Decentralized Formation Control of Multi-agent Robot Systems based on Formation Graphs

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Abstract: Formation control is an important issue of motion coordination of Multi-agent Robots Systems. The goal is to coordinate a group of agents to achieve a desired formation pattern. The control strategies are decentralized because every robot does not possess information about the positions and goals of all the other robots. Based on the formation graphs properties and the local potential functions approach, we obtain a formal result about global convergence to the desired pattern for any formation graph. Also, we characterize the topologies of the formation graphs where the centroid of positions remains stationary. Finally, the control approach is extended to the case of unicycle-type robots.

Keywords: Mobile robots, Decentralized control, Formation control, Graph theory, Unicycles.

1. Introduction

The term Multi-agent robots systems (MARS) means groups of autonomous robots coordinated to achieve cooperative tasks. Formation control is an important issue of motion coordination of MARS, specifically applied to groups of mobile wheeled robots. Applications include toxic residues cleaning, transportation and manipulation of large objects, exploration, searching and rescue tasks and simulation of biological entities behaviors [1]. The goal is to guarantee the convergence of the agents or robots to a particular formation pattern. The problem is complex because it is assumed that every robot does not possess global information. Therefore, the control strategies are decentralized and the main intention is to achieve desired global behaviors through local interactions [2].

Some advantages of decentralized approaches are greater autonomy for the robots, less computational load in control implementations and its applicability to large scale groups [3]. Decentralized formation control strategies includes behavior-based [4], [5], [6] swarms stability [7], virtual structures [3] and Local Potential Functions (LPF) [8], [9]. The LPF method consists of applying the negative gradient of a potential function as control inputs of agents. The LPF's are designed according to the desired inter-agent distances and steer all agents to the desired formation. Formation Graphs (FG), are an important tool to guarantee convergence to the desired pattern [10], [11], [12]. The application of different FG's to the same group of robots produces different dynamic behaviors of the group in the closed-loop system. For example, [13] analyze the convergence of the complete FG, where every robot measures the position of the rest of the group. The cyclic pursuit FG is studied in [2] where every robot pursues the next robot and the last robot pursues the first one making a closed-chain configuration. A FG with bidirectional communication in the cyclic pursuit is analyzed in [14]. An analysis of convergence of all undirected FG's is presented in [10] where the communication between pair of robots is bidirectional. The convergence of some leader-followers schemes is analyzed in [15] for the case of the FG centered on a virtual leader and [16] for the open-chain or convoy configuration. Another approaches of leader-followers schemes are found in [8], [17], [18]. Although the LPF and FG approaches are used commonly in the literature, there does not exist a general result about the convergence of the closed-loop system using an arbitrary formation graph. Inspired in [2], we analyze the convergence to the desired formation for any FG based on the Laplacian matrix of the FG and the Gershgorin circles Theorem [19]. Also, we analyze the conditions of the FG such that the centroid of positions remains constant for all time. To the best of our knowledge, the unique similar result is exposed in [2] for the cyclic pursuit FG only. The results originally were presented in [20] and selected for publication in this journal.

The paper is organized as follows. Section 2 introduces the problem statement and defines the notion of FG. Section 3 describes the formation control strategy based on LPF for the case of point-robots and the main result about the convergence to the desired formation. The analysis of the centroid of positions is given in Section 4. The approach is extended to the case of unicycle-type robots in Section 5, together with some numerical simulations. Finally, concluding remarks are offered in Section 6.

2. Problem Statement and Formation Graphs

Denote by $N = \{R_1, ..., R_n\}$, a set of n agents moving in plane with positions $z_i(t) = [x_i(t), y_i(t)]^T$, $i = 1, ..., n$. The kinematic model of each agent or robot R_i is described by

$$\dot{z}_i = u_i, \; i = 1, ..., n \tag{1}$$

where $u_i = [u_{i1}, u_{i2}]^T \in \Re^2$ is the velocity of $i-th$ robot along the X and Y axes. Let N_i denote the subset of positions of the robots which are detectable for R_i, where $N_i \neq \emptyset, i = 1, ..., n$. Let $c_{ij} = [h_{ij}, v_{ij}]^T$, $\forall j \in N_i$ denote a vector which represents the desired position of R_i with respect to R_j in a particular formation. Thus, we define the desired relative position of every R_i in the formation by

$$z_i^* = \varphi_i(N_i) = \frac{1}{n_i} \sum_{j \in N_i} (z_j + c_{ji}), \tag{2}$$

where n_i is the cardinality of N_i. Thus, the desired relative position of R_i can be considered as a combination of the desired positions of z_i with respect to the positions of all elements of N_i.

Problem Statement. The control goal is to design a control law $u_i(t) = f_i(N_i(t))$ for every robot R_i, such that

$$\lim_{t \to \infty} (z_i - z_i^*) = 0, \; i = 1, ..., n.$$

Remark 1. It is important to point out that the inter-agent collisions problem is not considered in this work. Some analysis of the non-collision for some FG's are presented in [10], [14] and [15].

According to [10], [11], [12], the desired formations of a group of agents can be represented by a FG defined by:

Definition 1. A Formation Graph $G = \{Q, E, C\}$ is a triplet that consists in (i) a set of vertices $Q = \{R_1, ..., R_n\}$ related to the team members, (ii) a set of edges $E = \{(i, j) \in Q \times Q\}, i \neq j,$ containing pairs of nodes that represent inter-agent communications, therefore $(j, i) \in E$ iff $j \in N_i$ and (iii) a set of vectors $C = \{c_{ji}\}, \forall (j, i) \in E$ that specify the desired relative position between agents i and j, i.e. $z_i - z_j = c_{ji} \in \Re^2$, $\forall i \neq j, j \in N_i$ in a desired formation pattern.

If $(j, i) \in E$, then the vertices i and j are said to be *adjacent*. The degree g_i of the $i-th$ vertex is defined as the number of its adjacent vertices. A path from a vertex i to a vertex j is a sequence of distinct vertices starting with i and ending with j such that consecutive vertices are adjacent. The underlying graph of a FG, is a new graph where for every edge $(i, j) \in E,$ a new edge (j, i) is added, if it did not appear on the original FG. If there is a path between any two vertices of the underlying graph of FG, then the FG is *connected*. A FG is said to be well defined if it satisfies the following conditions: *(1)* the graph is connected and *(2)* the desired vectors of positions establish a closed-formation, i.e., if there exist the vectors $c_{jm_1}, c_{m_1m_2}, c_{m_2m_3}, ..., c_{m_rj}$, then thy must satisfy

$$c_{jm_1} + c_{m_1m_2} + c_{m_2m_3} + ... + c_{m_rj} = 0. \tag{3}$$

The previous condition establishes that some position vectors form closed-polygons and it is related to the feasibility formation [10]. Note that (3) implies that if $c_{ij}, c_{ji} \in C,$ then $c_{ij} = -c_{ji}$.

The *Laplacian matrix* of a FG captures many fundamental topological properties of the graph and it is defined bellow:

Definition 2. The Laplacian matrix of a FG G is the matrix

$$L(G) = \Delta - A_d, \quad (4)$$

where $\Delta = diag\left[g_1, ..., g_n\right]$, where g_i is the degree of the vertex i, $A_d \in R^{n \times n}$ is called the adjacency matrix with elements

$$a_{ij} = \begin{cases} 1, & \text{if } (j,i) \in E \\ 0, & \text{otherwise} \end{cases}. \quad (5)$$

For a connected FG, the Laplacian has a single zero eigenvalue and the corresponding eigenvector is $[1,...,1]^T \in \Re^n$ [10]. Figure 1 shows an example of FG. The vertices are represented by circles and the arrows are the vectors c_{ji}. The circled elements of the Laplacian matrix are the degrees g_i. It is clear that $g_i = n_i$, i=1,...,n.

A FG is said to be *directed* if $\forall (j,i) \in E$, then $(i,j) \notin E$ (or $j \in N_i$ implies $i \notin N_j$), *undirected* if $\forall (j,i) \in E$, then $(i,j) \in E$ (or $j \in N_i$ implies $i \in N_j$) and *mixed* otherwise. For instance, the FG of Figure 1 is mixed. For the case of undirected FG, the Laplacian is always a symmetric positive semidefinite matrix. Figure 2 shows some examples of FG topologies commonly found in the literature and their respective Laplacian matrices. The cyclic pursuit FG (Figure 2b) is directed and the rest are undirected. Also, we observe that the Laplacian matrix describes completely the configuration of every FG.

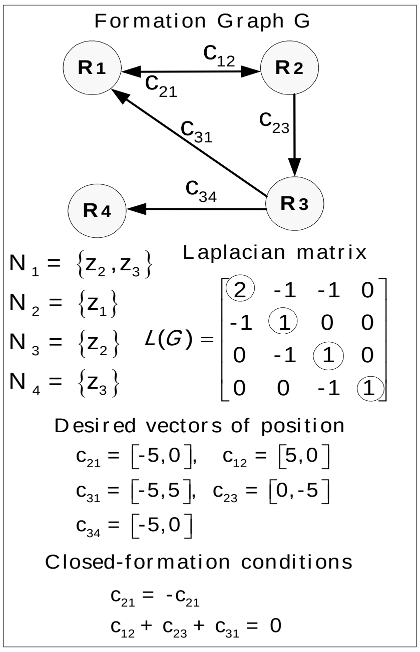

Figure 1. Example of a Formation Graph.

3. Control Strategy

For system (1), LPF's are defined by

$$\gamma_i = \sum_{j \in N_i} \left\| z_i - z_j - c_{ji} \right\|^2, \forall j \in N_i, i = 1, ..., n. \quad (6)$$

The functions γ_i are always positive and reach their minimum ($\gamma_i = 0$) when $z_i - z_j = c_{ji}, i = 1, ..., n, j \in N_i$. Using these LPF's, we define a control law given by

$$u_i = -\frac{1}{2} k \frac{\partial \gamma_i}{\partial z_i}, i = 1, ..., n, k \in \Re, \quad (7)$$

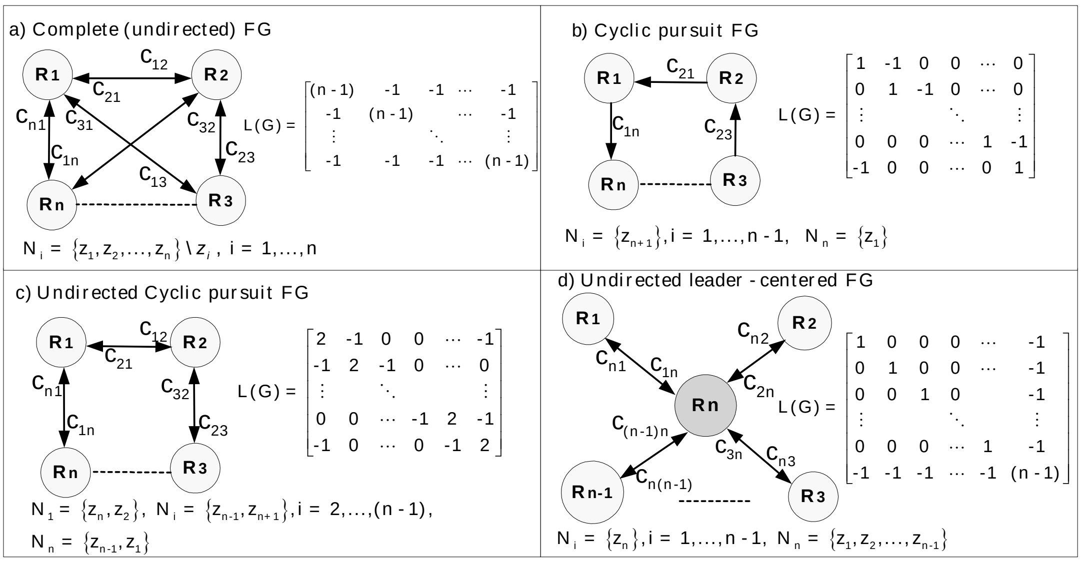

Figure 2. Topologies of a Formation Graphs.

Theorem 1. Consider the system (1) and the control law (7). Suppose that $k>0$ and the desired formation is based on a well defined FG. Then, in the closed-loop system (1)-(7) the agents converge exponentially to the desired formation, i.e. $\lim_{t\to\infty}\left(z_i - z_i^*\right)=0,\ i=1,...,n.$

The proof requires some preliminary lemmas.

Lemma 1. (Gershgorin circles Theorem [19]) Let $A\in\Re^{n\times n}$, if around of every principal diagonal element a_{ii} we draw a circle with radius the sum of the absolute values of the other elements on the same row, i.e. $r_i=\sum_{j=1,\,j\neq i}^{n}\left|a_{ij}\right|$, then every eigenvalue of A lies in one of these circles.

Lemma 2. If $A\in\Re^{n\times n}$ has eigenvalues $\lambda_1,...,\lambda_n$, then, the eigenvalues of matrix $kA,\ k\in\Re$ are given by $k\lambda_1,...,k\lambda_n$.

Proof of Theorem 1. The closed-loop system (1)-(7) has the form

$$\dot{z}=-k\left[\left(L(G)\otimes I_2\right)z-c\right], \tag{8}$$

where $L(G)$ is the Laplacian matrix of the FG, $z=\left[z_1,...,z_n\right]^T$, $\otimes$ denotes the Kronecker product [10], I_2 is the 2×2 identity matrix and $c=\left[\sum_{j\in N_1}c_{j1},...,\sum_{j\in N_n}c_{jn}\right]^T$. Define the formation errors as

$$\begin{aligned} e_i &= z_i - z_i^* \\ &= \frac{1}{g_i}\sum_{j\in N_i}\left(z_i - z_j - c_{ji}\right), i=1,...,n. \end{aligned} \tag{9}$$

The dynamics of every error e_i is given by

$$\dot{e}_i=-k\sum_{j\in N_i}\left(e_i-\frac{g_j}{g_i}e_j\right), i=1,...,n. \tag{10}$$

Rewritten in matrix form, we obtain

$$\dot{e}=-k\left(B\otimes I_2\right)e, \tag{11}$$

where $e=\left[e_1,...,e_n\right]^T$ and the matrix $B\in\Re^{n\times n}$ has elements:

$$b_{ij}=\begin{cases} g_i, & \text{if } i=j \\ -\dfrac{g_j}{g_i}, & \text{if } i\neq j,\ j\in N_i \\ 0, & \text{if } i\neq j,\ j\notin N_i \end{cases} \tag{12}$$

It is clear that $e=0$ is an equilibrium point of the closed-loop system (11) and that

$$PBP^{-1}=L(G), \tag{13}$$

where the similarity matrix is simply given by $P=diag\left[g_1,...,g_n\right]$. Thus, the convergence of the formation errors can be analyzed through the eigenvalues of $L(G)$. Based on Lemma 1, from matrix $L(G)$, we can draw n circles where the center of every circle $i,\ i=1,...n$ is $g_i>0$ and its radio is equal to $\left|g_i\right|$. On the other hand, as mentioned above, a connected FG has a Laplacian matrix with exactly one eigenvalue equal to zero. Therefore, if $\lambda_1,...,\lambda_n$ are eigenvalues of $L(G)$, then $\lambda_1=0$ and $\text{Re}(\lambda_2),...,\text{Re}(\lambda_n)>0$ where $\text{Re}(\lambda_i)$ denote real part of eigenvalue λ_i. Thus, the matrix $L(G)$ is always positive semidefinite with rank equal to $(n-1)$. Using Lemma 2, we know that the eigenvalues of matrix $-kL(G)$ are multiple of the eigenvalues of $L(G)$. If we choose $k>0$, the matrix $-kL(G)$ has eigenvalues $k\lambda_1,...,k\lambda_n$ where $k\lambda_1=0$ and $\text{Re}(k\lambda_2),...,\text{Re}(k\lambda_n)<0.$ Thus, the remaining $(n-1)$ eigenvalues lie on the open left-half complex plane and the formation errors of the system (11) converge exponentially to zero.

Figure 3 and 4 show an example of the convergence to the desired formation with $n=4,\ k=1$ using the FG and desired vectors of positions given by Figure 1. The initial positions in Figure 3 (denoted by circles) are $z_1(0)=[0,-1]$, $z_2(0)=[-1,0]$, $z_3(0)=[-4,-1]$ and $z_4(0)=[1,-3]$. We observe that the formation errors shown in Figure 4 converge to zero and therefore, all agents converge to the desired formation. The eigenvalues of $-kL(G)$ are $\{0,-1,-2,-2\}$.

4. Analysis of the Centroid of Positions

Definition 3. The centroid of positions $\bar{z}(t)$ is the mean of the positions of all robots in the group, i.e.

$$\bar{z}(t)=\frac{1}{n}\left(z_1(t)+\ldots+z_n(t)\right) \quad (14)$$

We observe in Figure 3a that the centroid of positions (denoted by X) does not remain constant. However, in some cases of FG, this centroid remains time-invariant. This property is interesting because, regardless of the individual goals of the agents, the dynamics of group remains centered at the position of the centroid. The second main contribution of this paper is to establish the condition of the FG topology to comply with the previous property.

Proposition 1. Consider the system (1) and the control law (7). Suppose that $k>0$ and the desired formation is based on a well defined FG. Then, in the closed-loop system (1)-(7), the centroid of positions remains constant, i.e. $\bar{z}(t)=\bar{z}(0), \forall t\geq 0$ iff the FG topology satisfies the condition

$$[1,\ldots,1]L(G)=[0,\ldots,0]. \quad (15)$$

Proof. The dynamics of every R_i in the closed-loop system (1)-(7) are given by

$$\dot{z}_i(t)=-k\left(g_i z_i-\sum_{j\in N_i} z_j-\sum_{j\in N_i} c_{ji}\right), \quad (16)$$
$$i=1,\ldots,n$$

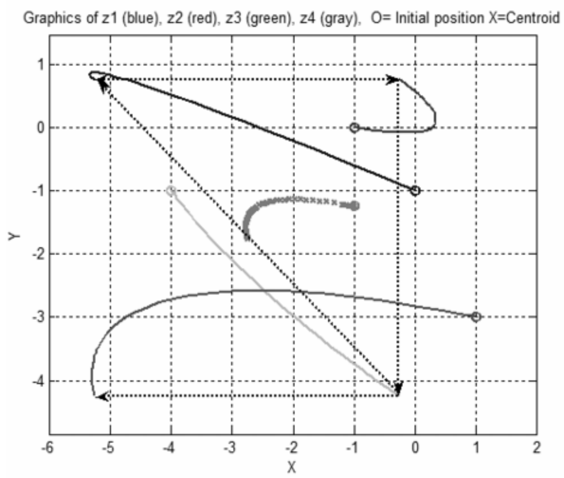

Figure 3. Formation using the FG of Figure 1.

Thus, the dynamics of the centroid of positions is given by

$$\dot{\bar{z}}(t)=-\frac{k}{n}\left(\sum_{i=1}^{n} g_i z_i-\sum_{i=1}^{n}\sum_{j\in N_i} z_j-\sum_{i=1}^{n}\sum_{j\in N_i} c_{ji}\right) \quad (17)$$

Since that the FG satisfy the closed-formation condition (3), then $\sum_{i=1}^{n}\sum_{j\in N_i} c_{ji}=0.$

Thus equation (17) can be reduced to

$$\dot{\bar{z}}(t)=-\frac{k}{n}\sum_{i=1}^{n}\left(g_i z_i-\sum_{j\in N_i} z_j\right). \quad (18)$$

The term $g_i z_i-\sum_{j\in N_i} z_j, i=1,\ldots,n,$ corresponds to the $i-th$ element of the column vector $\left(L(G)\otimes I_2\right)z.$ Thus, Eq. (18) is the sum of the elements of $\left(L(G)\otimes I_2\right)z$ multiplied by $-\frac{k}{n}.$ Therefore Eq. (18) is equivalent to (19)

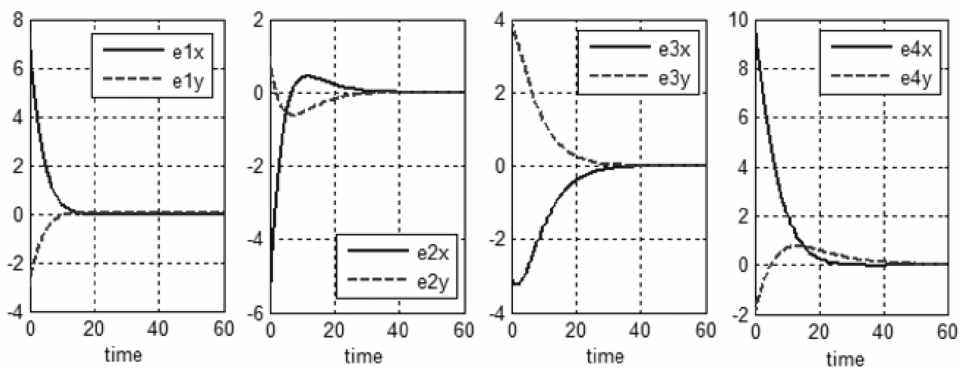

Figure 4. Formation errors

$$\dot{\bar{z}}(t) = -\frac{k}{n}\left([1,...,1]\left(L(G)\otimes I_2\right)z\right) \quad (19)$$

It is clear that $\dot{\bar{z}}(t)=0, \forall t \geq 0$ iff condition (15) holds.

All the undirected graphs, the cyclic pursuit FG and some mixed FG satisfy the condition (15). Figure 5 and 6 show a numerical simulation with the undirected cyclic pursuit FG of Figure 2c with $n=5$ and $k=1$. The desired vectors of position are given by

$c_{21}=[-10,0], c_{32}=\left[-10\cos\frac{2\pi}{5}, -10\sin\frac{2\pi}{5}\right]$,

$c_{43}=\left[10\sin\frac{3\pi}{10}, -10\cos\frac{3\pi}{10}\right], c_{54}=\left[10\sin\frac{3\pi}{10}, 10\cos\frac{3\pi}{10}\right]$,

and $c_{15}=\left[-10\cos\frac{2\pi}{5}, 10\sin\frac{2\pi}{5}\right]$ (pentagon with side equal to 10). The eigenvalues of $L(G)$ result in $\{0,-1.38,-1.38,-3.61,-3.61\}$. We observe that the formation errors of Figure 6 converge to zero and therefore the agents converge to the desired formation. Also, the FG satisfies the condition (15), therefore, the centroid of positions remains constant at $\bar{z}(t)=[0,-2]$.

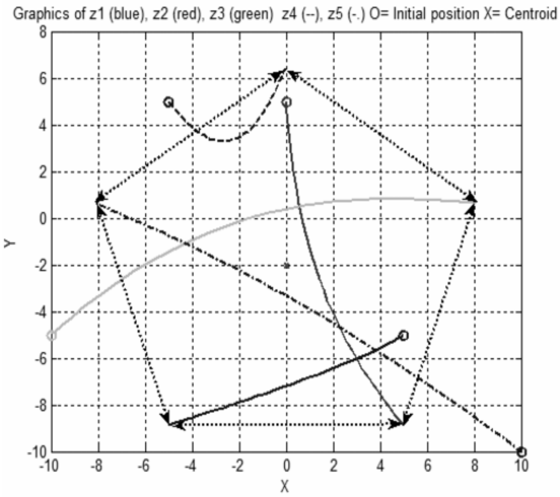

Figure 5. Formation control using the FG of Figure 2c.

5. Extension to the Case of Unicycles

In this section, the control laws developed so far are extended to the case of unicycles-type robot formations. The kinematic model of each agent or robot R_i, as shown in Figure 7, is given by (20)

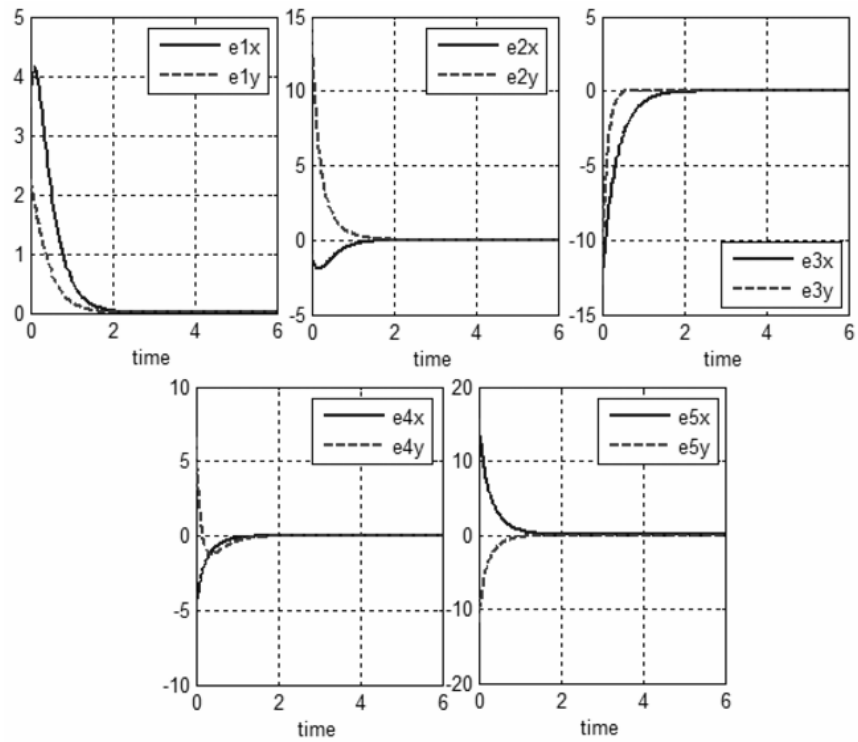

Figure 6. Formation errors.

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \cos\theta_i & 0 \\ \sin\theta_i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_i \\ w_i \end{bmatrix}, i = 1,...,n, \quad (20)$$

where u_i is the longitudinal velocity of the midpoint of the wheels axis and w_i is the angular velocity of the robot. It is known [21] that the dynamical system (20) cannot be stabilized by continuous and time-invariant control law. Because of this restriction, we will analyze the dynamics of the coordinates $\alpha_i = (p_i, q_i)$ shown in Figure 7 instead coordinates (x_i, y_i). The coordinates α_i are given by

$$\alpha_i = \begin{bmatrix} p_i \\ q_i \end{bmatrix} = \begin{bmatrix} x_i + \ell\cos\theta_i \\ y_i + \ell\sin\theta_i \end{bmatrix}. \quad (21)$$

The dynamics of (20) are given by

$$\dot{\alpha}_i = A_i(\theta_i)[u_i, w_i]^T, \quad A_i(\theta_i) = \begin{bmatrix} \cos\theta_i & -\ell\sin\theta_i \\ \sin\theta_i & \ell\cos\theta_i \end{bmatrix} \quad (22)$$

where the so-called decoupling matrix $A_i(\theta_i)$ is non-singular. The idea of controlling coordinates α_i instead of the center of the wheels axis is frequently found in the mobile robot literature in order to avoid singularities in the control law. Following the control strategy of the Section 3, the desired position of R_i is given by $\alpha_i^* = \frac{1}{g_i}\sum_{j\in N_i}(\alpha_j + c_{ji})$. Then, the formation control strategy is defined as

$$\begin{bmatrix} u_i \\ w_i \end{bmatrix} = -\frac{1}{2}kA_i^{-1}(\theta_i)\frac{\partial\tilde{\gamma}_i}{\partial\alpha_i}, i = 1,...,n, \quad (23)$$

where $\tilde{\gamma}_i = \sum_{j\in N_i}\tilde{\gamma}_{ij}$ with $\tilde{\gamma}_{ij}$ similar to (6) but related to coordinates α_i.

Corollary 1. Consider the system (20) and the control law (23). Suppose that $k > 0$ and the desired formation is related to a well defined FG. Then, in the closed-loop system (20)-(23), the robots converge to the desired formation, i.e. $\lim_{t\to\infty}(\alpha_i - \alpha_i^*) = 0$.

Proof. The dynamics of the coordinates α_i for the closed-loop system (20)-(23) is given by $\dot{\alpha}_i = -\frac{1}{2}k\left(\frac{\partial\tilde{\gamma}_i}{\partial\alpha_i}\right)^T, i = 1,...,n.$

The closed-loop dynamics of the coordinates α_i has the form

$$\dot{\alpha} = -k\left[\left(L(G)\otimes I_2\right)\alpha - c\right], \quad (24)$$

where $\alpha = [\alpha_1,...,\alpha_n]^T$ and $L(G)$ and C were previously defined for system (8). It is clear that the closed-loop system (24) is the same that (8) for the case of point agents. The result follows.

Remark. The control law (23) steers the coordinates α_i to a desired position. However, the angles θ_i remain uncontrolled. These angles do not converge to any specific value. Thus, the control law (23) is to be considered as a formation control without orientation.

Figures 8 and 9 show a simulation for the closed-loop system (20)-(23) for $n = 4$, $\ell = 1$, and $k = 1$ and the complete FG shown in Figure 2a where the desired formation is a square with $c_{21} = [0,10]$, $c_{31} = [-10,10]$, $c_{41} = c_{32} = [-10,0]$, $c_{42} = [-10,-10]$ and $c_{43} = [0,-10]$. The initial conditions are given by $[x_{10}, y_{10}, \theta_{10}] = \left[-10,-10,\frac{3\pi}{2}\right]$,

$$[x_{20}, y_{20}, \theta_{20}] = \left[10,-\frac{5}{2},0\right],$$

$$[x_{30}, y_{30}, \theta_{30}] = \left[\frac{10}{3},10,\frac{\pi}{2}\right],$$

$$[x_{40}, y_{40}, \theta_{40}] = \left[-10,-\frac{5}{2},\pi\right].$$

Thus, the initial positions of coordinates α_i are given by $\alpha_{10} = [-10,-11]$, $\alpha_{20} = [11,-2.5]$, $\alpha_{30} = [6.6667,11]$ and $\alpha_{40} = [-11,-2.5]$. We observe in Figure 9 that the formation errors, now related with the coordinates α_i converge to zero. Therefore, the coordinates α_i converge to the desired formation and the centroid of positions remains constant. It is important to note that the agents do not converge with the same orientation.

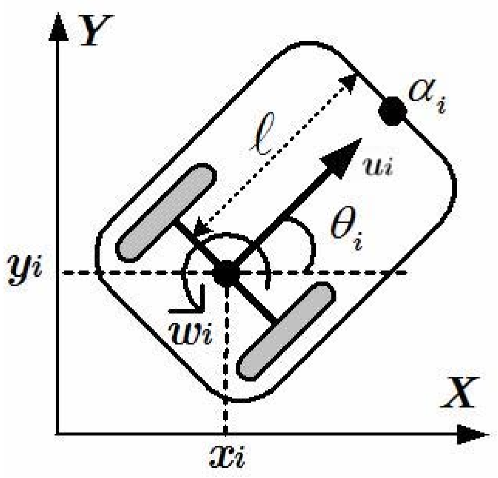

Figure 7. Kinematic model of unicycles.

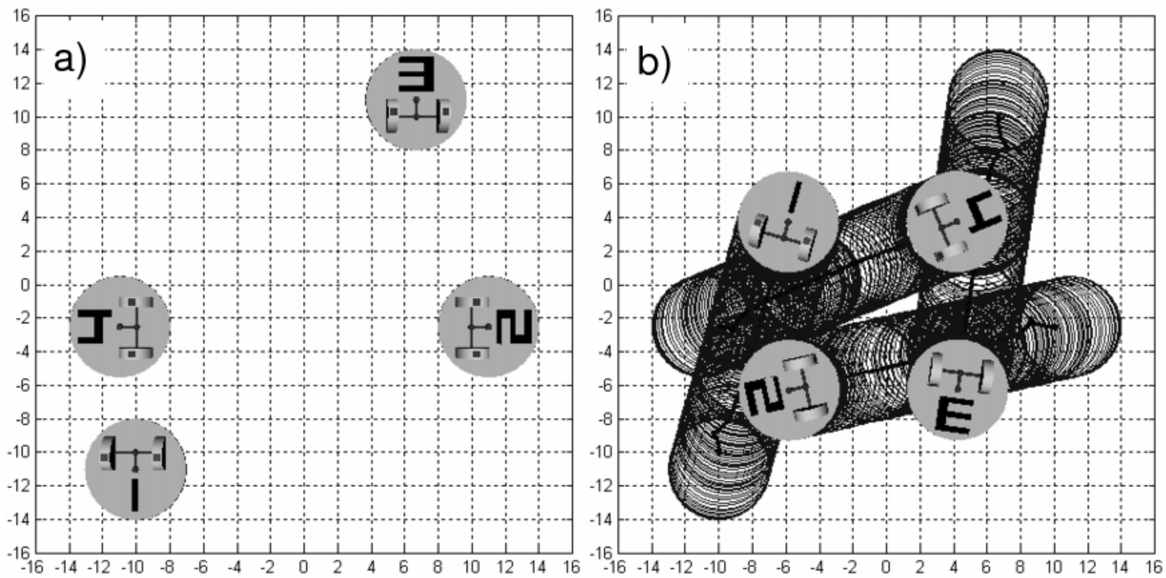

Figure 8. Agent trajectories in plane at a) $t=0$, b) $t=1.5$. Continuous line represents the position of (x_i, y_i).

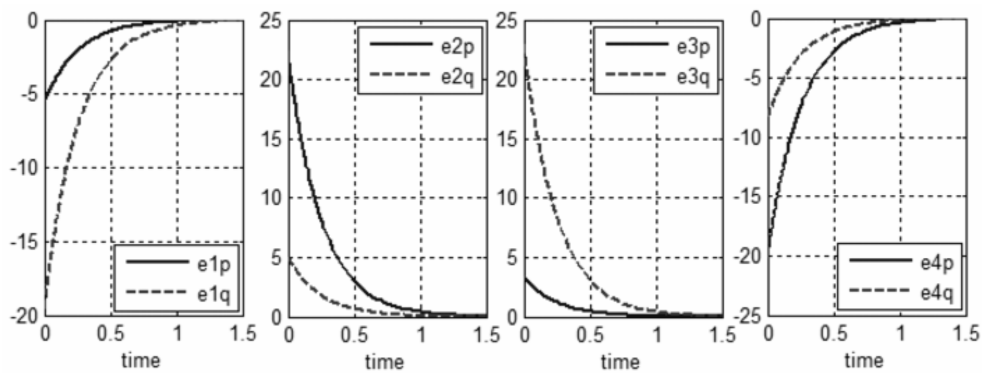

Figure 9. Formation errors.

6. Conclusions

This paper presents a formation control strategy based on LPF's and the FG approaches. The main contribution is a formal proof about the global convergence to the desired formation applied to any well-defined FG based on the properties of the Laplacian matrix. Also, the topological feature of a FG which ensures that the centroid of positions remains stationary is established. This property is interesting because the dynamics of the group remains centered at the position of the centroid, although every agent obeys a decentralized control strategy.

The main contributions are extended to the case of unicycle-type robots including some numerical simulations. In further research the problem of inter-agent collision and experimental work will be addressed.

Acknowledgements

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