Stabilizability Conditions for Switched Linear Systems with Constant Input via Switched Observer

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Abstract: In this paper, stabilizability conditions for switched linear systems with constant input via two types of switched rule which depends on the state of switched observer are presented. The obtained results provide stabilizability conditions via state feedback switched rule. Further, two illustrative numerical examples are also given.

Keywords: Switched Systems, Switched Observer, Stabilizability, Constant Input

1. Introduction

Switched system is one of the so-called hybrid systems which consist of a family of subsystems and a switching rule among them. The aspect of the switched system is found in various fields such as aircraft industry, mobile robot, animal world and Ethernet etc[3], [7].Further, the idea of switching has also been used to design intelligent control which is based on the switching between different controllers. An important problem in such switched systems is the stability problem with arbitrary switching and the stabilization problem via appropriate switching rule. Until now many results on stability and stabilization problems for various types of switched linear systems without input have been studied (e.g., [1],[2],[5], [6], [8]-[20]).

In addition, it is also important to consider the case which contains the control input for practical applications. In particular, Deaecto et al. [4] gave some conditions for some equilibrium point to be globally asymptotically stable via state feedback switched rule. The conditions are related to continuous-time switched linear system with constant input. The results were applied to DC-DC converters control design. However, the same problems via switched observer which contains information of the outputs instead of the state for the switched systems have not been investigated.

The objective of this paper is to study conditions under which equilibrium points are globally asymptotically stable via the switched observer. The conditions are related to continuous-time switched linear systems with constant input. In Section 2 the main results of this paper are given. In Section3two illustrative numerical examples are shown. Finally, concluding remarks are given in Section 4.

2. Stabilizability Conditions

At first, the following notations which will be needed throughout this study are given.

Notations

– $\lambda_{\max}(M)$ is the maximum eigenvalue of a symmetric matrix $M \in \mathbb{R}^{n\times n}$.

– $\|M\|$ is the maximum singular value of a matrix $M \in \mathbb{R}^{n\times m}$ (i.e., $\|M\|^2 = \lambda_{\max}(M^{\mathrm{T}}M)$).

– For two matrices M_1, M_2, $M_1 > M_2$ implies that $M_1 - M_2$ is positive definite (i.e., $M_1 - M_2 > 0$).

– $\Lambda := \left\{ \lambda = (\lambda_1, \lambda_2, \cdots, \lambda_N) \mid \sum_{i=1}^{N} \lambda_i = 1, \lambda_i \geq 0 \right\}$.

– $A_\lambda := \sum_{i=1}^{N} \lambda_i A_i,\ \lambda = (\lambda_1, \lambda_2, \cdots, \lambda_N) \in \Lambda$.

– $\mathbb{N} := \{1, 2, \cdots N\}$.

– $\arg\min_{i\in\mathbb{N}} S := \min\left\{ i : s_i = \min_{j\in\mathbb{N}}\left(s_j\right) \right\}$ That is the minimum index$i$ such that s_i is equal to the smallest element of the ordered set $S = \{s_1, s_2, \cdots, s_N\}$.

Next, consider the following continuous-time switched linear system

$$\Sigma_\sigma : \begin{cases} \dot{x}(t) = A_{\sigma(\hat{x},t)}x(t) + B_{\sigma(\hat{x},t)}u(t),\ x(0) = x_0, \\ y(t) = C_{\sigma(\hat{x},t)}x(t), \end{cases}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) = u \in \mathbb{R}^m$ is the constant input, $y(t) \in \mathbb{R}^p$ is the output, $\hat{x}(t) \in \mathbb{R}^n$ is the state of the following switched linear observer.

$$\frac{d}{dt}\hat{x}(t) = A_{\sigma(\hat{x},t)}\hat{x}(t) + L_{\sigma(\hat{x},t)}\{y(t) - C_{\sigma(\hat{x},t)}\hat{x}(t)\} + B_{\sigma(\hat{x},t)}u, \qquad (1)$$

where $\sigma(\hat{x},t) : \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{N}$ is the switching strategy which depends on the observer state $\hat{x}$ and $L_{\sigma(\hat{x},t)}$ is the observer gain.

Now, consider the following closed-loop system $\tilde{\Sigma}_\sigma$ combined by the switched system Σ_σ and the switched observer (1).

$$\tilde{\Sigma}_\sigma : \frac{d}{dt}\tilde{x}(t) = \begin{bmatrix} A_{\sigma(\hat{x},t)} & L_{\sigma(\hat{x},t)}C_{\sigma(\hat{x},t)} \\ 0 & A_{\sigma(\hat{x},t)} - L_{\sigma(\hat{x},t)}C_{\sigma(\hat{x},t)} \end{bmatrix}\tilde{x}(t) + \begin{bmatrix} B_{\sigma(\hat{x},t)} \\ 0 \end{bmatrix}u,$$

where $\tilde{x}(t) := [\hat{x}^{\mathrm{T}}(t)\ \ (x(t) - \hat{x}(t))^{\mathrm{T}}]^{\mathrm{T}}$ is the extended state vector.

The following lemma will be used to prove our main results.

Lemma1. [14] Suppose that $\varepsilon > 0,\ \eta > 0,$ $P_1 \in \mathbb{R}^{n\times n}(> 0)$ is a positive-definite matrix and $L_\sigma \in \mathbb{R}^{n\times p}, C_\sigma \in \mathbb{R}^{p\times n}$.

If $H_\sigma := -P_1 L_\sigma C_\sigma$ and

$$\alpha > \frac{\max_{\sigma\in\mathbb{N}} \|H_\sigma\|^2}{\varepsilon\eta},$$

then the following matrix

$$\tilde{P}_\sigma := \begin{bmatrix} \varepsilon I & -P_1 L_\sigma C_\sigma \\ -(P_1 L_\sigma C_\sigma)^{\mathrm{T}} & \alpha\eta I \end{bmatrix}$$

is positive-definite. □

Then, the following theorem can be obtained.

Theorem 1. Suppose that a switched system Σ_σ with constant input $u(t) = u$ is given. Let $x_e \in \mathbb{R}^n$ be given. Suppose that the following two conditions (i) and (ii) are satisfied:

(i) There exist $\lambda \in \Lambda$, a positive-definite matrix $P_1\ (> 0)$ and $\varepsilon > 0$ such that

$$A_\lambda^{\mathrm{T}} P_1 + P_1 A_\lambda < -\varepsilon I, \qquad (2)$$

$$A_\lambda x_e + B_\lambda u = 0. \qquad (3)$$

(ii)There exist a positive-definite matrix $P_2\ (> 0)$ and $Y_i \in \mathbb{R}^{n\times p}\ (i = 1, \cdots, N)$ such that

$$\begin{cases} A_1^{\mathrm{T}} P_2 + P_2 A_1 - C_1^{\mathrm{T}} Y_1^{\mathrm{T}} - Y_1 C_1 < -\eta I \\ \vdots \\ A_N^{\mathrm{T}} P_2 + P_2 A_N - C_N^{\mathrm{T}} Y_N^{\mathrm{T}} - Y_N C_N < -\eta I \end{cases} \qquad (4)$$

for some $\eta > 0$.

Then $\lim_{t\to\infty} x(t) = x_e$ for an arbitrary initial state $x_0 \in \mathbb{R}^n$ via the switching rule

$$\sigma(\hat{x},t) = \arg\min_{i\in\mathbb{N}} \hat{\xi} P_1 (A_i\hat{x} + B_i u),\ \ \hat{\xi} := \hat{x} - x_e \qquad (5)$$

which depends on the state $\hat{x}$ of switched observer (1) with observer gain matrices $L_i := P_2^{-1} Y_i\ \ (i = 1, \cdots, N)$.

Proof. Define $\tilde{\xi} := [\hat{\xi}^{\mathrm{T}}\ (\xi - \hat{\xi})^{\mathrm{T}}]^{\mathrm{T}}\ (\xi := x - x_e)$ and a quadratic form $V(\tilde{\xi})$ as $V(\tilde{\xi}) = \tilde{\xi}^{\mathrm{T}} \tilde{P} \tilde{\xi}$ .Denote $H_\sigma := -P_1 L_\sigma C_\sigma$ and

$$\tilde{P} := \begin{bmatrix} P_1 & 0 \\ 0 & \alpha P_2 \end{bmatrix},$$

where $\tilde{P}$ is a positive-definite matrix. Suppose that

$$\alpha > \frac{\max_{\sigma\in\mathbb{N}} \|H_\sigma\|^2}{\varepsilon\eta}\ \ (> 0\,),$$

Then, it follows from (2), (3), (4) and (5) that the time derivative of $V(\tilde{\xi})$ satisfies the following equations:

$$\dot{V}(\tilde{\xi}) = \dot{\tilde{x}}^{\mathrm{T}} \begin{bmatrix} P_1 & 0 \\ 0 & \alpha P_2 \end{bmatrix} \tilde{\xi} + \tilde{\xi}^{\mathrm{T}} \begin{bmatrix} P_1 & 0 \\ 0 & \alpha P_2 \end{bmatrix} \dot{\tilde{x}}$$

$$= \{A_\sigma \hat{x} + L_\sigma C_\sigma (x - \hat{x}) + B_\sigma u\}^{\mathrm{T}} P_1 \hat{\xi} + \alpha\{(A_\sigma - L_\sigma C_\sigma)(x - \hat{x})\}^{\mathrm{T}} P_2 (\xi - \hat{\xi})$$

$$
\begin{aligned}
&+\hat{\xi}^{\mathrm{T}}P_1\{A_\sigma\hat{x}+L_\sigma C_\sigma(x-\hat{x})+B_\sigma u\}\\
&+\alpha(\xi-\hat{\xi})^{\mathrm{T}}P_2\{(A_\sigma-L_\sigma C_\sigma)(x-\hat{x})\}\\
&=2\hat{\xi}^{\mathrm{T}}P_1(A_\sigma\hat{x}+B_\sigma u)+2\hat{\xi}^{\mathrm{T}}P_1L_\sigma C_\sigma(x-\hat{x})\\
&\quad+2\alpha(\xi-\hat{\xi})^{\mathrm{T}}P_2(A_\sigma-L_\sigma C_\sigma)(x-\hat{x})\\
&=2\hat{\xi}^{\mathrm{T}}P_1(A_\sigma\hat{x}+B_\sigma u)\\
&\quad+2\hat{\xi}^{\mathrm{T}}P_1L_\sigma C_\sigma\{(\xi+x_e)-(\hat{\xi}+x_e)\}\\
&\quad+2\alpha(\xi-\hat{\xi})^{\mathrm{T}}P_2(A_\sigma-L_\sigma C_\sigma)\\
&\qquad\cdot\{(\xi+x_e)-(\hat{\xi}+x_e)\}\\
&=2\min_{i\in\mathbb{N}}\{\hat{\xi}^{\mathrm{T}}P_1(A_i\hat{x}+B_iu)\}\\
&\quad+2\hat{\xi}^{\mathrm{T}}P_1L_\sigma C_\sigma(\xi-\hat{\xi})\\
&\quad+2\alpha(\xi-\hat{\xi})^{\mathrm{T}}P_2(A_\sigma-L_\sigma C_\sigma)(\xi-\hat{\xi})\\
&=\min_{i\in\mathbb{N}}[2\hat{\xi}^{\mathrm{T}}P_1\{A_i(\xi+x_e)+B_iu\}]\\
&\quad+2\hat{\xi}^{\mathrm{T}}P_1L_\sigma C_\sigma(\xi-\hat{\xi})\\
&\quad+\alpha(\xi-\hat{\xi})^{\mathrm{T}}\{(A_\sigma-L_\sigma C_\sigma)^{\mathrm{T}}P_2\\
&\qquad+P_2(A_\sigma-L_\sigma C_\sigma)\}(\xi-\hat{\xi})\\
&=\min_{i\in\mathbb{N}}\{\hat{\xi}^{\mathrm{T}}(A_i^{\mathrm{T}}P_1+P_1A_i)\hat{\xi}\\
&\qquad+2\hat{\xi}^{\mathrm{T}}P_1(A_ix_e+B_iu)\}\\
&\qquad+2\hat{\xi}^{\mathrm{T}}P_1L_\sigma C_\sigma(\xi-\hat{\xi})\\
&\quad+\alpha(\xi-\hat{\xi})^{\mathrm{T}}(A_\sigma^{\mathrm{T}}P_2+P_2A_\sigma\\
&\qquad-C_\sigma^{\mathrm{T}}Y_\sigma^{\mathrm{T}}-Y_\sigma C_\sigma)(\xi-\hat{\xi})\\
&<\min_{\lambda\in\Lambda}\{\hat{\xi}^{\mathrm{T}}(A_\lambda^{\mathrm{T}}P_1+P_1A_\lambda)\hat{\xi}\\
&\quad+2\hat{\xi}^{\mathrm{T}}P_1(A_\lambda x_e+B_\lambda u)\}\\
&\quad+2\hat{\xi}^{\mathrm{T}}P_1L_\sigma C_\sigma(\xi-\hat{\xi})-\alpha\eta(\xi-\hat{\xi})^{\mathrm{T}}(\xi-\hat{\xi})\\
&<-\varepsilon\hat{\xi}^{\mathrm{T}}\hat{\xi}+2\hat{\xi}^{\mathrm{T}}P_1L_\sigma C_\sigma(\xi-\hat{\xi})\\
&\qquad-\alpha\eta(\xi-\hat{\xi})^{\mathrm{T}}(\xi-\hat{\xi})\\
&=-[\hat{\xi}^{\mathrm{T}}\ (\xi-\hat{\xi})^{\mathrm{T}}]\begin{bmatrix}\varepsilon I & -P_1L_\sigma C_\sigma\\ -(P_1L_\sigma C_\sigma)^{\mathrm{T}} & \alpha\eta I\end{bmatrix}\\
&\qquad\cdot\begin{bmatrix}\hat{\xi}\\ \xi-\hat{\xi}\end{bmatrix}.
\end{aligned}
$$

Since $\alpha>\dfrac{\max_{\sigma\in\mathbb{N}}\|H_\sigma\|^2}{\varepsilon\eta}$ (>0), it follows from Lemma1 that

$$
\tilde{P}_\sigma:=\begin{bmatrix}\varepsilon I & -P_1L_\sigma C_\sigma\\ -(P_1L_\sigma C_\sigma)^{\mathrm{T}} & \alpha\eta I\end{bmatrix}
$$

is positive-definite. Hence $\dfrac{d}{dt}V(\tilde{\xi})<0$ which implies that $V(\tilde{\xi})$ is a Lyapunov function for the extended switched system $\tilde{\Sigma}_\sigma$ which implies

$$
\lim_{t\to\infty}\hat{\xi}(t)=\lim_{t\to\infty}(\hat{x}(t)-x_e)=0
$$

and

$$
\lim_{t\to\infty}(\xi(t)-\hat{\xi}(t))=\lim_{t\to\infty}(x(t)-\hat{x}(t))=0.
$$

Thus, $\lim_{t\to\infty}x(t)=x_e$ for an arbitrary initial state $x_0\in\mathbb{R}^n$ via the switching rule (2).This completes the proof of Theorem1. □

The following theorem says that the state x_e is asymptotically stable via the switched rule which is linear on $\hat{x}$ if there exists a common solution P_1 (>0) satisfying Lyapunov inequalities for stable subsystems matrices A_i $(i=1,\cdots,N)$.

Theorem 2. Suppose that a switched system $\sum_\sigma$ with constant input $u(t)=u$ and $x_e\in\mathbb{R}^n$ be given. If the following two conditions (i) and (ii) are satisfied, then $\lim_{t\to\infty}x(t)=x_e$ for an arbitrary initial state $x_0\in\mathbb{R}^n$ via the switching rule

$$
\sigma(\hat{x},t)=\arg\min_{i\in\mathbb{N}}\hat{\xi}P_1(A_ix_e+B_iu),\ \hat{\xi}:=\hat{x}-x_e \tag{6}
$$

which depends on the state $\hat{x}$ of switched observer (1) with $L_\sigma:=P_2^{-1}Y_\sigma$.

(i) There exist $\lambda\in\Lambda$, a positive-definite matrix P_1 (>0) and $\varepsilon>0$ such that

$$
A_i^{\mathrm{T}}P_1+P_1A_i<-\varepsilon I, \tag{7}
$$

$$
A_\lambda x_e+B_\lambda u=0. \tag{8}
$$

(ii)There exist a positive-definite matrix P_2 (>0) and $Y_i\in\mathbb{R}^{n\times p}$ such that

$$\begin{cases} A_1^{\mathrm{T}} P_2 + P_2 A_1 - C_1^{\mathrm{T}} Y_1^{\mathrm{T}} - Y_1 C_1 < -\eta I \\ \vdots \\ A_N^{\mathrm{T}} P_2 + P_2 A_N - C_N^{\mathrm{T}} Y_N^{\mathrm{T}} - Y_N C_N < -\eta I \end{cases} \tag{9}$$

for some $\eta > 0$.

Proof. Define $\tilde{\xi} := [\hat{\xi}^{\mathrm{T}}\ (\xi - \hat{\xi})^{\mathrm{T}}]^{\mathrm{T}}$ $(\xi := x - x_e)$ and a quadratic form $V(\tilde{\xi})$ as

$$V(\tilde{\xi}) = \tilde{\xi}^{\mathrm{T}} \tilde{P} \tilde{\xi},\ \tilde{P} := \begin{bmatrix} P_1 & 0 \\ 0 & \alpha P_2 \end{bmatrix},$$

where $\tilde{P}$ is apositive-definite matrix and

$$\alpha > \frac{\max_{\sigma \in \mathbb{N}} \|H_\sigma\|^2}{\varepsilon \eta} \ (>0), \qquad H_\sigma := -P_1 L_\sigma C_\sigma.$$

Then, it follows from (6), (7), (8) and (9) that the time derivative of $V(\tilde{\xi})$ satisfies the equations:

$$\begin{aligned} \dot{V}(\tilde{\xi}) &= \dot{\tilde{x}}^{\mathrm{T}} \begin{bmatrix} P_1 & 0 \\ 0 & \alpha P_2 \end{bmatrix} \tilde{\xi} + \tilde{\xi}^{\mathrm{T}} \begin{bmatrix} P_1 & 0 \\ 0 & \alpha P_2 \end{bmatrix} \dot{\tilde{x}} \\ &= \{A_\sigma \hat{x} + L_\sigma C_\sigma (x - \hat{x}) + B_\sigma u\}^{\mathrm{T}} P_1 \hat{\xi} \\ &\quad + \alpha \{(A_\sigma - L_\sigma C_\sigma)(x - \hat{x})\}^{\mathrm{T}} P_2 (\xi - \hat{\xi}) \\ &\quad + \hat{\xi}^{\mathrm{T}} P_1 \{A_\sigma \hat{x} + L_\sigma C_\sigma (x - \hat{x}) + B_\sigma u\} \\ &\quad + \alpha (\xi - \hat{\xi})^{\mathrm{T}} P_2 \{(A_\sigma - L_\sigma C_\sigma)(x - \hat{x})\} \\ &= 2\hat{\xi}^{\mathrm{T}} P_1 (A_\sigma \hat{x} + B_\sigma u) + 2\hat{\xi}^{\mathrm{T}} P_1 L_\sigma C_\sigma (x - \hat{x}) \\ &\quad + 2\alpha (\xi - \hat{\xi})^{\mathrm{T}} P_2 (A_\sigma - L_\sigma C_\sigma)(x - \hat{x}) \\ &= 2\hat{\xi}^{\mathrm{T}} P_1 A_\sigma \xi + 2\hat{\xi}^{\mathrm{T}} P_1 (A_\sigma x_e + B_\sigma u) \\ &\quad + 2\hat{\xi}^{\mathrm{T}} P_1 L_\sigma C_\sigma \{(\xi + x_e) - (\hat{\xi} + x_e)\} \\ &\quad + 2\alpha (\xi - \hat{\xi})^{\mathrm{T}} P_2 (A_\sigma - L_\sigma C_\sigma) \\ &\qquad \cdot \{(\xi + x_e) - (\hat{\xi} + x_e)\} \\ &= \hat{\xi}^{\mathrm{T}} (A_\sigma^{\mathrm{T}} P_1 + P_1 A_\sigma) \xi \\ &\quad + 2 \min_{i \in \mathbb{N}} \hat{\xi}^{\mathrm{T}} P_1 (A_i x_e + B_i u) \\ &\quad + 2\hat{\xi}^{\mathrm{T}} P_1 L_\sigma C_\sigma (\xi - \hat{\xi}) \\ &\quad + 2\alpha (\xi - \hat{\xi})^{\mathrm{T}} P_2 (A_\sigma - L_\sigma C_\sigma)(\xi - \hat{\xi}) \\ &< -\varepsilon \hat{\xi}^{\mathrm{T}} \hat{\xi} + 2 \min_{\lambda \in \Lambda} \hat{\xi}^{\mathrm{T}} P_1 (A_i x_e + B_i u) \\ &\quad + 2\hat{\xi}^{\mathrm{T}} P_1 L_\sigma C_\sigma (\xi - \hat{\xi}) \\ &\quad + \alpha (\xi - \hat{\xi})^{\mathrm{T}} \{(A_\sigma - L_\sigma C_\sigma)^{\mathrm{T}} P_2 \\ &\qquad + P_2 (A_\sigma - L_\sigma C_\sigma)\} (\xi - \hat{\xi}) \\ &= -\varepsilon \hat{\xi}^{\mathrm{T}} \hat{\xi} + 2\hat{\xi}^{\mathrm{T}} P_1 L_\sigma C_\sigma (\xi - \hat{\xi}) \\ &\quad + \alpha (\xi - \hat{\xi})^{\mathrm{T}} (A_\sigma^{\mathrm{T}} P_2 + P_2 A_\sigma \\ &\qquad - C_\sigma^{\mathrm{T}} Y_\sigma^{\mathrm{T}} - Y_\sigma C_\sigma)(\xi - \hat{\xi}) \\ &< -\varepsilon \hat{\xi}^{\mathrm{T}} \hat{\xi} + 2\hat{\xi}^{\mathrm{T}} P_1 L_\sigma C_\sigma (\xi - \hat{\xi}) \\ &\qquad - \alpha \eta (\xi - \hat{\xi})^{\mathrm{T}} (\xi - \hat{\xi}) \\ &= -[\hat{\xi}^{\mathrm{T}}\ (\xi - \hat{\xi})^{\mathrm{T}}] \begin{bmatrix} \varepsilon I & -P_1 L_\sigma C_\sigma \\ -(P_1 L_\sigma C_\sigma)^{\mathrm{T}} & \alpha \eta I \end{bmatrix} \\ &\qquad \cdot \begin{bmatrix} \hat{\xi} \\ \xi - \hat{\xi} \end{bmatrix}. \end{aligned}$$

Hence, it follows from Lemma1 that $\frac{d}{dt} V(\tilde{\xi}) < 0$ which implies that $V(\tilde{\xi})$ is a Lyapunov function for the extended switched system $\tilde{\Sigma}_\sigma$. Thus, $\lim_{t \to \infty} x(t) = x_e$ for an arbitrary initial state $x_0 \in \mathbb{R}^n$ via the switching rule (6). □

3. Illustrative Numerical Examples

In this section, two numerical examples for Theorem1and Theorem2 will be shown.

3.1 An example for Theorem 1

Consider the two-dimensional switched linear system Σ_σ with constant input which consists of two subsystems and switched observer (1). Here, each subsystem's matrices A_i, input matrices B_i and output matrices C_i, are as follows. In this example, we note that A_1 and A_2 are unstable.

$$A_1 = \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix},\ B_1 = \begin{bmatrix} 3 \\ 3 \end{bmatrix},\ C_1 = \begin{bmatrix} 4 & 2 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix},\ B_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix},\ C_2 = \begin{bmatrix} 6 & 1 \end{bmatrix}.$$

Now, if we choose an input $u = 1$, a positive definite matrix $P_1 = I_2$, $\varepsilon = 1/2$, parameters

$$\lambda_1 = 0.4, \quad \lambda_2 = 0.6 \quad (\lambda_1 + \lambda_2 = 1)$$

and $x_e = [2\ 1]^{\mathrm{T}}$, then the condition (i) of Theorem1 is satisfied.

Next, if we choose the observer gain

$$L_1 := P_2^{-1} Y_1,\ L_2 := P_2^{-1} Y_2,$$

where $P_1 = I_2$, $Y_1 := \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, $Y_2 := \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ and $\eta = 1/8$, then the condition (ii) of Theorem1 is also satisfied. Thus, x_e is globally asymptotically stable via the following switched rule

$$\sigma(\hat{x}, t) = \arg\min_{i \in \{1,2\}} \hat{\xi} P_1 (A_i \hat{x} + B_i u). \tag{10}$$

In fact, for this example, if we choose an initial state $x_0 = [3\ 6]^{\mathrm{T}}$ of the system Σ_σ and $x_0 =$ $[3\ 5.5]^{\mathrm{T}}$ of the switched observer (1), then we have

$$\begin{cases} \hat{\xi}_0 P_1 (A_1 \hat{x}_0 + B_1 u) = 26.7500 \\ \hat{\xi}_0 P_1 (A_2 \hat{x}_0 + B_2 u) = -59.2500 \end{cases}, \hat{\xi}_0 := \hat{x}_0 - x_e.$$

According to the switching rule $\sigma(\hat{x}, t)$ in (10), $\sigma(\hat{x}_0, 0) = 2$ is chosen. Further, if we choose the number of subsystems according to the switching rule in the same way, then we can see the states $x(t)$ and $\hat{x}(t)$ go to x_e as t tends to ∞ (see Figure 1).

3.2 An example for Theorem 2

Consider the two-dimensional switched linear system Σ_σ with constant input which consists of two subsystems and switched observer (1). Here, each subsystem's matrices A_i, input matrices B_i and output matrices C_i, are as follows.

$$A_1 = \begin{bmatrix} -2 & -1 \\ 0 & -3 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 7 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 2 & 1 \end{bmatrix}.$$

Now, if we choose an input $u = 1$, a positive definite matrix $P_1 = I_2$, $\varepsilon = 1/2$, parameters

$$\lambda_1 = 0.2, \quad \lambda_2 = 0.8 \quad (\lambda_1 + \lambda_2 = 1)$$

and $x_e = [2\ \ 1]^{\mathrm{T}}$, then the condition (i) of Theorem2 is satisfied.

Next, if we choose the observer gain

$$L_1 := P_2^{-1} Y_1, \quad L_2 := P_2^{-1} Y_2,$$

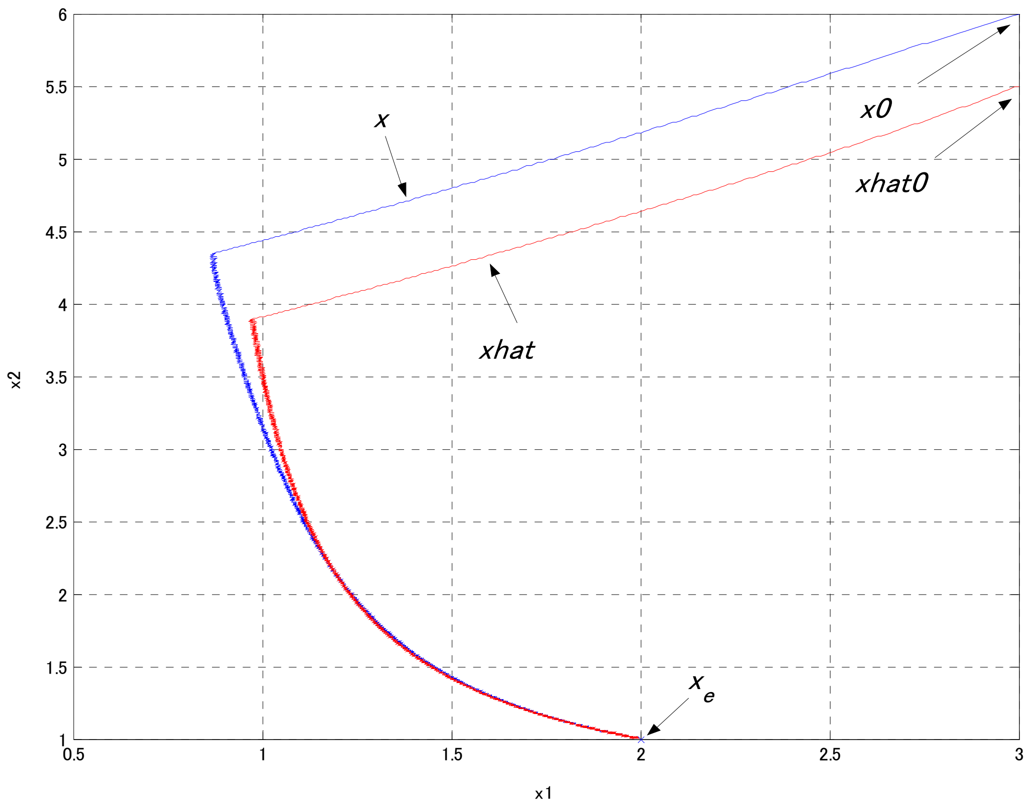

Figure 1. State trajectories of $x(t)$ and $\hat{x}(t)$

where $P_1 = I_2, Y_1 := \begin{bmatrix} 1 \\ 0 \end{bmatrix}, Y_2 := \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and a parameter $\eta = 1$, then the condition (ii) of Theorem2 is satisfied. Thus, x_e is globally asymptotically stable via the following switched rule

$$\sigma(\hat{x},t) = \arg \min_{i\in\{1,2\}} \hat{\xi} P_1 (A_i x_e + B_i u). \tag{11}$$

In fact, for this example, if we choose an initial state $x_0 = [6\ \ 3]^{\mathrm{T}}$ of the system Σ_σ and $x_0 =$ $[6\ \ 2.7]^{\mathrm{T}}$ of the switched observer (1), then we have

$$\begin{cases} \hat{\xi}_0 P_1 (A_1 \hat{x}_0 + B_1 u) = -9.2000 \\ \hat{\xi}_0 P_1 (A_2 \hat{x}_0 + B_2 u) = -2.3000 \end{cases}, \quad \hat{\xi}_0 := \hat{x}_0 - x_e.$$

According to the switching rule $\sigma(\hat{x},t)$ in (11), $\sigma(\hat{x}_0, 0) = 1$ is chosen. Further, if we choose the number of subsystems according to the switching rule in the same way, then we can see the states $x(t)$ and $\hat{x}(t)$ go to x_e as t tends to ∞ (see Figure 2).

4. Concluding Remarks

In this paper, stabilizability for continuous-time switched linear systems with constant input via switched observer was investigated. Firstly, the conditions for equilibrium points related to the switched linear system with constant input to be globally asymptotically stable via switched observer were presented. Next, two numerical examples to illustrate the main results (Theorem 1 and Theorem 2) were also shown, respectively. As future studies we need to investigate parameter insensitive stabilization problems for switched linear systems, with constant input via switched observer.

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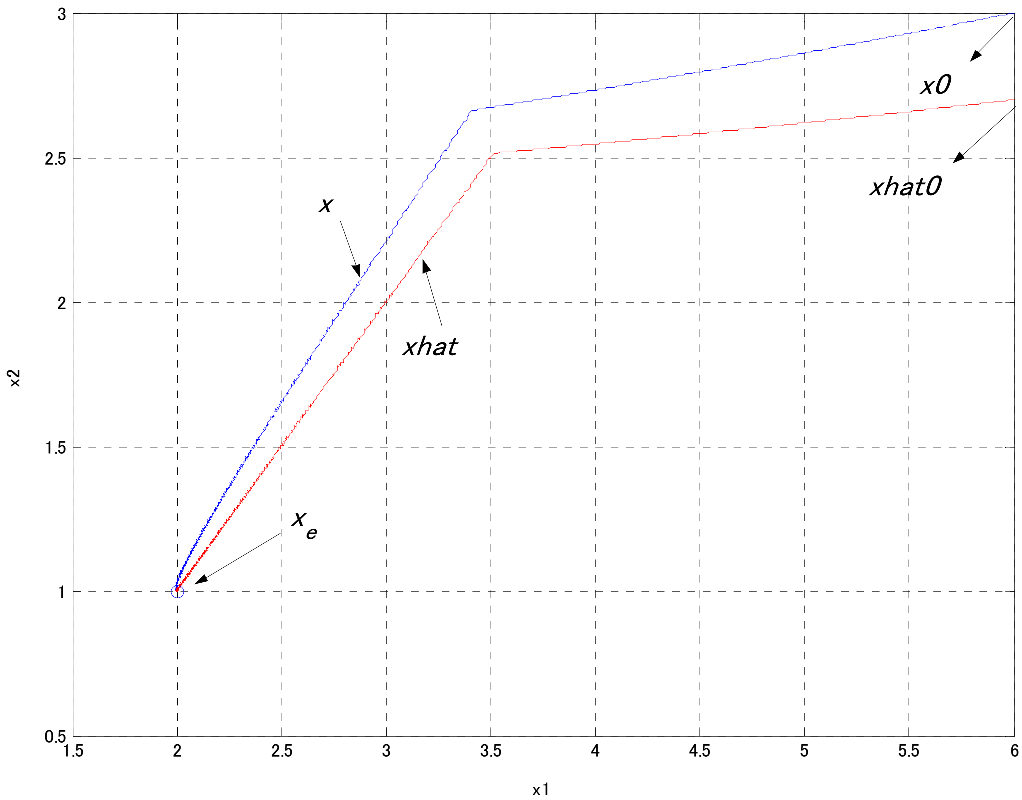

Figure 2. State trajectories of $x(t)$ and $\hat{x}(t)$

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