is computed as the sum of the slow and fast control signals. A two-time-scale digital controller is derived by using the singular perturbation theory and is applied for motor position servoing [2]. Recently, a robust two-time-scale control based on the singular perturbation method and time delay control is proposed for a pneumatic vibration isolator [4]. A decentralised two-time-scale motions control of linear time-invariant plants with unstable decentralised fixed modes (UDFM) is designed in [5]. The method used generalizes the sampled-data hold function introduced by Kabamba [6] to eliminate UDFM and to decouple the discrete-time plant model into independent input-output channels.

An alternative approach for two-time-scale controller design is proposed in [3] using the theory of the sampled-data systems with piecewise functioning (SPF) [3]. The developed controller, however, needs full

information for the plant state which limits its practical application.

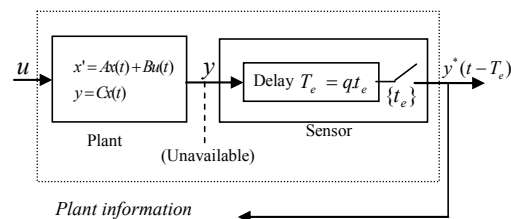


Figure 1. System to be controlled

In this paper we deal with the case when the only available plant information is delivered from the plant output via a digital sensor introducing a delay T_e corresponding to the time needed to process the information (see Figure 1). Further on we consider that $T_e = qt_e$, where t_e is the sampling period. Such systems are frequently encountered in industry when digital technology is used for measurement (e.g. camera) and/or control [8]. Their dynamics can be described as

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (1a)$$

$$y = Cx(t) \quad (1b)$$

$$y^*(t - T_e) = y^*(t - qt_e) \quad (1c)$$

where $A \in \mathfrak{R}^{n \times n}$, $B \in \mathfrak{R}^{n \times r}$ and $C \in \mathfrak{R}^{m \times n}$ are constant matrices and * represents sampling operation with constant period t_e .

To ensure efficient tracking control of the considered systems, we propose a new type

of SPF based two-time-scale sampled-data controllers (SDC) which use sampled and delayed plant output measurements.

The paper is organized as follows. In Section 2 the SPF theory and the existing full state information SDC are briefly presented. In Section 3 we develop SDC which use delayed state measurements and in Section 4 we generalize these controllers for the case when only delayed plant output measurements are available. The performance analysis of the new class of SDC by numerical simulation is presented in Section 5. Finally, in Section 6 some concluding remarks are given.

2. Sampled-data Systems and Controllers with Piecewise Functioning

The basic concept in SPF theory is the two-time scale illustrated in Figure 2 [3].

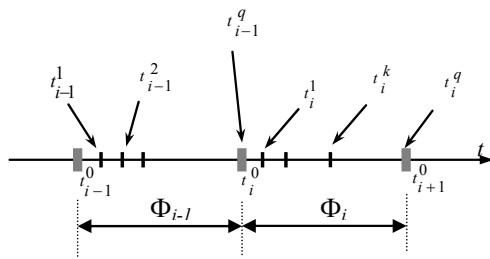


Figure 2. Sampled-data two-time scale

The discrete instants $t_i^k = iT_e + kt_e$ are characterized by the indices:

i - indicating the time scale relative to switching instants iT_e , $i = 0, 1, 2, \dots$, and

k - specifying the time scale relative to the system evolution between two switching instants.

Two successive switching instants t_i^0 and t_{i+1}^0 delimit an interval noted Φ_i . In Φ_i the index i does not change while k increases from 0 to q . One has $t_{i-1}^q = t_i^0$.

A linear sampled-data system with piecewise functioning is a system whose dynamics can be described over all intervals Φ_i as

$$x_i^0 = B_s v_i^0 \quad (2a)$$

$$x_i^{k+1} = A_d x_i^k + B_d u_i^k, \quad k = 0, 1, \dots, q-1 \quad (2b)$$

$$y_i^k = C x_i^k \quad (2c)$$

where $x_i^k \in \mathfrak{R}^n$ is the state vector, $u_i^k \in \mathfrak{R}^r$ is the control vector in Φ_i , $v_i^0 \in \mathfrak{R}^s$ is the control vector at the switching instant t_i^0 and $y_i^k \in \mathfrak{R}^m$ is the output vector.

Denote the time t in the interval Φ_i as $\tau + iT_e$, where $\tau \in [0, T_e]$. Thus we have

$$x(t) = x(iT_e + \tau) = x_i(\tau) \quad (3)$$

$$u(t) = u_i(\tau) \quad (4)$$

and in Φ_i the system (1) can be written as

$$\dot{x}_i(\tau) = A x_i(\tau) + B u_i(\tau) \quad (5a)$$

$$y_i(\tau) = C x_i(\tau) \quad (5b)$$

$$y^*(t - T_e) = y_{i-1}(\tau) = C x_{i-1}(\tau). \quad (5c)$$

Discretizing (5) we obtain the SPF

$$x_i^{k+1} = A_d x_i^k + B_d u_i^k, \quad k = 0, 1, \dots, q-1 \quad (6a)$$

$$y_i^k = C x_i^k \quad (6b)$$

$$y_{i-1}^k = C x_{i-1}^k \quad (6c)$$

with

$$A_d = e^{A T_e}, \quad B_d = \int_0^{T_e} e^{A(t_e - \tau)} B d\tau \quad \text{and} \quad v_i^k = x_i^k.$$

Based on the SPF theory, a two-time-scale sampled-data controller (SDC) has been developed for plants with full state information realizing state trajectory tracking with one switching period of delay [3]. On the interval Φ_i , the SDC is defined as

$$\lambda_i^0 = \beta_s \psi_i^0 \quad (7a)$$

$$\lambda_i^{k+1} = \alpha \lambda_i^k + \beta a_i^k, \quad k = 0, 1, \dots, q-1 \quad (7b)$$

$$u_i^k = \gamma \lambda_i^{k+1} \quad (7c)$$

where $\lambda_i^k \in \mathfrak{R}^n$ and $u_i^k \in \mathfrak{R}^r$ denote respectively the SDC state and the output at the instant t_i^k , $\psi_i^0 \in \mathfrak{R}^s$ is the input at switching instants, and $\alpha \in \mathfrak{R}^{n \times n}$, $\beta \in \mathfrak{R}^{n \times r}$, $\beta_s \in \mathfrak{R}^{n \times s}$ and $\gamma \in \mathfrak{R}^{r \times n}$ are constant matrices with appropriate dimensions. The matrix α is chosen so as to ensure the stability of SDC. For simplified SDC β is a zero matrix and γ is an arbitrary matrix. The SDC representation is given in Figure 3a and Figure 3b. Generally, $\lambda_i^0 \neq \lambda_{i-1}^q$ which implies discontinuity at switching instants, as illustrated in Figure 3c.

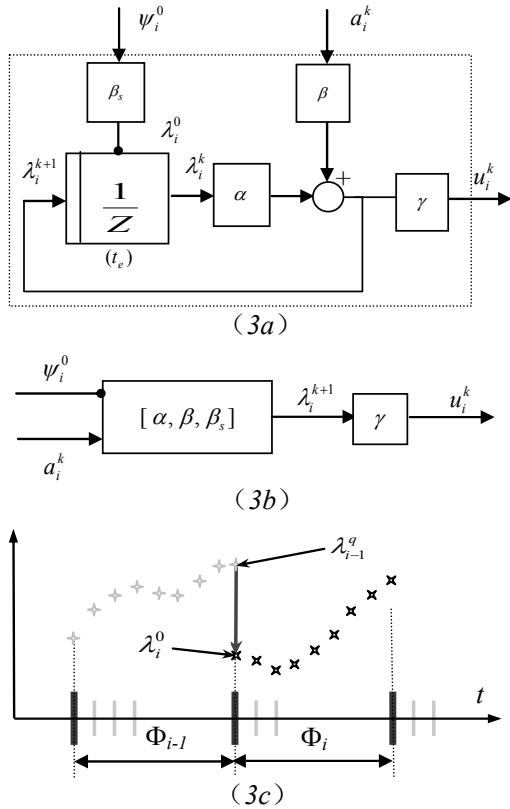


Figure 3. Two-time-scale SDC

- Detailed representation
- Symbolic representation
- Evolution of the controller state

Denote by $c(t)$ the desired plant state trajectory. The tuning of simplified SDC consists of determining ψ_i^0 and β_s in order to ensure sampled trajectory tracking with one switching period of delay:

$$x_{i+1}^0 = c_i^0 \quad \forall i = 0, 1, 2, \dots \quad (8)$$

From (6) and (7) one obtains

$$x_i^q = A_d^q x_i^0 + M \lambda_i^0 \quad (9)$$

with

$$M = \begin{bmatrix} A_d^{q-1} B_d, A_d^{q-2} B_d, \dots, A_d^0 B_d \\ \gamma \alpha^0 \\ \gamma \alpha^1 \\ \vdots \\ \gamma \alpha^{q-1} \end{bmatrix}. \quad (10)$$

Taking into account (8) one has

$$\beta_s \psi_i^0 = M^{-1} [c_i^0 - A_d^q x_i^0] \quad (11)$$

and thus

$$\beta_s = M^{-1}, \quad \psi_i^0 = c_i^0 - A_d^q x_i^0. \quad (12)$$

The SDC performances can be improved minimizing the oscillation effects as shown in [11], [3]. Define the cost function

$$J = \frac{1}{2} \sum_{k=0}^{q-1} \left[(c_{i-1}^k - x_i^k)^T Q (c_{i-1}^k - x_i^k) + u_i^{kT} R u_i^k \right] \quad (13)$$

where $Q \in \mathfrak{R}^{n \times n}$ and $R \in \mathfrak{R}^{r \times r}$ are positive definite matrices. The minimization of J ensures the reduction of intersample ripples and the moderation of control magnitude. Denote the Hamiltonian of the optimal control problem (6a), (13) by

$$H = -\frac{1}{2} \sum_{k=0}^{q-1} \left[(c_{i-1}^k - x_i^k)^T Q (c_{i-1}^k - x_i^k) + u_i^{kT} R u_i^k \right] + \sum_{k=0}^{q-1} l_i^{(k+1)T} [A_d x_i^k + B_d u_i^k] \quad (14)$$

where $l_i^k \in \mathfrak{R}^n$ is the Lagrange multiplier. Using the Pontryagin maximum principle one obtains the optimal control law

$$l_i^{k+1} = A_d^{-T} l_i^k - A_d^{-T} Q (c_{i-1}^k - x_i^k) \quad (15)$$

$$u_i^k = R^{-1} B_d^T l_i^{k+1} \quad (16)$$

which can be realized by a SDC (7) with

$$\lambda_i^k = l_i^k, \quad \alpha = A_d^{-T}, \quad \beta = A_d^{-T} Q, \quad \gamma = R^{-1} A_d^{-T}. \quad (17)$$

In this case

$$\beta_s = \Theta_{12}^{-1} \quad (18)$$

$$\psi_i^0 = [c_i^0 - \Theta_{11} x_i^0 - I_h] \quad (19)$$

where

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{21} & \Theta_{22} \end{bmatrix} = \begin{bmatrix} A_d - B_d R^{-1} B_d^T A_d^{-T} Q & B_d R^{-1} B_d^T A_d^{-T} \\ -A_d^{-T} Q & -A_d^{-T} \end{bmatrix}^q \quad (20)$$

$$\begin{bmatrix} I_h \\ I_b \end{bmatrix} = \begin{bmatrix} H^{q-1} K & H^{q-2} K & \dots & H^0 K \end{bmatrix} \begin{bmatrix} c_{i-1}^0 \\ c_{i-1}^1 \\ \vdots \\ c_{i-1}^{q-1} \end{bmatrix} \quad (21)$$

and

$$H = \begin{bmatrix} A_d + B_d R^{-1} B_d^T A_d^{-T} Q & B_d R^{-1} B_d^T A_d^{-T} \\ A_d^{-T} Q & A_d^{-T} \end{bmatrix} \quad (22)$$

$$K = \begin{bmatrix} -B_d R^{-1} B_d^T A_d^{-T} Q \\ -A_d^{-T} Q \end{bmatrix}. \quad (23)$$

In the next two sections we shall generalize the presented full state information SDC for the cases of delayed state and output plant information.

3. Sampled Tracking with Delayed State Feedback

In this section, we assume that for system (6) delayed state measurements x_{i-1}^k are only available. To solve the sampled tracking problem in this case we propose a new class of SDC incorporating an estimator of x_i^k based on the measurements x_{i-1}^k and the control signals u_i^k .

Using the system equation (6a) the state vector x_i^k can be estimated as

$$x_i^k = A_d^q x_{i-1}^k + \begin{bmatrix} A_d^{q-1} B_d & A_d^{q-2} B_d & \dots & A_d^0 B_d \end{bmatrix} \begin{bmatrix} u_i^{k-q} \\ u_i^{k-q+1} \\ \vdots \\ u_i^{k-1} \end{bmatrix} \quad (24)$$

Thus the sampled tracking with delayed state information can be realized by a controller consisting of a serial connexion of the estimator (24) and the standard full state information SDC (7).

When simplified full state information SDC is used, the controller parameters are determined from (12) and the corresponding controller structure is represented in Figure 4.

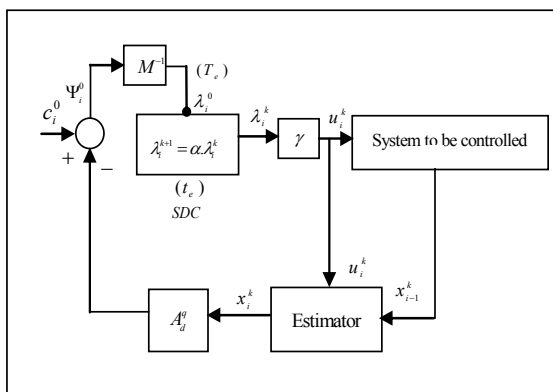


Figure 4. Simplified SDC with delayed state information

The controller performances can be optimized using the optimal full state information SDC presented in Section 2. In this case the controller parameters are determined from (17)-(24). The optimal

controller structure is represented in Figure 5, where $\Sigma(\cdot)$ is defined by equations (21)-(23).

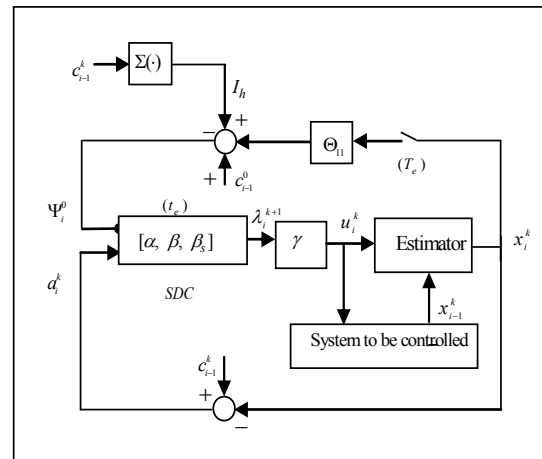


Figure 5. Optimal SDC with delayed state information

4. Sampled Tracking with Delayed Output Feedback

Consider now the general sampled tracking problem when delayed plant output measurements y_{i-1}^k are only available. This problem can be solved incorporating an observer in the SDC developed in Section 3 in order to estimate the delayed plant state x_{i-1}^k based on y_{i-1}^k .

The x_{i-1}^k estimation can be realized by using a discrete reduced-order Luenberger observer [12] with inputs y_{i-1}^k and u_i^k . Rewrite the equations (6) of the system to be controlled as

$$x_i^{k+1} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{21} \end{bmatrix} \begin{bmatrix} x_{1,i}^k \\ x_{2,i}^k \end{bmatrix} + \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} u_i^k \quad (26)$$

$$y_{i-1}^k = C x_{i-1}^k = x_{1,i-1}^k \quad (27)$$

where $x_{1,i}^k \in \mathfrak{R}^m$ and $x_{2,i}^k \in \mathfrak{R}^{n-m}$. The vector $x_{2,i-1}^k$ can be estimated by the reduced-order Luenberger observer

$$z_i^{k+1} = F z_i^k + G x_{1,i}^k + H u_i^k \quad (28a)$$

$$z_i^k = \hat{x}_{2,i}^k - L x_{1,i}^k \quad (28b)$$

$$\hat{x}_{2,i}^k = z_i^k + L x_{1,i}^k \quad (28c)$$

where

$$F = (f_{22} - L f_{12}) \quad (29a)$$

$$G = (f_{22} - L f_{12}) L + (f_{21} - L f_{11}) \quad (29b)$$

$$H = (f_{22} - L f_{12}) \quad (29c)$$

and the matrix $L \in \mathfrak{R}^{(n-m) \times m}$ is chosen in (29a) so that the matrix F to have zero spectrum. Thus we obtain the estimate

$$\hat{x}_{i-1}^k = \begin{bmatrix} x_{1,i-1}^k & \hat{x}_{2,i-1}^k \end{bmatrix}^T.$$

Replacing x_{i-1}^k by \hat{x}_{i-1}^k in (24) we can realize an output trajectory tracking with two switching period of delay:

$$y_{i+2}^0 = Cc_i^0, \quad \forall i = 0, 1, 2, \dots$$

For simplified SDC with delayed plant output information, the controller parameters are determined as

$$\beta^d = M^{-1}, \quad \psi_i^0 = [c_i^0 - A_d^q \hat{x}_i^0].$$

We can also replace x_i^k by \hat{x}_i^k in (15) and (19) obtaining thus an optimal SDC with delayed plant output information, which parameters are

$$\begin{aligned} \alpha &= A_d^{-T} \\ \beta &= A_d^{-T} Q \\ \gamma &= R^{-1} A_d^{-T} \\ \beta_i^d &= \Theta_{12}^{-1} \\ a_i^k &= c_{i-1}^k - \hat{x}_i^k \\ \psi_{i+1}^0 &= c_i^0 - \Theta_{11} \hat{x}_i^0 - I_n. \end{aligned} \quad (30)$$

The corresponding controller structure is illustrated in Figure 6.

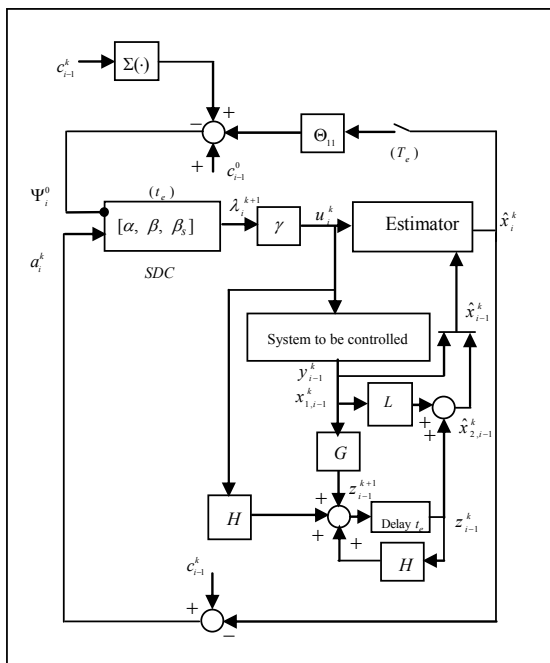


Figure 6. Optimal SDC with delayed plant output information

5. Numerical Simulations

The performances of the proposed SDC have been analysed by numerical simulation for the second order plant model

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (31)$$

$$y = Cx(t) \quad (32)$$

with $A = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$, $C = [1 \ 0]$, and

delayed output measurements

$$y^*(t - T_e) = y^*(t - qt_e). \quad (33)$$

The corresponding two-time-scale sampled-data system is

$$x_i^{k+1} = A_d x_i^k + B_d u_i^k, \quad k = 0, 1, \dots, q-1 \quad (33)$$

$$y_{i-1}^k = Cx_{i-1}^k. \quad (34)$$

The switching period $T_e = qt_e = 1s$ and $q = 2, 4, 20$; $t_e = 0.5, 0.25, 0.05$ have been chosen corresponding to following sampled-data system matrices:

1) $q = 2$, $t_e = 0.5$

$$A_d = \begin{bmatrix} 1.222 & 0.427 \\ 0.854 & 0.795 \end{bmatrix}, \quad B_d = \begin{bmatrix} 0.222 \\ 0.854 \end{bmatrix};$$

2) $q = 4$, $t_e = 0.25$

$$A_d = \begin{bmatrix} 1.058 & 0.226 \\ 0.452 & 0.832 \end{bmatrix}, \quad B_d = \begin{bmatrix} 0.058 \\ 0.452 \end{bmatrix};$$

3) $q = 20$, $t_e = 0.05$

$$A_d = \begin{bmatrix} 1.003 & 0.049 \\ 0.098 & 0.954 \end{bmatrix}, \quad B_d = \begin{bmatrix} 0.003 \\ 0.098 \end{bmatrix}.$$

The desired state trajectories has been defined as

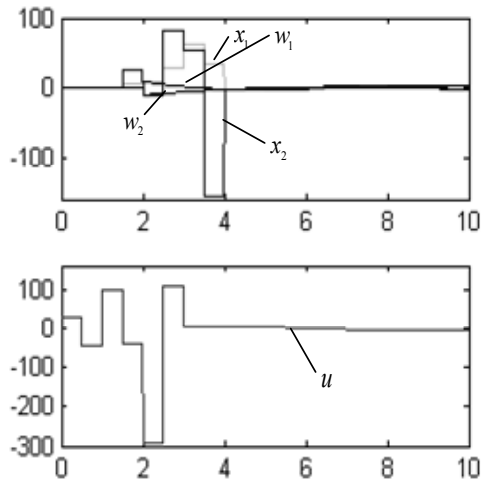
$$c(t) = \begin{bmatrix} c_1(t) \\ c_2(t) \end{bmatrix} = \begin{bmatrix} -0.17t^3 + 2.5t^2 - 10t + 10 \\ -0.51t^2 + 5 * t - 10 \end{bmatrix}.$$

First, sampled tracking with delayed state measurements has been simulated for simplified and optimal SDC. The simplified SDC parameters were determined from

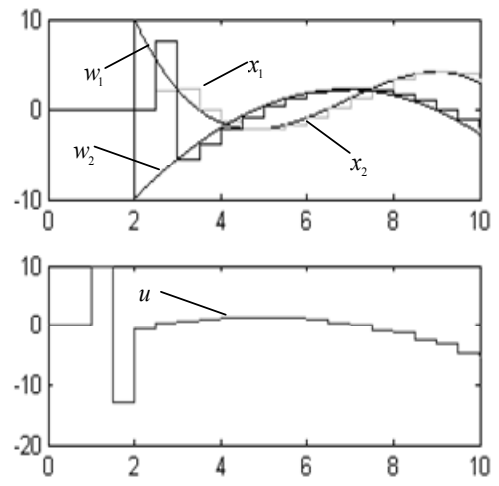
$$\alpha = \exp(\delta t_e), \quad \delta = \begin{bmatrix} -2 & 8 \\ -7 & 10 \end{bmatrix}$$

and $\gamma = [10 \ 10]$.

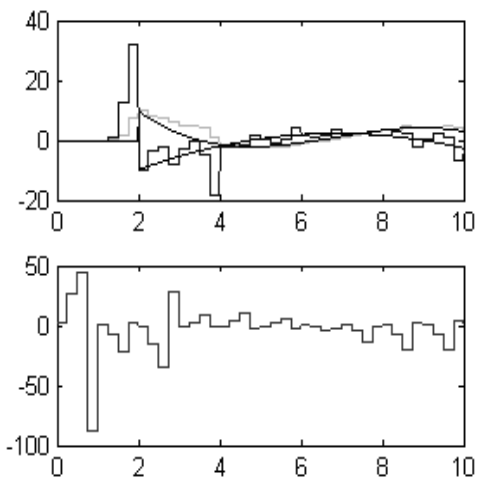
The results obtained are given in Figure 7.



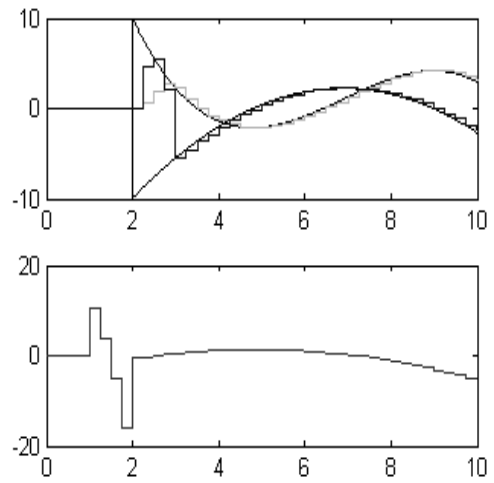
(a) $q = 2, t_e = 0.5 \text{ s}, T_e = 1 \text{ s}.$



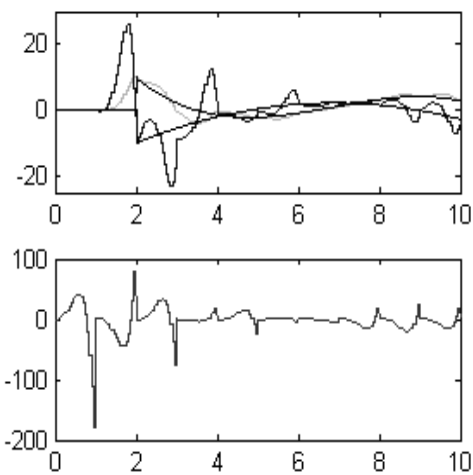
(a) $q = 2, t_e = 0.5 \text{ s}, T_e = 1 \text{ s}.$



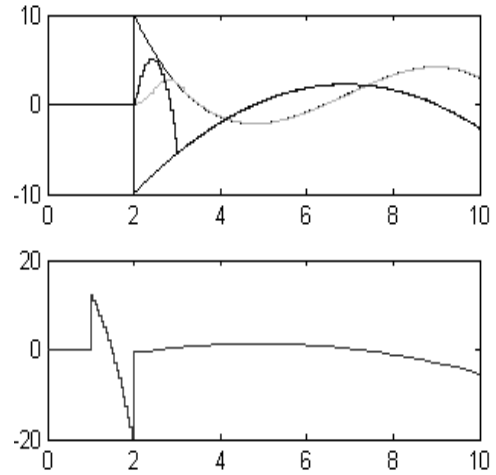
(b) $q = 4, t_e = 0.25 \text{ s}, T_e = 1 \text{ s}.$



(b) $q = 4, t_e = 0.25 \text{ s}, T_e = 1 \text{ s}.$



(c) $q = 20, t_e = 0.05 \text{ s}, T_e = 1 \text{ s}.$



(c) $q = 20, t_e = 0.05 \text{ s}, T_e = 1 \text{ s}.$

Figure 7. Optimal SDC with delayed state information

Figure 8. Optimal SDC with delayed plant state information

Note that to facilitate the tracking performance evaluation, in Figures 7 – 10 the trajectory

$$w(t) = \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix} = \begin{bmatrix} c_1(t-2T_e) \\ c_2(t-2T_e) \end{bmatrix}$$

is presented instead of $c(t)$.

For the optimal SDC with delayed state measurements, $Q = 0.01 I_2$ and $R = 100$ have been chosen and the controller parameters have been determined by (17)-(23). The corresponding simulation results are given in Figure 8.

Then sampled tracking with delayed plant output measurements has been simulated for the optimal SDC. The controller parameters have been determined from (30) and (29) for $Q = 0.01 I_2$ and $R = 100$. The simulation results are presented in Figure 9.

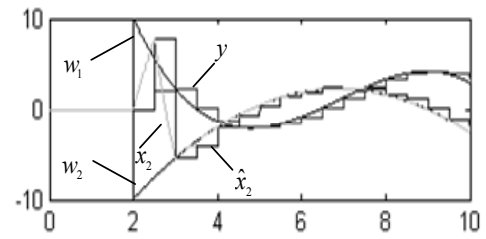
From Figure 7 and Figure 8 it can be seen that the delayed plant state x_{i-1}^k follows perfectly the desired state trajectory in switching instants with two switching periods of delay. In turn, Figure 9 shows that the delayed plant output y_{i-1}^k strictly follows the desired output trajectory $c_1(t)$ with two switching periods of delay. In the trajectory tracking using simplified SDC there exist oscillations between two switching instants except in the case where q is equal to the order of the system. In the trajectory tracking by optimal SDC both for delayed state and output measurements, these oscillations are significantly reduced. Figure 9 shows also that the observer state $\hat{x}_{2,i-1}^k$ tracks well the desired state trajectory $c_2(t)$ with two switching periods of delay.

Finally, to analyse the robustness of the optimal SDC with delayed output information, the parameter variations in the plant model have been realized as

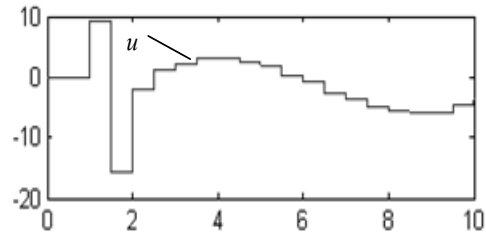
$$A = \begin{bmatrix} 0 & 1 \\ 2 + 0.1\sin(5t) & -1 + 0.2\sin(10t) \end{bmatrix}$$

and

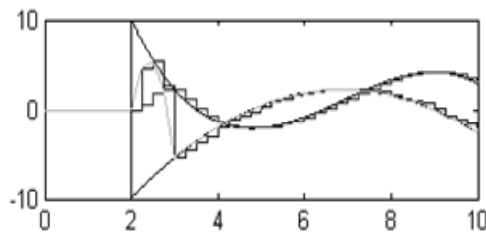
$$B = \begin{bmatrix} 0 \\ 2 + 0.1\sin(5t) \end{bmatrix}.$$



(a) $q = 2, t_e = 0.5 \text{ s}, T_e = 1 \text{ s}$.



(b) $q = 4, t_e = 0.25 \text{ s}, T_e = 1 \text{ s}$.



(c) $q = 20, t_e = 0.05 \text{ s}, T_e = 1 \text{ s}$.

Figure 9. Optimal SDC with delayed

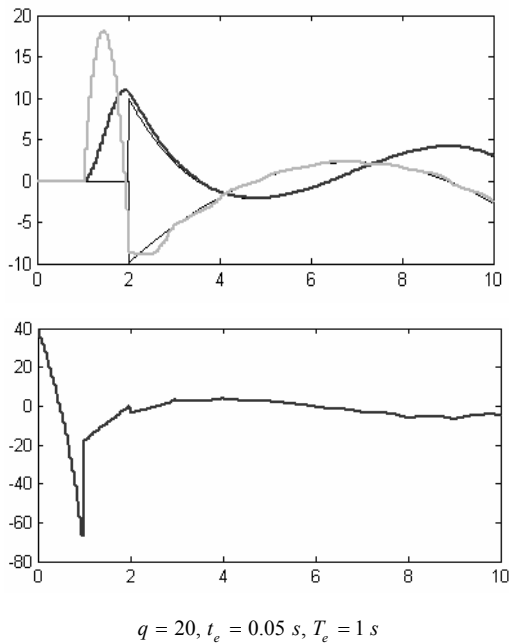


Figure 10. Optimal SDC with delayed output information in the case of system parameter variations

The corresponding trajectory tracking results, given in Figure 10, show the robustness and the efficiency of the proposed SDC.

6. Conclusions

In this paper, simplified and optimal two-time-scale sampled-data controllers are proposed for trajectory tracking control of linear plants with delayed state or output measurements. These controllers can achieve perfect trajectory tracking in switching instants with two switching periods of delay and are sufficiently robust against variations of the plant parameters. The proposed optimal controllers enable also to minimize the oscillations between switching instants.

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