Feedback Linearization and Model Reference Adaptive Control of a Magnetic Levitation System

Luiz H. S. Torres[1], Leizer Schnitman[1], Carlos A. V. V. Júnior[1], J. A. M. Felippe de Souza[2]

[1] Centro de Capacitação Tecnológica em Automação Industrial (CTAI),
Universidade Federal da Bahia,
Rua Aristides Novis, nº 02, Escola Politécnica, 2º andar, 40.210-630,
Salvador, Bahia, Brasil.
luizhenrique@ieee.org; leizer@ufba.br; carlos@trecon.com.br.

[2] Electromechanical Engineering Dept, University of Beira Interior,
Covilhã, Portugal.
felippe@ubi.pt

Abstract: The aim of this paper is to combine two techniques to control a nonlinear Magnetic Levitation System (MLS). Firstly, a feedback linearization technique (here, exact linearization with state feedback) is applied to obtain a linear system. Secondly, the linearization is made via direct cancellation of nonlinear functions, which represent the phenomenological model of the system. Finally, to deal with the presence of uncertainty in the system model, an adaptive controller is used. The controller is based on model reference adaptive control to estimate the functions that contain the nonlinearities of the system. The exact linearization and the adaptive controller were implemented in a simulated environment (Matlab Simulink ©). The linear adaptive controller structure guarantees the parameters adaptation and the overall stability of the system. The results show that the controller output signal tracks a reference input signal with a small error.

Keywords: Adaptive Control; Direct Approach; Exact Linearization; Magnetic Levitation; Model Reference.

1. Introduction

In recent years, due to computational developments that have enabled more complex applications of nonlinear problems, the area of nonlinear control systems has been the subject of many studies (Soltanpour and Shafie, 2010). The present paper shows a combination of an adaptive controller and a feedback linearization technique to control a Magnetic Levitation System (MLS). This system was chosen since it has nonlinear dynamics and a didactic kit of the physical system is available to continue with future work.

The MLS used is manufactured by ECP – Educational Control Products (www.ecp.com) and will be described in more detail in section II. Here one desires to control a magnet displacement over a glass stick as a result of the application of an electrical current on a coil (ECP, 1999).

The relationship between the electrical flow and the magnetic disc movement is given by a second order nonlinear ordinary differential equation. This nonlinear relationship belongs to a class of engineering systems of the type $\dot{x} = \phi(t,x) = Ax + B[F(x) + G(x)u]$. Several nonlinear control strategies can be used to control the disc position, such as, for example: fuzzy, neural network, adaptive control, feedback linearization (Khalil, 1996; Abdel-Hady and Abuelenin, 2008; Torres *et al.*, 2010a). Here both the exact feedback linearization technique and adaptive control will be used.

Exact feedback linearization can enable a transformation from a nonlinear system to a linear through the addition of nonlinear compensators. Thus, this transformation allows designing a linear controller for the system linearized. However, the exact linearization technique with state feedback requires a mathematic model that represents the dynamics of the real plant (Slotine, 1991). Furthermore, the uncertainties in the phenomenological model can compromise better results. To deal with some uncertainties in the system's model, an adaptive controller is used. The controller is based on direct model reference approach (Narendra and Valavani, 1978; Ioannou and Sun, 1996) to provide a control law that will be used to the system after the linearization.

The aim of this paper is to use the combination of two techniques to control the MLS: exact linearization with state feedback and Model Reference Adaptive Control (MRAC). These two techniques combined will enable a linear controller structure to deal with some uncertainties in the system model to MSL´s control disk position (a typical nonlinear dynamical system).

In section 2 the MLS is briefly explained .The proposal of the exact linearization with state feedback and its application over the MLS are presented in section 3. In section 4, the adaptive controller structure with a linear control law is presented. The simulation and analysis results are discussed in section 5. Finally, some remarks about the controller performance are presented in section 6. It is important to mention that a first version of this paper was published in (Torres *et al*., 2010b).

2. The Model

Magnetic levitation system

In this paper the MLS made by ECP was used and is shown in Figure 1. It comprises two magnetic discs, a glass stick, two laser sensors and two coils. The sensors are used to obtain the system response associated with the disc positions. The system input is given by the application of an electrical current to the coils. The physical system communicates with a computer via Digital Signal Processing (DSP) and a black box is responsible for the electrical current drivers and the energy supply.

This MLS can be classified according to two modes, SISO (Single Input Single Output) or MIMO (Multiple Input Multiple Output), which depends on the desired system configuration. In the SISO mode only one disc is used whereas in the MIMO mode two discs are used. Here the MLS was configured to operate as SISO and in the repulsive mode over the disc. In other words, only the bottom coil was used in this work.

Figure 1. Magnetic Levitation System made by ECP (ECP, 1999).

The MLS manual (ECP, 1999) shows the mathematic model, based on the physical laws, which allows us to obtain its differential equation model. The development of the mathematic model is beyond the scope of this paper. Through the balance of forces, the equation is given by (Laithwaite, 1965)

$$\ddot{y}+\frac{c}{m}\dot{y}=\frac{F_m}{m}-g\ , \qquad (1)$$

where:

y - magnetic disc position

$\dot{y}$ - first derivative of the magnetic disc position

$\ddot{y}$ - second derivative of the magnetic disc position

c - air viscosity coefficient

m - magnetic disc mass

F_m - magnetic force applied to the magnetic disc.

The magnetic force can be written in the following way (ECP, 1999)

$$F_m=\frac{i}{a(y+b)^4}\ , \qquad (2)$$

where:

i - electrical current applied on the coil;

a and b - are constants related with the coil properties.

By substituting (2) in (1), a nonlinear relationship between the magnetic disc position and electrical current applied to the coil gives

$$\ddot{y}=-g-\frac{c}{m}\dot{y}+\frac{1}{ma(y-b)^4}i \qquad (3)$$

Parameter estimation

There are five parameters in (3): g, c, m, a, and b. The parameters g = 9.81 [m/s^2], m = 0.12 [Kg] and c = 0.15 [Ns/m] are considered as constant and known values (ECP, 1999). The parameters a and b are constants related with magnetic coil properties and must be estimated. In (Silva, 2009), the least square and Monte Carlo methods were used to estimate a and b. Accordingly with (Silva, 2009), based on a cost function, the Monte Carlo method presented the best values for these parameters. The values are a = 0.95 and b = 6.28. These values will be used in this paper.

3. Feedback Linearization

Exact linearization with state feedback

Feedback linearization can be applied to a certain class of nonlinear systems, including the MLS studied, and enables to transform the original system models into equivalent models of a simpler form. The control scheme uses the exact linearization with state feedback based on the cancellation of nonlinear functions. However, to enable the application of the technique, the system dynamic must be represented by (Guadarbassi and Savaresi, 2001)

$$\dot{X} = F(X) + G(X)u \,, \tag{4}$$

where the functions *F(X)* and *G(X)* represent the nonlinearities of the states, *u* is the control system input and *X* is the state vector. Furthermore, two conditions must be satisfied. The first one is that the system must be controllable. For this first condition the matrix formed by vectorial fields in (5) must has order n, where n is the system order (Khalil, 1996)

$$[ad_F^0 G \; ad_F^1 G ... ad_F^{n-1} G] \,, \tag{5}$$

where $ad_F^n G$ is the notation of Lie's bracket (Slotine, 1991).

The second one is that the system should be involutive. It means that the distribution expressed in (6) also should be involutive (Guadarbassi and Savaresi, 2001)

$$D = span\{ad_F^0 G \; ad_F^1 G ... ad_F^{n-1} G\} \,, \tag{6}$$

where D is the involutive distribution of *G(X)* expanded in Taylor's series (represented here by the notation *span{.}*) on an equilibrium state X_0 (Nam *et al.*, 1993) The order of D is given by $n-1$.

In order to the distribution in (6) to be involutive, it is necessary that the order *n* of the expression in (7) be equal to dimension of D in (6)

$$[ad_F^0 G \; , ad_F^{n-1} G] \,, \tag{7}$$

Once the conditions are satisfied it is possible to determine a diffeomorphism $Z = T(X)$. After this, the dynamic of the system given by (4) can be transformed into the form

$$\dot{Z} = EZ + F\beta^{-1}(Z)[u_f - \alpha(Z)] \,, \tag{8}$$

where $\alpha(Z)$ and $\beta(Z)$ represent the state feedback, u_f is the feedback signal, and E and F are, in this work, considered as constant with known values. A feedback signal u_f for the nonlinear system is chosen in the form in (9)

$$u_f = \alpha(Z) + \beta(Z)u \,, \tag{9}$$

Thus, the linear system can be written as

$$\dot{Z} = EZ + Fu \tag{10}$$

where *u* is the control signal (control law) for the system after linearization. The determination of *u* will be discussed in next section.

Linearization of the MLS

The model of the MLS made by ECP was presented in (3) and the two conditions for application of the exact linearization were presented in the last subsection. The feedback signal u_f and the variables of states can be set as

$$u_f = i \qquad x_1 = y \qquad x_2 = \dot{y} \,, \tag{11}$$

The dynamic of the system given by (3) can be rewritten in the form given in (4)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -g - \frac{c}{m} x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{ma(x_1 + b)^4} \end{bmatrix} u_f \cdot , \tag{12}$$

The functions *F(X)* and $G(X)$ that contain the nonlinearities of the system can be set as follows in (13) and (14)

$$F(X) = \begin{bmatrix} x_2 \\ -g - \frac{c}{m} x_2 \end{bmatrix} \tag{13}$$

$$G(X) = \begin{bmatrix} 0 \\ \frac{1}{ma(x_1 + b)^4} \end{bmatrix} . \tag{14}$$

The transformation $Z = T(X)$ can be set in the form given by (15) (Khalil, 1996)

$$Z = \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = T(X) \,, \tag{15}$$

The functions $\alpha(Z)$ and $\beta(Z)$ can be calculated in the form given by

$$\alpha(Z) = (mga + caZ_2)(Z_1 + b)^4 \,, \tag{16}$$

$$\beta(Z) = ma(Z_1 + b)^4 \tag{17}$$

Finally, the feedback signal u_f can be rewritten as

$$u_f = (mga + caZ_2)(Z_1 + b)^4 + ma(Z_1 + b)^4 u \quad (18)$$

The application of the feedback signal u_f over the system given by (12) will cancel the nonlinearities and the system will be transformed into a linear system given by (10). In the same way but using (15) (Slotine, 1991)

$$\dot{Z} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}. \quad (19)$$

A block diagram was implemented in Matlab Simulink© which simulates the exact linearization technique applied in the MLS (see Figure 2).

The block diagram in Figure 3 may explain the general idea of a system using MRAC.

MRAC applied to the MSL after the linearization

In this work MRAC classical technique using the stability theory from the input-output view is applied to the MSL after the exact linearization. Once the dynamics are now linear, the control problem will be formulated as model-following. The derivation of the MRAC will follow the 3 steps below (Aström and Wittenmark, 2008)

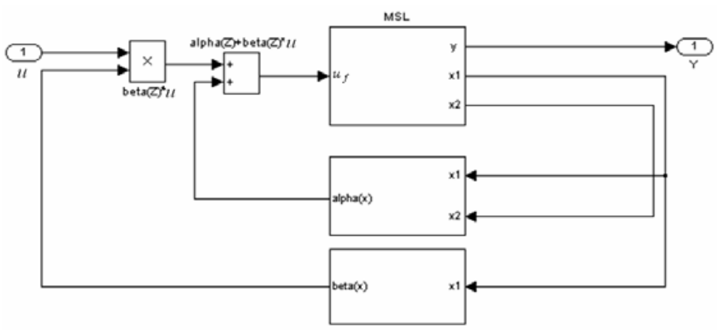

Figure 2. Block diagram for the exact linearization in the MLS.

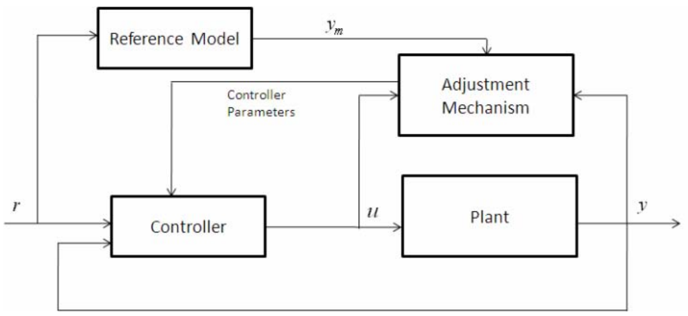

Figure 3. General idea of a system using MRAC.

4. Adaptive Controller

Model Reference – Direct Approach

The Model Reference Adaptive Control (MRAC) is one of main techniques in adaptive control. The changes in the controller parameters are provided by the adjustment mechanism with the objective to minimize the error between the system under control and a model reference output (that is the desired response). MRAC uses integral laws to adaptation which can introduce a slow and oscillatory transitory, however in steady state there is a soft control signal (Narendra and Valavani, 1978; Ioannou and Sun, 1996).

Step 1: Find a controller structure that admits perfect output tracking;

Step 2: Derive an error model of the form

$$\varepsilon = G_1(p)\{\phi^T(t)(\theta^0 - \theta)\} \quad (20)$$

where $G_1(p)$ is a Strictly Positive Real (SPR) transfer function in *p*, θ^0 is the process parameters (or the true controller parameters), and θ is the controller parameters (or the adjustable controller parameter).

Step 3: Use the parameter adjustment law

$$\dot{\theta}(t) = \gamma\phi\varepsilon \quad (21)$$

where γ is the adaptation gain, ϕ an auxiliary vector of filtered signals and ε the error signal.

All these parameters and variables in (20) and (21) will be defined in the next subsection.

The controller structure

The desired response given by a continuous-time model reference signal could be written as follows

$$A_m y_m(t) = B_m r(t) \tag{22}$$

where A_m and B_m are polynomials to obtain the model transfer function in the form

$$G_m(s) = \frac{B_m}{A_m} \tag{23}$$

The desired response (in this case here is the height value) and the input reference signal $r(t)$ is bounded. The continuous-time plant (the MSL after exact linearization) could be described as follows

$$Ay(t) = b_0 Bu(t) \tag{24}$$

It is assumed that the polynomials A and B do not have common factors and the polynomial B is monic with all zeros in the left half-plane. The parameter b_0 is called the high-frequency gain. The variable $y(t)$ is the measured output and it is also bounded. The plant is a minimum-phase and, in this work, it is assumed the sign of b_0 is known. If the parameter b_0 is not known, it must be estimated. A linear controller can be written as

$$Ru(t) = -Sy(t) + Tr(t) \tag{25}$$

where R, S and T are polynomials. Since the polynomial B is stable, the corresponding poles can be canceled by the controller. In other words, $R = R_1 B$. The closed-loop system when (25) is applied becomes

$$(AR_1 + b_0 S)y = b_0 Tr \tag{26}$$

where T will be chosen as $T = t_0 A_0$, and A_0 is a stable monic polynomial and R_1 and S satisfy

$$AR_1 + b_0 S = A_0 A_m \tag{27}$$

One way to achieve the perfect model-following is set

$$A_m y_m(t) = b_0 t_0 r(t) \tag{28}$$

and the error equation (the difference between the plant response and the desired response given by the reference model) is set as

$$e = y(t) - y_m(t) \tag{29}$$

Let introduce the polynomials P_1, P_2 and P: $P = P_1 P_2$, $P_1 = p^n + \delta_1 p^{n-1} + ... + \delta_n$, and $P_2 = p^k + \lambda_1 p^{k-1} + ... + \lambda_k$, where n = *degree* (A_m) = *degree* (P_1), and k = *degree* (R) = *degree* (P_2). It is assumed that *degree* (P_1) > *degree* (P_2) and the polynomial P_2 is a stable monic polynomial.

From (27) and after some algebraic manipulations, let define the filtered error e_f as

$$e_f = \frac{b_0 Q}{A_0 A_m}\left(\frac{R}{P}u + \frac{S}{P}y - \frac{T}{P}r(t)\right) \tag{30}$$

where Q is also a polynomial whose degree is not greater than *degree* $(A_0 A_m)$ such that $\frac{b_0 Q}{A_0 A_m}$ is SPR. This filtered error e_f will be necessary to obtain the error signal ε in the form of (20). One can writes $\frac{R}{P}$ as

$$\frac{R}{P} = \frac{R - P_2 + P_2}{P_1 P_2} = \frac{1}{P_1} + \frac{R - P_2}{P} \tag{31}$$

The filtered error then becomes

$$e_f = \frac{b_0 Q}{A_0 A_m}\left(\frac{1}{P_1}u + \frac{R - P_2}{P} + \frac{S}{P}y - \frac{T}{P}r(t)\right) \tag{32}$$

Let l and m be the degrees of the polynomials S and T, respectively. Let introduce a vector of true controller parameters

$$\theta^0 = (r_1 ... r_k \; s_0 ... s_l \; t_0 ... t_m)^T \tag{33}$$

where r_i are the coefficients of the polynomial $R - P_2$. Also define the auxiliary vector ϕ of filtered input, output and input reference signals

$$\phi^T = \left(\frac{p^{k-1}}{P(p)}u ... \frac{1}{P(p)}u \quad \frac{p^l}{P(p)}y ... \frac{1}{P(p)}y \quad -\frac{p^m}{P(p)}r ... -\frac{1}{P(p)}r\right) \tag{34}$$

The filtered error in (32) can then be rewritten as

$$e_f = \frac{b_0 Q}{A_0 A_m}\left(\frac{1}{P_1}u + \phi^T \theta^0\right) \tag{35}$$

To obtain an error model, one must introduce a parameterization of the controller. The control law $u(t)$ is given by

$$u(t) = -\theta^T (P_1\phi) \tag{36}$$

By using this control law and (35) the filtered error can be written as

$$e_f = \frac{b_0 Q}{A_0 A_m}\left(\phi^T \theta^0 - \phi^T \theta - \frac{1}{P_1}\theta^T (P1\phi) + \phi^T \theta\right) \tag{37}$$

Let introduce the signals η and ε. After some algebraic manipulation by using (37) these signals can be defined by

$$\eta = -\left(\frac{1}{P_1}u + \phi^T \theta\right) \tag{38}$$

$$\varepsilon = \frac{Q}{P}(y - y_m) + \frac{b_0 Q}{A_0 A_m}\eta \tag{39}$$

$$e_f = \frac{Q}{P}e = \frac{Q}{P}\left(y - y_m\right) \tag{41}$$

$$\eta = -\left(\frac{1}{P_1}u + \phi^T \theta\right) \tag{42}$$

$$\varepsilon = e_f + \frac{b_0 Q}{A_0 A_m}\eta \tag{43}$$

$$\dot{\theta}(t) = \gamma\phi\varepsilon \tag{44}$$

$$u(t) = -\theta^T (P_1\phi) \tag{45}$$

5. Simulation and Analysis Results

The Matlab Simulink© was used to simulate the proposed controller for the MLS in the present work. The block diagram designed in Simulink is shown in Figure 4. The model parameters of the MSL used here were presented in subsection 2.2.

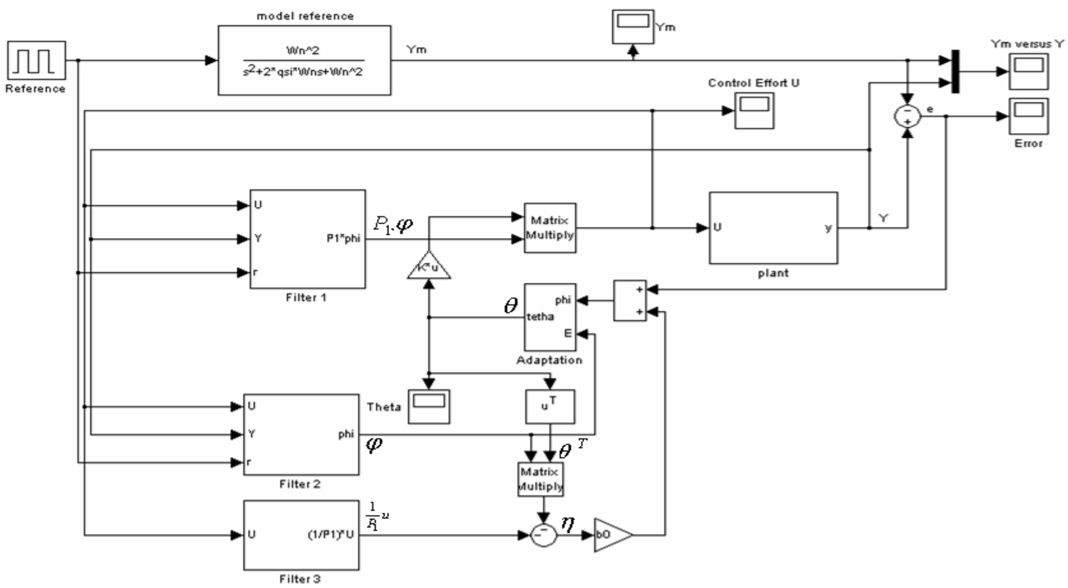

Figure 4. Block diagram in Simulink.

The signal ε is called the augmented error, and η is called the error augmentation. The error model in (39) is the same defined in (20). It is also linear in the parameters and satisfies the requirements of the *step 2*, and the parameters will be updated by (21). The stability of the closed-loop system is obtained by considering that $b_0 Q/(A_0 A_m)$ is SPR and that signals in ϕ are bounded. Finally, the equations needed to implement the MRAC system can be summarized as follows:

$$ym = \frac{B_m}{A_m} r \tag{40}$$

Many different model reference adaptive systems can be obtained by different choices of the design parameters. For sake of simplicity, in this work the polynomials were chosen as follow: $P_1(s) = A_m(s)$, $P_2(s) = A_0(s)$, and $Q(s) = A_0(s).A_m(s)$.

The transfer functions of the reference model was chosen as

$$G_m(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}; \quad \Rightarrow \xi = 0.1 \quad \omega_n = 0.75 \tag{46}$$

The polynomials A_0, R, S, and T were chosen as follow below and by using the solution given by the Diophantine equation (27)

$$A_0(s) = s + a_0 \Rightarrow a_0 = 2 \qquad (47)$$

$$R(s) = s + r_1 \Rightarrow r = 2.15 \qquad (48)$$

$$S(s) = s_0 s + s_1 \Rightarrow s_0 = 0.43; \quad s_1 = 0.56 \qquad (49)$$

$$T(s) = t_0 s + t_1 \Rightarrow t_0 = 0.28; \quad t_1 = 0.56 \qquad (50)$$

The simulations were performed regarding $r(t)$ (reference) as a square wave signal with amplitude equal to 1 and the adaption gain γ equal to 0.8. The value of b_0 was chosen equal to 0.5. Figure 5 shows the compared response between the model reference output and the plant output. Figure 6 and Figure 7 show the error signal and the control effort, respectively.

According to (3) the reference signal r is the electrical current applied on the coil $i(t)$ (that is the manipulated variable). It could be observed in Figure 5 that the model reference output given by (3) was tracked by the plant output. The process variable $y(t)$ that is the magnetic disc position could be observed in Figure 5 with oscillations in transitory, but stable in steady state. The error e in steady state is little according to Figure 6 and the control effort associated with the electrical current is bounded as shown in Figure 7.

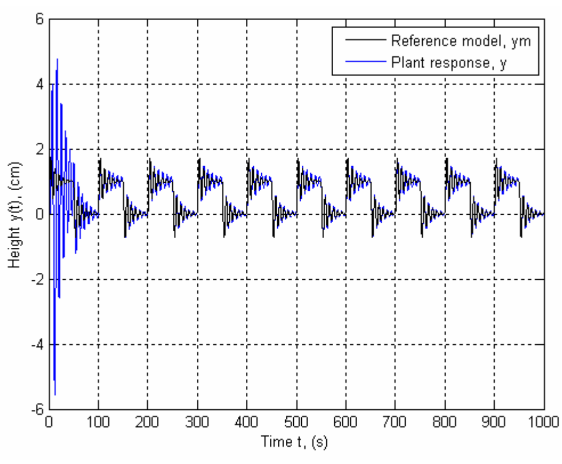

Figure 5. Comparison between the desired response (model reference) and plant response.

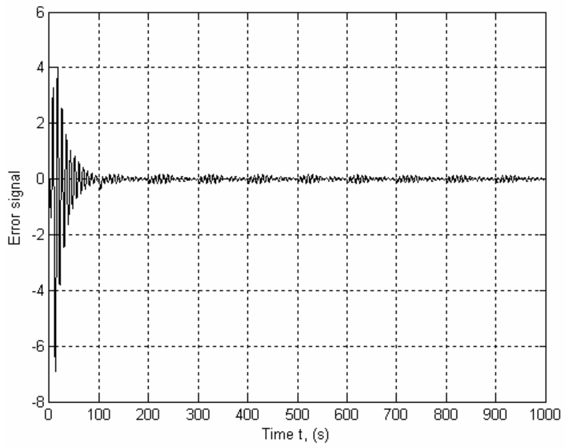

Figure 6. Error signal.

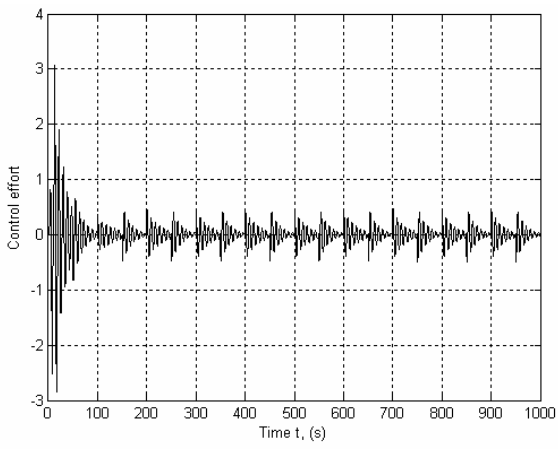

Figure 7. Control effort.

6. Conclusions

In this paper the combination of two techniques to control a MLS were presented: exact linearization with state feedback and MRAC – direct approach. It could be observed through the simulations and analysis results that the desired response (output signal of the model reference) was tracked by the plant response. The error signal could be seen as bounded and near to zero and the control effort could be seen also as bounded. The simulation results show that the adaptive controller is able to control satisfactorily the magnetic disc position, in spite of the presence of model uncertainties. For future work this adaptive controller should be implemented in the real physical system.

Acknowledgements

The authors wish to acknowledge the support with infrastructure from CTAI, and CAPES for the financial support.

REFERENCES

1. ASTRÖM, K., B. WITTENMARK, **Adaptive control**, 573p, Dovler: New York, USA, 2008.
2. ABDEL-HADY, F., S. ABUELENIN, **Design and Simulation of a Fuzzy-Supervised PID Controller for a Magnetic Levitation System**. Studies in Informatics and Control, Vol. 17, No. 3, 2008, pp. 315-328.
3. ECP, 1999. **Manual for Model 730: Magnetic Levitation System**. Bell Canyon, USA.
4. GUADARBASSI, G., S. SAVARESI, **Approximate Linearization via Feedback – An Overview**, Automatica, Vol. 37, 2001, pp. 1-15.
5. IOANNOU, P. A., J. SUN, **Robust Adaptive Control**. Prentice Hall: New York, 1996, 833p.
6. KHALIL, H., **Nonlinear Systems**, 2nd ed., Prentice Hall: Michigan, 1996.
7. LAITHWAITE, E. R., **Electromagnetic Levitation**, in Proceedings of Institution of Electrical Engineers, Vol. 112, no. 12, 1965, pp. 2361-2375.
8. NAM, K., A. ARAPOSTATHIS, K. K. LEE, **Some Numerical Aspects of Approximate Linearization of Single Input Nonlinear Systems**. IEEE Transactions on Automatic Control, Vol. 57, 1993, pp. 463-472.
9. NARENDRA, K. S., L. S. VALAVANI, **Stable Adaptive Controller Design – Part I – Direct Control**. IEEE Transactions on Automatic Control, Vol. AC-23, no. 4, 1978, pp. 570-583.
10. SILVA, E. B., **Modeling and Controlling a Magnetic Levitation System**, B. Sc. thesis, Dept. Elect. Eng., UFBA Univ., Salvador Bahia, Brazil, 2009.
11. SOLTANPOUR, M. R., E. S. SHAFIE, **Design and Stability Analysis of a Robust Impedance Control System for a Robot Manipulator**. Studies in Informatics and Control, Vol. 19, No. 1, pp. 5-16.
12. SLOTINE, J. J., W. LI, **Applied Non-linear Control**. Prentice Hall: Englewood Cliffs, New Jersey, 1991.
13. TORRES, L. H. S., L. SCHNITMAN, C. A. V. VASCONCELOS JÚNIOR, J. A. M. FELIPPE DE SOUZA, **Exact Linearization and Fuzzy Logic Applied to the Control of a Magnetic Levitation System**, in Proc. 2010 IEEE World Congress on Computational Intelligence, WCCI 2010, Barcelona-Spain, 2010, pp. 1347-1351.
14. TORRES, L. H. S., L. SCHNITMAN, C. A. V. VASCONCELOS JÚNIOR, J. A. M. FELIPPE DE SOUZA, **Feedback Linearization and Model Reference Adaptive Control of a Magnetic Levitation System**, in Proc. XIV Latin American Congress of Automatic Control / XIX Congress of the Chilean Association of Automatic Control – ACCA, Santiago-Chile, 2010.