Algebraic State Estimation for a Class of Switched Linear Systems

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Abstract: In this paper, an algebraic state estimation method for a class of switched linear systems is derived. This approach is based on algebraic tools and distribution theory. Firstly, the unknown switching instant and the active mode are identified on-line, and then the process of state estimation is given by an explicit algebraic formula, rather than by an auxiliary dynamic system, which can be implemented formally and estimated very fast in computer. Numerical example and Simulations illustrate the efficiency of the proposed techniques.

Keywords: Algebraic approach, State estimation, Switching instants identification, Switched linear systems.

1. Introduction

Hybrid dynamical systems (HDS), in which continuous dynamics and discrete events coexist and interact between each other, can be used to model a large number of practical systems. Switched systems as higher--level abstractions of HDS, obtained by neglecting the details of the discrete behaviour. A family of dynamical linear subsystems and a switching law, which orchestrates the switching between them, can compose a switched linear system (see [1] for surveys).

In the recent years, there has been an increasing interest in the control problems of switched linear systems due to their significance from both a theoretical and practical point of view. Important results for switched systems have been achieved for problems of stability analysis [2], stabilization [3-5], tracking design [6] or controllability [7, 8].

Observability and state estimation is a very challenging problem for such systems since both the active mode and the continuous state have to be estimated during a finite time interval. The notion of state estimation for switched systems was firstly introduced in [9]. Observability notions for some classes of hybrid systems such as switched linear systems has been discussed and characterized in recent works such as [1,10,11].The problem is to recover from available measurements the state of the system and/or the switching signal, and eventually the switching time. Different observation and identification methods have been performed during the last years [12-20]: state estimation for nonlinear switched system using Petri Net [12], for linear switched systems with unknown inputs [13], or an original and effective sampled and delayed output observer design which is based on hybrid switching systems [16] etc. Usually, the hybrid observer consists of two parts: an index estimator of the current active sub-model and a continuous observer that estimates asymptotically in most cases, the continuous state of the hybrid system.

The aim of this paper is to estimate the switching instants, active mode and the continuous state of a class of switched linear systems with the knowledge of the first active mode. The possibility to have finite time estimate for this kind of systems is clearly important. The approach considered here takes root in recent works developed in [21] for parameter identification of linear time-invariant systems. This method is based on algebraic tools (differential algebra, module theory and operational calculus) and results in finite time estimates given by explicit algebraic formula that can be implemented in a straightforward manner using standard tools from computational mathematics. Those results have been extended to the problems of closed-loop parametric estimation for continuous-time linear systems in [22], state estimation of linear systems in [23] or with time-varying parameters in [24], fault diagnosis in [25], nonlinear systems with unknown inputs in [26] or nonlinear feedback control in [27], switched systems estimation with Zeno phenomenon in [28]. This approach was also applied in [29] for the estimation of the index corresponding to the current active subsystem, and the state variable of this subsystem. Based on the result in [30, 31], finite time identification of the switching instants and the active mode are firstly studied, and the switching time estimation is given by an explicit formula, as a function of the integral of the output, in order to attenuate the influence of measurement noises. Then, combining our results of state estimation for linear time invariant systems by algebraic approach [23], we give the main approach of current active mode estimation and the continuous state estimation in real time.

This paper is organized as follows: Section 2 gives the problem statement and the mathematical formulation. The main result is derived in Section 3 & Section 4. First, the switching instant identification of one commutation between two modes is analyzed. Then, the result is extended to the case of commutations among an arbitrary number of modes. In Section 4, with the estimated switching instants sequence and the algebraic state estimator for each mode ([23]), the state estimation of the autonomous switched linear systems is achieved. In section 5, simulation results that illustrate the proposed approach are provided. Finally, the last section is devoted to main conclusions and future works.

2. Preliminary

2.1 Problem statement

In this work, we study a class of switched linear systems called **autonomous switched systems** [1], i.e. the evolution of system determined by a collection of linear subsystems (Q modes) with continuous state connected by switches among a number of discrete state $q \in I_Q \triangleq \{1,\dots,Q\}$, modelled by:

$$\dot{x} = A_q x, \tag{2.1}$$

where $x \in \mathbb{R}^n$ is continuous state, and $A_q \in \mathbb{R}^{n\times n}$ are constant matrices. For such a system, the switches are arbitrary and independent of the systems state variable. For the sake of convenience and without loss of generality, it is assumed that at each time $t \in \mathbb{R}$, only one discrete event acts on the system.

The necessary and sufficient conditions of the state observability of the autonomous switched linear systems (continuous state and the active mode) are derived in [1]. Note that the systems studied in this paper are assumed satisfied these conditions and observable. The objective is to estimate the state of the switched system (2.1) assuming that the first active mode q is known. To do this, the crucial step is to estimate the unknown switching instant and the sequence of active modes. Hereafter, we will apply the algebraic method which involves high order time derivatives of $x(t)$. Due to the presence of non-smooth dynamics, derivation in this article has to be understood in the distribution sense.

2.2 Mathematical formulation

We recall here some standard definitions and results from distribution theory developed in [32], and fix the notations to be used. The space of C^∞ - functions having compact support in an open subset Ω of $\mathbb{R}$ is denoted by $\mathcal{D}(\Omega)$, and $\mathcal{D}'(\Omega)$ is the space of distributions on Ω, i.e., the space of continuous linear functional on $\mathcal{D}(\Omega)$. For $u \in \mathcal{D}'$, $<u,\varphi>$ denotes the real number which linearly and continuously depends on $\varphi \in \mathcal{D}$. This number is defined as $<u,\varphi> = \int_{-\infty}^{\infty} f\varphi$ for a locally Lebesgue integrable function $u = f$. The support of a distribution u, denoted as $\operatorname{supp} u$, is defined as the complement of the largest open subset of Ω in which the distribution u vanishes.

For the Dirac distribution $u = \delta$ and its derivative $\dot{u} = \dot{\delta}$, the functional is defined as $<u,\varphi> = \varphi(0)$ and $<\dot{u},\varphi> = -\dot{\varphi}(0)$, respectively. More generally, every distribution is indefinitely differentiable, by virtue of its definition:

$$<\dot{u},\varphi> = -<u,\dot{\varphi}>, \quad \varphi \in \mathcal{D}(\Omega). \tag{2.2}$$

In this paper, functions (locally Lebesgue integrable) are considered through the distributions they define, $\dot{u}$ or $u^{(1)}$ denote the distributional derivative of u, and du/dt stands for the distribution stemming from the usual derivative of u defined almost everywhere. Hence, if u is a continuous function except at point a with a finite jump σ_a, its distributional derivative can be written as

$$\dot{u} = du/dt + \sigma_a \delta_a,$$

where δ_a is the Dirac distribution located at point $\{a\}$. This result can be generalized to arbitrary derivation orders and discontinuity points as follows: let $\{t_\nu\}$ be an increasing sequence of points that are finite in every finite time interval. Assume that both left hand and right hand derivatives $\frac{d^p u}{dt^p}(t_\nu)$ exist and denote the corresponding jump $\sigma_\nu^p = \frac{d^p u}{dt^p}(t_\nu +) - \frac{d^p u}{dt^p}(t_\nu -)$. Then one has:

$$u^{(p)} = \frac{d^p u}{dt^p} + \sum_\nu \sum_{k=0}^{p-1} \sigma_\nu^{p-1-k} \delta_{t_\nu^{(k)}}. \tag{2.3}$$

In distribution sense, the class of differential equations we shall encounter will exhibit singular terms. The singularities, stemming from the origin $t_0 = 0$, will be gathered into a single distribution denoted ψ_0 with support $\{0\}$. The latter distribution ψ_0 will be referred to as initial condition term.

Generally, the multiplication of two distributions (α and u) is well-defined when at least one of the two terms (α here) is a smooth function. By definition:

$$< \alpha u, \varphi > = < u, \alpha\varphi > \tag{2.4}$$

With the properties (2.2), (2.4) and the reversed Leibniz rule, one can transform $\alpha u^{(n)}$ into linear combinations of derivatives of products $\alpha^{(k)} u$:

$$\alpha u^{(n)} = \sum_{k=0}^{n} (-1)^{2n-k} C_n^k w_k^{(n-k)}, \quad w_k := \alpha^{(k)} u \tag{2.5}$$

Finally for every smooth function α, one has:

$$\alpha\delta_\tau = \alpha(\tau)\delta_\tau,$$

if there exists $\alpha^{(k)}(\tau) = 0, (k = 0, \cdots, r)$, then one has:

$$\alpha\delta_\tau^{(r)} = 0 \tag{2.6}$$

This nice property will be used later for the annihilation of singular distributions.

3. Algebraic Switching Instant Estimation

The goal of this section is to obtain an algebraic relation of the measured variables in order to explicitly estimate the unknown switching instants sequence $\{t_j\}_{j=1}^{\infty}$. Firstly, a state transformation is used to get an intermediate differential algebraic relation which parameters are not depending anymore on the switching instant but only on a Dirac distribution at this instant. Then, an annihilating algebraic manipulation based on (2.6) is provided to get the desired differential algebraic relation where the unknown switching instants explicitly appear. Finally, thanks to the integral operator, the expression of the switching instant is obtained in terms of time integrals of the state variables.

3.1 The case of two modes and one commutation

In this case, $q \in \{1, 2\}$. Assume that the system (2.1) switches from one mode to the other one at instant t_c and the continuous state of the active mode is known. (Note that the algebraic continuous state estimation will be detailed in Section 4.) Then, the dynamical behaviour of the system can be written as follows:

$$\dot{x} = \Gamma(t)x, \qquad \Gamma(t) \in \{A_1, A_2\}. \tag{3.1}$$

Under the change of variable $z = e^{Gt}x$ (constant matrix G will be defined later), the system (3.1) is transformed as follow:

$$\dot{z} = M(t)z \tag{3.2}$$

with $M(t) = G + e^{Gt}\Gamma(t)e^{-Gt}$.

The matrix G is chosen such that:

M_1(t)+M_2(t)=0 (3.3)

with $M_i(t) = G + e^{Gt}A_i e^{-Gt}, \quad i = 1, 2$.

Since G and e^{Gt} commute, equation (3.3) implies that $G = -\frac{A_1 + A_2}{2}$ and

$$\dot{z} = \sigma(t)M_1(t)z \tag{3.4}$$

with $\sigma(t) \in \{-1,1\}$ and without loss of generality, $\sigma(0+) = 1$ if $\Gamma(0+) = A_1$.

Remark 1

When matrices A_1 and A_2 commute, $M_1(t) = -M_2(t) = \dfrac{A_1 - A_2}{2}$ are constant. This case was treated in [27]}.

The determinant $\Delta_1(t)$ of $M_1(t)$ satisfies the following property:

$$\begin{aligned} 2^n \Delta_1(t) &= \det(2M_1(t)) = \det(M_1 - M_2) \\ &= \det(A_1 - A_2) = \text{cte.} \end{aligned} \tag{3.5}$$

Assume that the matrix ($A_1 - A_2$) is full rank (If not, see [30] for the non-full rank case, the principle idea is to find an impulsive mode corresponds by algebraic operate, Lie algebras etc.). Then, $M_1(t)$ is invertible and one can define

$$W_1(t) := \mathrm{Ad}(M_1(t)) = \Delta_1(t) M_1^{-1}(t) ,$$

Then the equation (3.4) becomes

$$W_1(t)\dot{z} = \sigma(t)\Delta_1(t) z \tag{3.6}$$

Note that every term in (3.6) is known but the evolution of $\sigma(t)$. Since $\sigma(t)$ is a constant outside the set $\{t_c\}$, the distributional derivative of the product $\sigma(t)u(t)$ is well-defined when $u(t)$ is a smooth function.

Denoting $\sigma_c = \sigma(t_c+) - \sigma(t_c-) = \pm 2$ and using the properties (2.3) and (2.5), one has:

$$\dot{\sigma}(t) = \sigma_c \delta_{t_c},$$

$$\begin{aligned} (\sigma(t)u(t))^{\cdot} &= \dot{\sigma}(t)u(t) + \sigma(t)\dot{u}(t) \\ &= \sigma_c u(t_c)\delta_{t_c} + \sigma(t)\dot{u}(t) \end{aligned} \tag{3.7}$$

By derivation of (3.6) by using property (3.7), one obtains:

$$W_1(t)\ddot{z} + \dot{W}_1(t)\dot{z} - \Delta_1(t)M_1(t)z = \dot{\sigma}(t)\Delta_1(t)z. \tag{3.8}$$

Then the equation (3.8) can be rewritten as:

$$\sum_{i=0}^{2} K_i(t) z^{(i)} = \gamma_c \delta_{t_c} \tag{3.9}$$

with $K_1(t) = \dot{W}_1(t)$, $K_0(t) = -\Delta_1(t)M_1(t)$, $K_2(t) = W_1(t)$ and $\gamma_c = \sigma_c \Delta_1(t_c) z(t_c)$.

Thus, using the change of variables $z = e^{Gt}x$, one obtains a differential system with a left-hand side that only contains known quantities. The right-hand side involves a Dirac distribution that will be annihilated using property (2.6), as shown in the next subsection.

3.2 Explicit computation of the switching instant

In this subsection, the estimation of switching instant is presented. The main idea is to identify the impulsive systems (3.9) based on two steps: (i) multiplication with smooth function α for the annihilation of singular distributions; (ii) integration by parts to express the switching instant. From the property (2.6), let's take an arbitrary smooth function $\alpha = f(t, t_c)$ with the following properties:

$$f(t,t_c)\delta_{t_c} = 0,\ f(0,t_c) = \dot{f}(0,t_c) = 0.$$

Multiplying (3.9) by $f(t,t_c)$, one obtains:

$$f(t,t_c)\left(\sum_{i=0}^{2} K_i(t) z^{(i)}\right) = 0. \tag{3.10}$$

Integrating by parts (3.10) twice from 0 to $t > t_c$ and using the property

$$\int_0^t \int_0^{t_{\nu-1}} \cdots \int_0^{t_1} x(\tau) dt_{\nu-1} \cdots dt_1 d\tau = \int_0^t \frac{(t-\tau)^{\nu-1}}{(\nu-1)!} x(\tau) d\tau,$$

one obtains:

$$\begin{aligned} &\int_0^t (t-\tau)\Big(\ddot{f}(\tau,t_c)W_1(\tau) + \dot{f}(\tau,t_c)\dot{W}_1(\tau) - \Delta_1(\tau)M_1(\tau)\Big) z(\tau) d\tau \\ &+ f(t,t_c)W_1(t)z(t) - 2\int_0^t \dot{f}(\tau,t_c)W_1(\tau)z(\tau)d\tau \\ &-\int_0^t f(\tau,t_c)\dot{W}_1(\tau)z(\tau)d\tau = 0 \end{aligned} \tag{3.11}$$

So the estimation of t_c can be obtained from (3.11). For example: $f(t,t_c) = t^2(t - t_c)$ which satisfies the properties of (i) and (ii).

Using the result of (3.11), the estimate of t_c is given by the following formula:

$$D(t,z,M_1)t_c = N(t,z,M_1)$$

with

$$N(t,z,M_1)=t^3W_1(t)z(t)-\int_0^t(6\tau^2W_1(\tau)+\tau^3\dot{W}_1(\tau))z(\tau)d\tau$$
$$+\int_0^t(t-\tau)(6\tau W_1(\tau)+3\tau^2\dot{W}_1(\tau)-\Delta_1(\tau)M_1(\tau))z(\tau)d\tau \quad ,$$
$$D(t,z,M_1)=t^2W_1(t)z(t)-\int_0^t(4\tau W_1(\tau)+\tau^2\dot{W}_1(\tau))z(\tau)d\tau$$
$$+\int_0^t(t-\tau)(2W_1(\tau)+2\tau\dot{W}_1(\tau))z(\tau)d\tau \tag{3.12}$$

Note that $N(t,z,M_1)$ and $D(t,z,M_1)$ are column vectors of dimension n. So one has

$$t_c=\frac{N_g(t,z,M_1)}{D_g(t,z,M_1)},$$

for any $1\le g\le n$, where $N_g(t,z,M_1)$ and $D_g(t,z,M_1)$ are the components of $N(t,z,M_1)$ and $D(t,z,M_1)$, respectively.

Remark 2

When the measured signal $z(t)$ is perturbed by noise, (3.11) can be integrated one or several more times in order to reduce the noise effect. In this case, the expression of $N(t,z,M_1)$ and $D(t,z,M_1)$ in (3.12) becomes

$$N(t,z,M_1)=\int_0^t\frac{1}{2}(t-\tau)^2(6\tau W_1(\tau)+3\tau^2\dot{W}_1(\tau)-\Delta_1(\tau)M_1(\tau))z(\tau)d\tau$$
$$+\int_0^t\tau^3W_1(\tau)z(\tau)d\tau-\int_0^t(t-\tau)(6\tau^2W_1(\tau)+\tau^3\dot{W}_1(\tau))z(\tau)d\tau$$
$$D(t,z,M_1)=\int_0^t\frac{1}{2}(t-\tau)^2(2W_1(\tau)+2\tau\dot{W}_1(\tau))z(\tau)d\tau$$
$$+\int_0^t\tau^2W_1(\tau)z(\tau)d\tau-\int_0^t(t-\tau)(4\tau W_1(\tau)+\tau^2\dot{W}_1(\tau))z(\tau)d\tau$$

The method of switching instant identification is given above. Next, this method is extended to the general case.

3.3 The case of Q modes and S commutations

For each pair of matrices A_i and A_j ($i,j\in I_Q$ and $i\neq j$) in system (3.1), assume that (A_i-A_j) is full rank. It has been seen that an estimator $E_{i,j}$ which computes the quantities $D(t,z,M_{i,j})$ and $N(t,z,M_{i,j})$, with the used calculation expressions defined as follows:

$$M_{i,j}=G_{i,j}+e^{G_{i,j}t}A_ie^{-G_{i,j}t},\quad G_{i,j}=-\frac{A_i+A_j}{2}$$

and these can be used to determine a switching instant $t_{i,j}$ that occurs between the mode i and j (either from mode i to mode j or from j to mode i). Hence, in order to identify all the switches among Q modes, one can use $C_Q^2=(Q(Q-1))/2$ estimators in parallel. Then, the output signals of each estimator can determine the occurrence of a switch and its associated modes. Furthermore, with the knowledge of the first active mode, the whole sequence of active mode can be identified.

4. Algebraic State Estimation

An algebraic online switching instant estimation of the linear switched system is introduced in Section 3, this section is devoted to the estimation of the active mode and the continuous state of this class of system.

Hereafter, the system modelled by (2.1) and satisfying the following constraints is studied:

- Each mode is observable;
- The initial mode i from which the system starts is known;

The matrix (A_i-A_j) is full rank for each pair of matrices A_i,A_j $(i\neq j)$ or each pair of matrices A_i and A_j commute (if not, see [30] to treat the non-full rank case).

The main steps of the proposed method are:

1) Estimate the continuous state variables $x_1,\dots,x_n$ of the mode i with algebraic approach presented in article [23] knowing that the initial mode is i. The states of the other modes are estimated in parallel in order to follow the trajectory of the state faster when switch occurs.
2) Estimation of the switching instant and the active mode j with the method proposed in section 3.
3) Once the index j of the current active mode is known, estimation the current state variable of the system $\rightarrow$ step 1 .

5. Numerical Example

Consider an autonomous switched system in continuous time which commutes among three linear time-invariant modes described by the following defined matrices:

$A_1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, $A_2 = \begin{pmatrix} 3 & 2 \\ 0 & 3 \end{pmatrix}$, $A_3 = \begin{pmatrix} 2 & 1 \\ 5 & 0 \end{pmatrix}$, and $C = \begin{pmatrix} 1 & 0 \end{pmatrix}$.

5.1 Continuous state estimation

Firstly, we obtain an explicit formulation of the output estimation y_e and its derivative $\dot{y}_e$ of each mode using the method proposed in [23]. Secondly, since each mode is observable then their observability matrix is invertible. Thus, it is possible to express the state in function of the output and its derivative. By replacing this relationship with the estimated values, we can reconstruct state variable in function of the integral of $y(t)$.

For mode A_1, one has:

$$\begin{cases} y_e(t) = \dfrac{6\int_0^t \tau y(\tau) e^{(t-\tau)} d\tau - 2t\int_0^t y(\tau) e^{(t-\tau)} d\tau}{t^2} \\ \dot{y}_e(t) = \dfrac{6\int_0^t \tau y(\tau) e^{(t-\tau)} d\tau - 2t\int_0^t y(\tau) e^{(t-\tau)} d\tau - 2\int_0^t y(\tau) e^{(t-\tau)} d\tau}{t^2} \end{cases}$$

$$\begin{cases} x_{1e} = y_e \\ x_{2e} = \dot{y}_e(t) - y_e(t) \end{cases}$$

For mode A_2, one has:

$$\begin{cases} y_e(t) = \dfrac{6\int_0^t \tau y(\tau) e^{3(t-\tau)} d\tau - 2t\int_0^t y(\tau) e^{3(t-\tau)} d\tau}{t^2} \\ \dot{y}_e(t) = \dfrac{18\int_0^t \tau y(\tau) e^{3(t-\tau)} d\tau - 6t\int_0^t y(\tau) e^{3(t-\tau)} d\tau - 2\int_0^t y(\tau) e^{3(t-\tau)} d\tau}{t^2} + \dfrac{2y_e(t)}{t} \end{cases}$$

$$\begin{cases} x_{1e} = y_e(t) \\ x_{2e} = \dfrac{1}{2}(\dot{y}_e(t) - 3y_e(t)) \end{cases}$$

For mode A_3, one has:

$$\begin{cases} y_e(t) = \dfrac{2\int_0^t \left(e^{(1-\sqrt{6})(t-\tau)} + e^{(1+\sqrt{6})(t-\tau)}\right) \tau y(\tau) d\tau}{t^2} \\ \quad - \dfrac{\frac{\sqrt{6}}{6}\int_0^t \left(e^{(1+\sqrt{6})(t-\tau)} - e^{(1-\sqrt{6})(t-\tau)}\right) y(\tau) d\tau}{t^2} \\ \dot{y}_e(t) = \dfrac{\frac{6-6\sqrt{6}}{3}\int_0^t e^{(1-\sqrt{6})(t-\tau)} \tau y(\tau) d\tau + \frac{6+6\sqrt{6}}{3}\int_0^t e^{(1+\sqrt{6})(t-\tau)} \tau y(\tau) d\tau}{t^2} \\ \quad - \dfrac{\left(\frac{6-\sqrt{6}}{6}\int_0^t e^{(1-\sqrt{6})(t-\tau)} y(\tau) d\tau + \frac{6+\sqrt{6}}{6}\int_0^t e^{(1+\sqrt{6})(t-\tau)} y(\tau) d\tau\right)}{t^2} + \dfrac{2y_e(t)}{t} \end{cases}$$

$$\begin{cases} x_{1e} = y_e(t) \\ x_{2e} = \dot{y}_e(t) - 2y_e(t) \end{cases}$$

5.2 Numerical simulation results

The switching instant is assumed to occur at the following instants of

$t_1 = 0.4s$, $t_2 = 1s$, $t_3 = 1.2s$, $t_4 = 1.4s$ and $t_5 = 1.5s$ (cf. Fig. 1, Fig. 2) and the initial mode is mode 1. Their corresponding switching sequence is designed as follows

$A_1 \to A_3 \to A_2 \to A_1 \to A_3 \to A_2$ (cf. Fig. 3).

Since estimator 1 ($E_{1,2}$) detects the switching which occurs between the mode A_1 and the mode A_2, the estimator 2 ($E_{2,3}$) corresponds to commutation between mode A_2 and mode A_3, the estimator 3 ($E_{1,3}$) detects the commutation between the mode A_1 and the mode A_3, it is known that estimator $3 \to 2 \to 1 \to 3 \to 2$ works respectively (cf. Fig. 4).

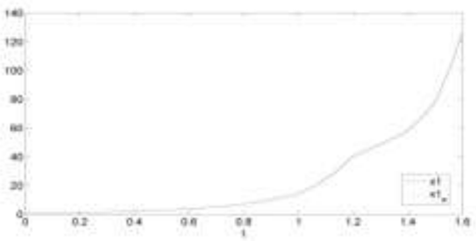

Figure 1. Real and estimated state value: x_1.

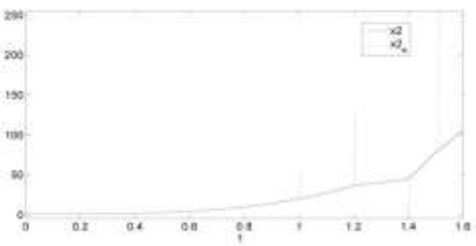

Figure 2. Real and estimated state value: x_2.

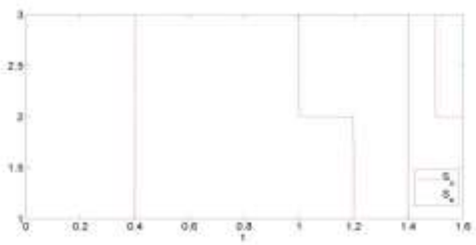

Figure 3. Real and estimated active mode S_o & S_e.

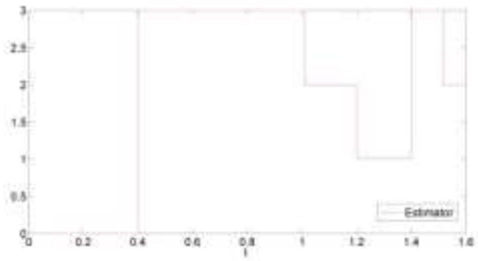

Figure 4. Sequences of the used estimator.

6. Conclusion

In this paper, an algebraic identification of switching instant, active mode and the continuous state estimation of autonomous switched linear systems has been introduced. Based on algebraic tools and distribution theory, an explicit algorithm computes the switching instants has been derived, then combining our previous results of algebraic state estimation for linear systems, the active mode and continuous state estimation in real time are given. Future works are concerned with the application to the fault diagnosis and state estimation of a class of the switched systems with time-varying modes.

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