Output Track Controller with Gravitational Compensation for a Class of Hyper-Redundant Robot Arms

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Abstract: The paper studies the output tracking control problem of a class of hyper-redundant robotic arms described by hyperbolic equations. The stability analysis and the resulting controllers are obtained by using the concept of boundary geometric control and an output tracking technique. A conventional PD control is proposed and analysed. Then, for a dynamic model with uncertain components, a robust algorithm is discussed. The output stability is analysed. Numerical simulations are also provided to verify the effectiveness of the approach presented.

Keywords: Hyper-redundant arm, gravitational compensation, controller.

1. Introduction

The goal of this paper is to implement a control system for a class of hyper-redundant robots with continuum components. This class of robots represents one of the most attractive domains of robotics during the last decades. In [1-4], were analyzed the kinematic models by the “backbone curve”. The papers [5-7] derived a new kinematic model by using the differential geometry, [8, 12] studied the manipulability of continuum robots. Cable-driven continuum robot control with variable stiffness was studied in [13]. In [14-16] were studied the kinematics of multi-section continuum robots. Several biomimetic robotic prototypes with undulating actuation have been developed in [17, 18]. The differential kinematic models of a class of continuum micro-robot for endovascular surgery applications are treated in [19-24]. Other papers [26-28,33] use the assumption that the arm bends with constant curvature and propose new control strategies.

In our paper, the main parameter, the system state, is determined by the position generalised variables. The dynamic model is inferred and the constraints of the state variables and nonliner components are proved. The estimation of gravitational terms is very difficult in a complex motion. For this reason, the gravitational forces are treated as uncertain components that satisfy the inequality constraints. An essential part of designing feedback controllers for these models is designing practical controllers that are implementable. The inequality constraints on the gravitational components allow to introduce a decoupled control system. A PD boundary control algorithm is used in order to achieve a desired shape of the arm. The stability analysis and the resulting controllers are obtained using Liapunov techniques. The exponential stability of the system (error-observer) was proved. Numerical simulations and experimental tests verify the effectiveness of the presented techniques.

The paper is organized as follows. In Section 2, the dynamic model is presented. Section 3 concerns the formulation of the output-feedback control and the design methodology of a PD output track controller. Section 4 presents the simulation results. Finally, a Conclusion Section ends the article.

2. Model Description

The technological model basis is a light weight arm with a distributed mass and friction. Although the conventional hyper-redundant models operate in 3-D space, the motion control will be first infer from the planar models. The 2D model basis from Figure 1 consists of a chain of vertebrae, elements, periodically spaced, each element having a special joint that ensures the rotation, elastic contact and a controllable friction force with the following element. All the joints are passive. All these elements determine a backbone type behavior of the arm. The motion of the arm, the bending, is determined by a pair of antagonistic cables (tendons) attached to the terminal point of the arm and that run through all the elements. The essence of the arm is the backbone curve C (Figure 2). The independent parameter s is related to the arc-length from origin of the curve C, $s \in \Omega$, $\Omega=[0,l]$, where l is the length of the arm. The position of a point s on curve C is defined by the position vector $r=r(s)$, $s \in [0,l]$. For a dynamic motion, the time variable will be introduced, $r=r(s,t)$. We denote by q the slope of the curve, $q=q(s)$ is the generalized coordinate, where $q \in L_2(\Omega)$ and $L_2(\Omega)$ is the Hilbert space of square integrable functions $q(s)$, $s \in [0,L]$ equipped with L_2 norm

$$\|q(\cdot)\|=\sqrt{\int_0^L q^2(s)ds}$$

Also, τ represents the equivalent moment at the end of the arm $(s=l)$ exercised by the cable forces.

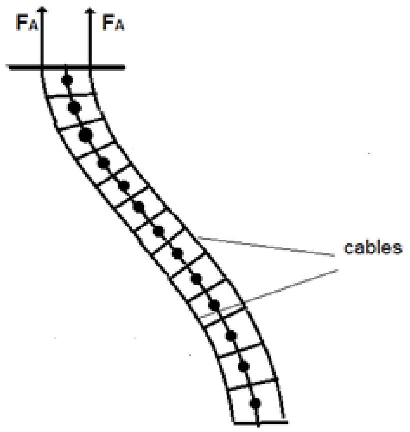

Figure 1. Technological arm

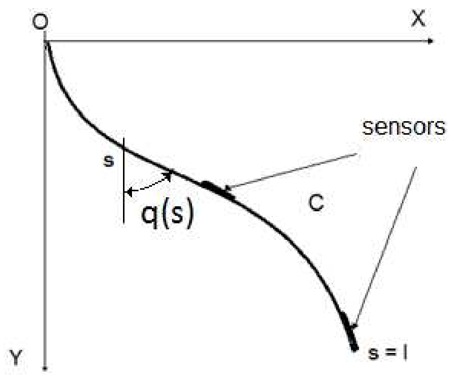

Figure 2. Ideal planar model

The dynamic model of the arm can be derived from the Hamiltonian principle [ ] as

$$I_\rho \ddot{q}=EIq_{ss}-b\dot{q}+cq-h(q) \tag{2.1}$$

where $q=q(t,s)$, s is a mono-dimensional (1-D) spatial variable, $s \in [0,l]$, $t \geq 0$ is time, $\dot{q}$ represents $(\partial q(t,s))/\partial t$ and $q_s=\partial q(t,s)/\partial s$, I_ρ is the rotational inertial density, *EI* is the bending stiffness coefficient, *b* is the equivalent damping matrix of the arm, *c* characterizes the elastic behaviour and $h(q)$ represents the nonlinear term determined by gravitational components. The state variables are $q \in L_2(0,l)$, $\dot{q} \in L_2(0,l)$.

We assume the following initial conditions

$$\begin{aligned} q(0,s)&=q_0(s) \in H^2(0,1) \\ \dot{q}(0,s)&=q_1(s) \in H^2(0,1) \end{aligned} \tag{2.2}$$

and the boundary conditions

$$q_s(t,0)=0\,,\ EIq_s(t,l)=\tau\,, \tag{2.3}$$

The gravitational component satisfies the inequality [ ]

$$\|h(q(s))\| \leq \eta\|q\|=\rho g L\|q\| \tag{2.4}$$

The output of the system is represented by the weighted average values defined by the relation

$$y(t)=\int_0^l w(s)q(t,s)ds,\ \ y \in C^2(0,l) \tag{2.5}$$

where $w(s)$ is spatial weighting measuring function, $w \in L_2(0,l)$. We assume that w satisfies the following conditions:

$$\begin{aligned} &a)\ w_{ss}=-\lambda w \\ &b)\ w(s)>0,\ s \in (0,l) \\ &c)\ w(0)=0;\ w_s=0 \end{aligned} \tag{2.6}$$

where λ is a positive constant.

3. A PD Output-Feedback Control

We consider a desired state $q^d(s)$, $q^d \in L_2(0,l)$ that satisfies the Eq. (2.1) with initial and boundary conditions (2.2), (2.3).

A desired output can be defined as

$$y^d(t)=\int_0^l w(s)q^d(t,s)ds \tag{3.1}$$

The control problem consists in the finding the control law $\tau(t)$, on the boundary $s=l$, such that the output $y(t)$ to track the "*a priori*" given desired output $y^d(t)$.

Definition 1. The control system is stable if

$$\lim_{t\to\infty}(t)=y^d(t) \tag{3.2}$$

In terms of this definition we can synthesize a PD (Proportional – Derivative) output-feedback controller that enforce output tracking and guarantee stability in the closed loop system (Figure 3).

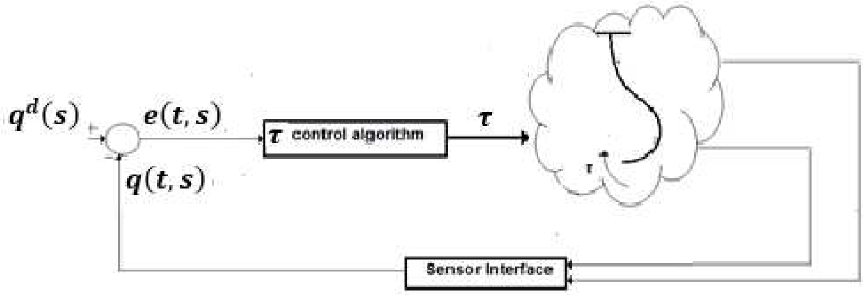

Figure 3. Control system

Theorem 1. An output – feedback control of the system (2.1) – (2.3) is stable, in the sense of Definition 1, if the control law is

$$\begin{aligned}\Delta\tau(t) = &-(EI)\,w_s(0)\big(q(t,0)-q^d(t,0)\big)-\\ &-k_1\int_0^l w(s)\big(q(t,s)-q^d(s)\big)ds-\\ &-k_2\int_0^l w(s)\dot q(t,s)\,ds\end{aligned} \quad (3.3)$$

where k_1, k_2 are the control coefficients, $k_1>0$, $k_2>0$

$$k_1>(c-\lambda EI)+\eta \quad (3.4)$$

$$k_2>\frac{\gamma}{\beta}\cdot I_\rho-b \quad (3.5)$$

α, β, and *γ* are positive constants that verify the conditions

$$\alpha>\gamma\cdot\frac{I_\rho}{4},\ \ \beta>\gamma \quad (3.6)$$

$$\begin{aligned}&\alpha+(c-\lambda EI)-\gamma b-\beta k_1-\gamma k_2+\beta\eta>0\\ &\Delta\tau(t)=\tau(t)-\tau^d\end{aligned} \quad (3.7)$$

τ^d is the desired moment applied at $s=l$.

Proof. Denote by $e(t)$ the weighted output error variable

$$e(t)=\int_0^l w(s)\big(q(t,s)-q^d(s)\big)ds \quad (3.8)$$

Substituting (3.8) into (2.1) and integrating by parts,

$$q_{ss}w=\frac{\partial}{\partial s}(q_s w)-\frac{\partial}{\partial s}(qw_s)+qw_{ss} \quad (3.9)$$

the error dynamics will be described by

$$\begin{aligned}I_\rho\ddot e=&-b\dot e+(c-\lambda EI)e-\Delta h^*(e)+\\ &+\Delta\tau+(EI)\,w_s(0)q(0)\end{aligned} \quad (3.10)$$

where

$$\Delta h^*(e)=\int_0^l w(s)\big(h(q)-h(q^d)\big)ds \quad (3.11)$$

(variables *s, t* are omitted in order to simplify the notation). From (2.4) can be inferred that

$$\left\|\Delta h^*(e)\right\|\le\sqrt{2}\,\eta\|q\| \quad (3.12)$$

Consider the following Liapunov function

$$V(t)=\frac{1}{2}\left(\alpha e^2+\beta I_\rho\dot e^2+\gamma I_\rho e\ddot e\right) \quad (3.13)$$

and using the conditions in (3.6), (3.13) is positive definite.

The time derivative of (3.13) will be

$$\dot V(t)=\alpha e\dot e+\beta I_\rho\dot e\ddot e+\gamma I_\rho\dot e^2+\gamma I_\rho e\ddot e \quad (3.14)$$

The following inequalities can be obtained by using the gravitational inequality (3.12)

$$\left|-\dot e\,\Delta h^*(e)\right|<\eta|\dot e||e| \quad (3.15)$$

$$\left|-\dot e\,\Delta h^*(e)\right|<\eta e^2 \quad (3.16)$$

Now, substituting the error dynamics (3.10) into (3.14), the control law $\Delta\tau(t)$ from (3.3), after simple additional manipulations, it is obtained

$$\begin{aligned}\dot V(t)<&\big(\alpha+(c-\lambda EI)-\lambda b-\beta k_1-\gamma k_2+\beta\eta\big)|\dot e||e|-\\ &-(\beta b-\gamma I_\rho+\beta k_2)\dot e^2-\\ &-\gamma\big(k_1-(c-\lambda EI)-\eta\big)e^2\end{aligned} \quad (3.17)$$

Using the conditions (3.4), (3.5), (3.7), this inequality is negative definite

$$\dot V(t)<0 \quad (3.18)$$

Remark 1 The control system (3.3) – (3.7) is exponentially stable.

Proof. The Liapunov function (3.13) satisfies the following inequality

$$V(t)<V^*(t)=\frac{M^*}{2}\left(e^2+\dot e^2\right) \quad (3.19)$$

where $M^*=max(\alpha,\beta I_\rho)$.

The inequality (3.17) can be rewritten as

$$\dot V(t)<-m^*(e^2+\dot e^2)=-\frac{2m^*}{M^*}V^*\le-\frac{2m^*}{M^*}V(t) \quad (3.20)$$

From (3.20) it can be then concluded that the system (3.6)-(3.7) is exponentially stable

$$V(t)\leq V(0)e^{-2\frac{m^*}{M^*}t} \quad (3.21)$$

Remark 2 The conditions (3.4), (3.5) can be rewritten as

$$k_1>c+\eta \quad (3.22)$$

$$k_2>\frac{\gamma}{\beta}I_\rho \quad (3.23)$$

Remark 3. The analyze of the motion stability can be obtained by the describing functions associated to the nonlinear components of the gravitational term. The describing functions of the nonlinear gravitational component Δh^* are represented by the fundamental components with respect to a sinusoidal input $A^*\sin(\omega^* t)$, where A^*, ω^* are the amplitude and frequency, respectively [35].

$$h_G=h_{G1}+jh_{G2} \quad (3.24)$$

where

$$h_{G1}=\frac{1}{\pi A^*}\cdot \int_0^{2\pi}\Delta h^* A^*\sin(\omega^* t)\sin(\omega^* t)d(\omega^* t) \quad (3.25)$$

$$h_{G1}=\frac{1}{\pi A^*}\cdot \int_0^{2\pi}\Delta h^* A^*\sin(\omega^* t)\cos(\omega^* t)d(\omega^* t) \quad (3.26)$$

The describing functions are computed for the mechanical parameters $I_\rho=0.001\,kg\cdot m^2$, $EI=1.2\,N\cdot m^3$, $b=0.06\,Nms/rad$. From (3.24) – (3.26) yields,

$$h_G=0.85+j\cdot 0 \quad (3.27)$$

For a desired position $y^d=0$ and the describing function (3.27), the transfer function (s^* is Laplace variable) of the arm with n segments can be approximated as

$$G(s^*)=\frac{e(s^*)}{\Delta\tau(s^*)}\cong \cong\frac{1}{I_\rho s^{*2}+bs^*+(\lambda EI-c+nh_{G1})} \quad (3.28)$$

The instability of the arm segment is determined by the elastic component *EI* and the gravitational component h_{G1}. This instability increases with respect to the number of segments (n) in the structure of the arm. The closed loop transfer function of the controller and the arm segment is evaluated as

$$G_{SA}(s^*)\cong\frac{k_1+k_2 s^*}{I_\rho s^{*2}+Bs^*+(\lambda EI-c+nh_{G1})} \quad (3.29)$$

The polar plots of $G_{SA}(j\omega^*)$ for the 1-segment, 2-segment, 3-segment arms, respectively, are represented in Figure 4. Clearly, by virtue of Nyquist stability criterion, the closed-loop system is stable because the number of counterclockwise encirclements of the (-1, 0) point is equal to one, the number of poles with positive real parts [34,35].

4. Numerical Simulations

Consider the dynamic model of a hyper-redundant continuum robotic arm described by (2.1) where the length of the arm is l=1, the rotational inertial density is I_ρ=1, the bending stiffness EI=1.5, the equivalent damping coefficient $b=-0.5$ and the elastic coefficient is c=15. These constants are scaled to realistic ratios for long thin arm. The initial and

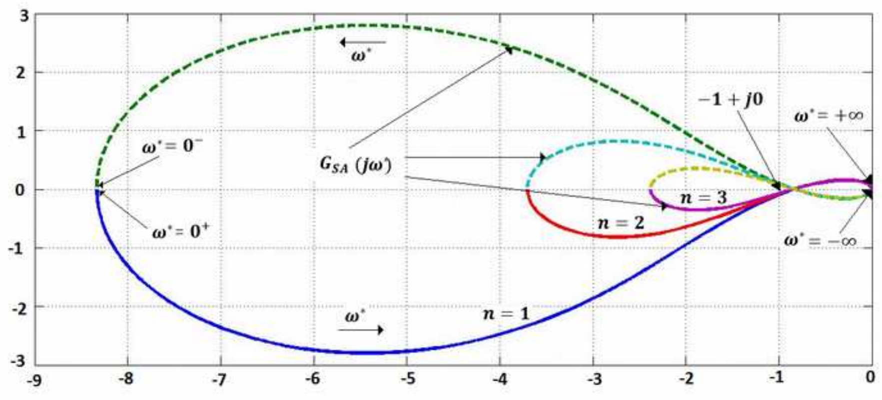

Figure 4. Polar plots of $G_{SA}(j\omega^*)$ for the 1-segment arm, 2-segment arm, 3-segment arm

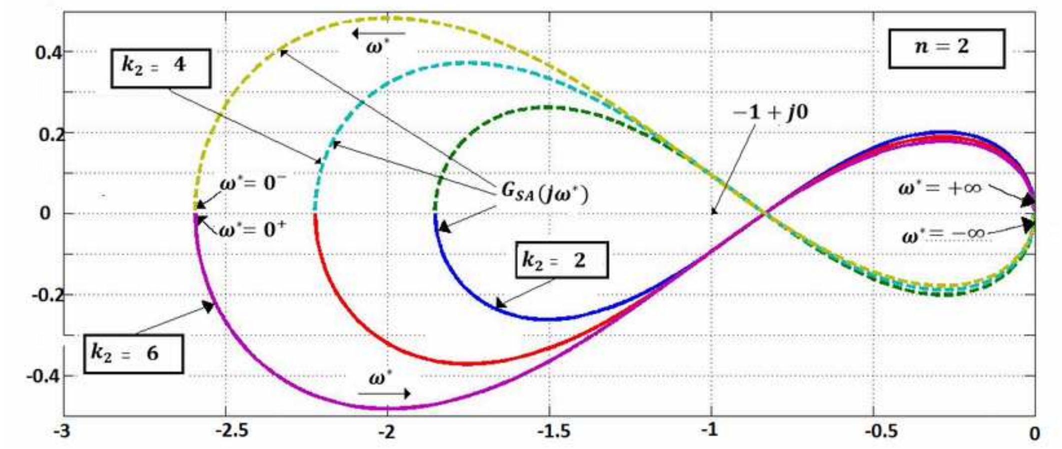

Figure 5. Polar plots of $G_{SA}(j\omega^*)$ for a 2-segment arm (control by k_2)

boundary conditions are $q_0(s)=0$, $q_1(s)=0$, $q_s(t,0)=0$, $EIq_s(t,l)=\tau$. The desired state is $q^d(s)=1.8\cos(1.5s)$, that satisfies the stationary desired state for $\tau^d=-3.8$.

A spatial weighting measuring function $w(s)=\sin\left(\frac{\pi s}{2}\right)$ that satisfies the conditions (2.6) for $\lambda=\left(\frac{\pi}{2}\right)^2$ and a control law (3.3) with k_1=18, k_2=2 are used. Figure 6 and Figure 7 reports three-dimensional plots of the solution *q*(*t*,*s*) and of the tracking weighted error *e*(*t*). Good performances of the proposed control algorithm is concluded from the graphics.

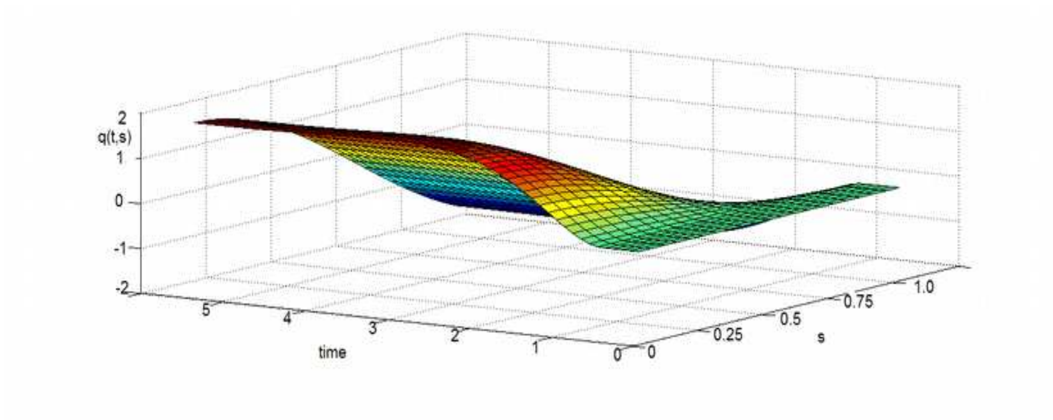

Figure 6. The state evolution *q*(*t*,*s*) for the desired state $q^d(s)=1.8\ \cos(1.5\,s)$

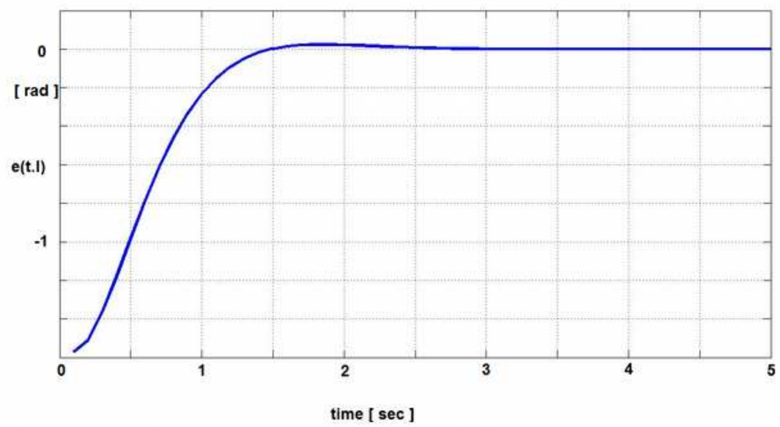

Figure 7. Tracking weighted error *e*(*t*)

5. Conclusion

This paper deals with the control problem of a class of robots constituted by a chain of continuum segments. The technological model basis is a central, long and thin, highly flexible and elastic backbone. The paper studies the output tracking control problem of a class of DPS described by hyperbolic DPE. The stability analysis and the resulting controllers are obtained by using the concept of boundary geometric control and an output track technique. A conventional PD control is proposed and analyzed. Numerical simulations are also provided to verify the effectiveness of the presented approach.

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