Algorithms for the Recognition of Net-free Graphs and for Computing Maximum Cardinality Matchings in Claw-free Graphs

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Abstract: During the last decades, numerous studies have been undertaken on the classes of net-free graphs, claw-free graphs, and the relationship between them. The notion of weakly decomposition (a partition of the set of vertices in three classes *A, B, C* such that *A* induces a connected graph and *C* is totally adjacent to *B* and totally non-adjacent to *A*) and the study of its properties allow us to obtain several important results such as: characterization of cographs, $\{P_4,C_4\}$-free and paw-free graphs. In this article, we give a characterization of net-free graphs, a characterization of claw-free graphs, using weakly decomposition. Also, we give a recognition algorithm for net-free graphs, an algorithm for determining a maximum matching in claw-free graphs, comparable with existing algorithms in terms of complexity, but using weakly decomposition.

Keywords: net-free graphs, claw-free graphs, asteroidal triple-free graphs, weakly decomposition, recognition algorithm, maximum matching in graphs.

1. Introduction

A graph is *claw-free* if it has no induced subgraph isomorphic to the claw, i.e., the four-vertex star $K_{1,3} = (\{a_1, a_2, a_3, b\}, \{ba_1.ba_2, ba_3\})$. A *net* is a graph obtained from a triangle by attaching to each vertex a new dangling edge.

The interval graphs [24], permutation graphs [16] and co-comparability graphs [18] have a linear structure. Each of these classes is a subfamily of the asteroidal triple graphs (AT-free graphs, for short). An independent set of three vertices is called an asteroidal triple if between any pair in the triple there exists a path that avoids the neighbourhood of the third. AT-free graphs were introduced by Lekkerkerker and Boland [24]. Corneil, Olariu and Stewart showed a number of results on the linear structure of AT-free [8, 9, 10].

A maximal subclass of a class of net-free graphs is the class (claw,net)-free graphs (CN-free graphs, for short). Also note that CN-free graphs are exactly the Hamiltonian-hereditary graphs[13] (was cited in [4]). CN-free graphs turn out to be closely related to AT-free graphs form their structure properties [4]. There are, however, few results about the structure of these graphs [4]. In [4] the authors give results on the linear and circular structure of CN-free graphs. AT-free graphs can be generalized in a manner obvious to admit circular structure [4]. CN-free graphs were introduced by Duffus [14]. Although CN-free graphs seems to be quite restrictive, it contains a couple of families of graphs that are interesting in their own right.

In this paper we give an algorithm for the recognition of net-free graph of complexity $O(n(n+ m^{1,63}))$. Also, we give an interesting property of claw-free graph that leads to a algorithm for the construction a maximum matching in claw-free graphs.

The content of the paper is organized as follows. In Preliminaries, we give the usual terminology in graph theory. In Section 3 we give a characterization of net-free graphs and a recognition algorithm using the weakly decomposition. In Section 4 we determine a maximum matching in the claw-free graph. Ideas for future work conclude the paper.

2. Preliminaries

Throughout this paper, *G=(V,E)* is a connected, finite and undirected graph ([3]), without loops and multiple edges, having *V=V(G)* as the vertex set and *E=E(G)* as the set of edges. $\overline{G}$ is the complement of *G*. If $U \subseteq V$, by *G(U)* we denote the subgraph of *G* induced by *U*. By *G-X* we mean the subgraph *G(V-X)*, whenever $X \subseteq V$, but we simply write *G-v*, when *X={v}*. If *e=xy* is an edge of a graph G, then *x* and *y* are adjacent, while *x* and *e* are incident, as are *y* and e. If $xy \in E$, we also use $x \sim y$, and $x \nsim y$ whenever x,y are not adjacent in G. If $A,B \subseteq V$ are disjoint and $ab \in E$ for every $a \in A$ and $b \in B$, we say that A,B are *totally adjacent* and we denote by $A \sim B$, while by $A \nsim B$ we mean that no edge of G joins some vertex of A to a vertex from B and, in this case, we say A and B are *totally non-adjacent*.

The *neighborhood* of the vertex $v \in V$ is the set $N_G(v)=\{u \in V: uv \in E\}$, while $N_G[v]=N_G(v) \cup \{v\}$; we denote $N(v)$ and $N[v]$, when G appears clearly from the context. The *degree* of v in G is $d_G(v)=|N_G(v)|$. The neighborhood of the vertex v in the complement of G will be denoted by $\overline{N}(v)$.

The neighborhood of $S \subseteq V$ is the set $N(S)=\cup_{v \in S} N(v)-S$ and $N[S]=S \cup N(S)$. A graph is complete if every pair of distinct vertices is adjacent.

By P_n, C_n, K_n we mean a chordless path on $n \geq 3$ vertices, a chordless cycle on $n \geq 3$ vertices, and a complete graph on $n \geq 1$ vertices, respectively.

Let F denote a family of graphs. A graph G is called *F-free* if none of its subgraphs are in F.

The *Zykov sum* of the graphs G_1, G_2 is the graph $G=G_1+G_2$ having:

$V(G)=V(G_1) \cup V(G_2)$,

$E(G)=E(G_1) \cup E(G_2) \cup \{uv: u \in V(G_1), v \in V(G_2)\}$.

3. A New Characterization of Net-free Graphs using the Weakly Decomposition

Formation control is an important issue of motion coordination of Multi-agent Robots Systems, it is based on the properties of the formation graphs ([20]).

We recall a characterization of the weakly decomposition of a graph.

Definition 1. ([32], [33]) *A set $A \subseteq V(G)$ is called a weakly set of the graph G if $N_G(A) \neq V(G)-A$ and G(A) is connected. If A is a weakly set, maximal with respect to set inclusion, then G(A) is called a weakly component. For simplicity, the weakly component G(A) will be denoted with A.*

Definition 2. ([32], [33]) *Let G=(V,E) be a connected and non-complete graph. If A is a weakly set, then the partition $\{A, N(A), V-A \cup N(A)\}$ is called a weakly decomposition of G with respect toA.*

The name of *weakly component* is justified by the following result.

Theorem 1.([32], [33]) *Every connected and non-complete graph G=(V,E) admits a weakly component A such that $G(V-A)=G(N(A))+G(\overline{N}(A))$.*

Theorem 2.([11], [12]) Let G=(V,E) be a connected and non-complete graph and $A \subseteq V$. Then A is a *weakly component of G if and only if G(A) is connected and $N(A) \sim \overline{N}(A)$.*

The next result, that follows from Theorem 2, ensures the existence of a weakly decomposition in a connected and non-complete graph.

Corollary 1. *If G=(V,E) is a connected and non-complete graph, then V admits a weakly decomposition (A,B,C), such that G(A) is a weakly component and G(V-A)=G(B)+G(C).*

Theorem 2 provides an *O(n+m)* algorithm for building a weakly decomposition for a non-complete and connected graph.

Algorithm for the weakly decomposition of a graph ([32])

Input: A connected graph with at least two nonadjacent vertices, G=(V,E).

Output: A partition V=(A,N,R) such that G(A) is connected, N=N(A), $A \nsim R=\overline{N}(A)$.

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Begin
    A:= any set of vertices such that A ∪ N(A) ≠ V
    N:= N(A)
    R := V-A ∪ N(A)
    While (∃ n∈N, ∃ r∈R such that nr∉ E) do
        Begin
            A:= A ∪ {n}
            N:= (N-{n}) ∪ (N(n) ∩ R)
            R:=R-(N(n) ∩ R)
        end
end
```

A new characterization of net-free graphs, using weakly decomposition, is given below.

Theorem 3. Let G=(V,E) be a connected and non-complete graph. Let (A,N,R) be a weakly decomposition with G(A) a weakly component. G=(V,E) is net-free if and only if:

i) does not exist P_4 in G(A) and n in N such that n is adjacent with the middle vertices of the P_4 specified;

ii) (does not exist P_4, with extremities in A and the middle vertices in N) or (does not exist t in N such that his neighbours t are not in P_4 specified);

iii) G(V-R), G(V-A) are net-free.

Proof.} Let G be net-free. Since the property of being net-free is hereditary as follows G(V-A) and G(V-R) net-free graphs, so iii) holds. If $\exists n \in N$, $\exists P_4 \subseteq G(A)$ such that n is adjacent to the middle vertices in P_4, so $\exists P_4$: a, b, c, d with a, b, c, d $\in$ A, ab, bc, cd $\in$E, ac, ad, bd $\notin$E and n b, n c $\in$E, then, because N~R, it follows that $\forall r \in R$: G({ a,b,c,d,n,r}) is net, a contradiction. So i) holds. If $\exists P_4$: ab, bc, cd with a,d $\in$A, b, c $\in$N and $\exists t \in N$ with ta,tb, tc, td $\notin$E then G({a,b,c,d,r,t}) is net $\forall r \in R$, a contradiction. So ii) holds.

Conversely, we suppose that i), ii), iii) hold and to show that G is net-free. From iii) it follows that G(A), G(N), G(R), G(A$\cup$N) and G(N $\cup$R) are net-free. Because G(A$\cup$R) is not connected it follows that G(A$\cup$R) is net-free. Suppose, however, that there is H=G({a,b,c,1,2,3}) an subgraph net, with the vertices a, b, c of the degree 1, the vertices 1, 2, 3 of degree 3, and a 1, b 2, c 3 $\in$ E.

Case 1. Let $|V(H)\cap R| = 1$. We assume 1.1. V(H) $\cap$ R = {a}. 1.2. V(H)$\cap$R ={1}. 1.1. From R~N it follows V(H) $\cap$N={1}. So V(H) $\cap$A={c,3,2,b}. But G({ c,3,2,b}) is P_4 and 1 is adjacent with the middle vertices in P_4, thereby contradicting with i). 1.2. From R~N it follows V(H) $\cap$N={a,2,3}. So V(H) $\cap$A= {b,c}. P=G({b,c,2,3}) is an P_4, with extremities b, c $\in$A and the middle vertices 2,3$\in$N. For t=a we have N(t) $\cap$V(P)=ϕ, thereby contradicting with ii). So, Case 1 holds not.

Case 2. Let $|V(H) \cap R|=2$. There are subcases 2.1. V(H) $\cap$R={a,1}; 2.2. V(H) $\cap$R={1,2}; 2.3. V(H) $\cap$ R={a,2}; 2.4. V(H) $\cap$R={a,b}.

2.1.cannot hold because the vertices a and 1 have no common neighbors. 2.2. cannot hold because the vertices 1 and 2 not have only common neighbors. 2.3. cannot hold because the vertices a and 2 not have only common neighbors. 2.4. cannot hold because the vertices a and b have no common neighbours.

Case 3. $|V(H) \cap R|=3$ cannot hold because the vertices in {a,1,2} and in {a,1,b} not have only common neighbours.

Case 4. Let $|V(H) \cap R|=4$. Any subset $X \subseteq V(H)$ with $|X|=4$ has the property that its vertices are not only common neighbours, that is $\exists v \in V-X$ such that v is adjacent some of the vertices of X, and with the rest of them, v it is not adjacent.

$|V(H) \cap R| \in \{5,6\}$ it is not possible, because $V(H) \cap A \neq \phi$ si $V(H) \cap N \neq \phi$.

Theorem 3 provides the following recognition algorithm for net-free graphs.

Algorithm Recognition

Input: A connected, non-complete graph G=(V,E).

Output: An answer to the question: "Is G net-free"?

Begin
1. $L_G \leftarrow$ {G}
2. *while* $L_G \neq \phi$ *do*
3. extract an element H in L
4. determine the weakly decomposition (A,N,R) with $[A]_H$ weakly component
5. *if* ($\exists P_4$ in G(A) and $\exists n \in N$ such that n is adjacent with the middle vertices in P_4) *then*
   G is not net-free *else*
6. *if* (($\exists P_4$, with the extremities in A and the middle vertices in N) and (there are t in N whose neighbourhood there crosses the vertices of P_4 specified)) *then*
   G is not net-free *else*
7. enter in L subgraphs [V-R], [V-A]
8. Return: G is net-free
9. *end*
EndRecognition

As Theorem 2 provides an *O(n+m)* algorithm for building a weakly decomposition for a non-complete and connected graph and as for the recognition P_4-free in *O(n+m)* ([17]) time (Finding small cycles in undirected graphs in $O(m^{1,63})$ ([1]) time), so step 5 and step 6 run in $O(n+ m^{1,63})$ time, it follows that, in total, the algorithm is run in $O(n(n+ m^{1,63}))$.

4. Determining a Maximum Matching within a Claw-free Graph

Software testing is an important process that helps to develop high quality software. Test data generation can be done using combinatorial optimizing techniques. ([28]).

Some interesting properties of claw-free graphs have been established in ([2], [6], [17], [22], [25], [30]). In [21] the authors present new algorithms for the recognition of claw-free graphs ($O(e^{1,69})$). In [7] the authors mention the structure theory of claw-free graphs.

In [5] the authors consider the algorithmic problem of finding a Hamiltonian path or a Hamiltonian cycle efficiently.

In what follows we characterize the claw-free graphs and we give an algorithm for constructing a cardinal maximum matching in a claw-free graph.

A similar result is found in ([32])

Theorem 4. *Let G=(V,E) be a connected and non-complete graph. Let (A,N,R) a weakly decomposition with G(A) a weakly component. G=(V,E) is claw-free if and only if:*

i) R and N(n)$\cap$A are cliques, $\forall$ n$\in$ N

ii) G(V-R), G(V-A) are claw-free.

Proof.} Let G be a claw-free graph. From he heredity of the claw-free graphs it follows that G(V-A) and G(V-R) are claw-free graphs. If there would be r_1, $r_2 \in$ R such that r_1 $r_2 \notin$ E, because R~N, $\forall$n $\in$N and a $\in$N(n) $\cap$A (N(n) $\cap A \neq \phi$, $\forall$ n $\in$N according to his N), G({a, n, r_1, r_2}) is isomorphic to claw.

Show of the reverse implication. We assume that there are { x,a,b,c} a claw with center in x. From ii) results that G(A$\cup$N) and G(N$\cup$R) are claw-free graphs. So x$\notin$A$\cup$R, that is x$\in$N. From i), two of the vertices a, b, c are necessarily in N, that is {x, a,b,c} are in A$\cup$N or in N$\cup$R, thereby contradicting with ii).

Theorem 4 leads to a recognition algorithm for a claw-free graph that shows the combinatorial structure of the graph the decomposition mode used. Because the complexity, of this algorithm is not better than the most efficient known, not present.

Sumner ([31]) and, independently, Las Vergnas ([23]) proved that every claw-free connected graph with an even number of vertices has a perfect matching.

It is known that if G is a connected graph there is a vertex v$\in$V(G) such that G-v is connected.

In [29], Sumner shows that in any connected claw-free graphs one can find a pair of adjacent vertices the removal of which leaves the remaining graphs connected.

An interesting question would be to determine the connected graphs with the $\exists$Q$\subset$V(G), Q the maximum clique in relation to inclusion so that G-Q is connected. This property is related to the existence of induced subgraphs claw. The following result.

Theorem 5. *Let G=(V,E) be a connected and non-complete graph. Let (A,N,R) be a weakly decomposition with G(A) a weakly component. G is claw-free if and only if for any induced subgraph H of G, H connected, there is a maximal clique Q in H such that H-Q is connected.*

Proof. Let G be a connected and claw-free graph. Because the property of being claw-free is hereditary, $\forall$H, induced subgraph of G, H connected, H is claw-free. From Theorem 7 it follows that R is clique. Let Q$\subseteq$N be a maximal clique in $[N]_H$. Then Q_1=Q$\cup$R is a maximal clique in H (in N~R it follows that Q_1 is clique; Q_1 is maximal because $\forall$x$\in$V(H)-Q_1, we have or x$\in$A and xr$\notin$E(H), $\forall$r$\in Q_1 \cap$R or x$\in$N-Q and because Q is maximal, $\exists$y$\in Q_1 \cap$N such that x y $\notin$E(H)). Because $[A]_H$ is connected and $\forall$x$\in$N-Q_1, $\exists$a$\in$A such that ax$\in$E(H), it follows that H-Q_1 is connected.

Show of the reverse implication. We assume that there is H isomorphic a induced claw in G, H= G({x,a,b,c}), x being the center of H. Then the maximum cliques of H are {x, a}, {x, b}, {x,c} and every one of them disconnects H.

As Edmonds ([15]) showed, a maximum matching in any graph may be found in polynomial time. Sbihi ([27]) extended this algorithm to one that computes a maximum independent set in any claw-free graph.Minty ([26]) (corrected by Nakamura, Tamura [27]) independently provided an alternative extension of Edmonds' algorithms to claw-free graphs.

Theorem 5 provides an algorithm for constructing a cardinal maximum matching in a claw-free graph.

Input: A connected, non-complete, claw-free graph

G=(V,E).

Output: A cardinal maximum matching in G

begin
1. M= ϕ
2. *If* G is complete *then*
3. Choose $\frac{|V(G)|}{2}$ independent edges and be M_1 set their
4. *Return* M=M $\cup$ M_1
5. *else*
6. Determine a weakly decomposition (A,N,R) for G with G(A) a weakly component
7. Determine Q, maximal clique in G(N)
8. Let Q_1 = Q $\cup$ R
9. Choose in the clique Q_1, $\frac{|Q_1|}{2}$ independent set and be M_1 set their
10. *Return* M = M $\cup$ M_1
11. Let X (the empty set or the set with the only vertex in Q_1)
12. G=(G-Q_1) $\cup$ X (connected and claw-free) and back to 2

end

Step 7 is run in *O(n(n+m))}*

Begin
71. $L_G \leftarrow$ {G(N)}
72. Q= ϕ
73. *while* $L_G \neq \phi$ *do*
74. extract an element H in L_G
75. determine the weakly decomposition (A,N,R) with $[A]_H$ weakly component
76. because G(N) is claw-free, R clique and R~N
77. Q $\leftarrow$ Q $\cup$ R

end}

End

As Theorem 2 provides an *O(n+m)* algorithm for building a weakly decomposition for a non-complete and connected graph, in total, the algorithm is run in *O(n(n(n+m)))*.

5. Conclusions and Future Work

In this paper we have given a recognition algorithm for net-free graphs and an algorithm for determining a maximum matching in claw-free graphs. Our future work is going to put forward some applications of {net,claw}-free.

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