1. Introduction

Although many complex and delicate control techniques have been developed, proportional, integral and derivative (PID) controllers have been the most commonly used in industrial sites. The ubiquity of PID controllers enables the designing and tuning operations to be made easier in comparison with other sophisticated instruments, and enables engineers in the field to operate them relatively easily. But despite these advantages, conventional linear PID (LPID) controllers display a conflicting relationship: a fast response requires large gains, which in turn give rise to a large overshoot. For this reason, a variety of feedback control forms and related tuning methods have been implemented to guarantee satisfactory performance. Recently, multiple studies which introduce nonlinearities into the structure of the standard PID controller have been performed to solve this conflicting relationship. This study proposes an EA-based nonlinear PID controller with a first-order filter added to the derivative action to achieve the desirable quick response with low overshoot. This is realized by introducing a new type of nonlinearities in the controller gains that are time-varying functions in terms of the error and/or error rate. In addition, the nonlinear controller is designed by considering a saturation element. Then, the parameters of the nonlinear PID controller are optimally tuned by an evolutionary algorithm. In tuning the nonlinear PID controller gains, the integral of time-weighted absolute error is used as the performance evaluation of the overall control system. A set of simulation works performed on two processes with actuator saturation shows the feasibility of the proposed method.

Keywords: Nonlinear PID controller, Nonlinear functions, Saturation, Evolutionary algorithm, ITAE.
for improvement because most of the proposed methods are based on complex hyperbolic secant or exponential functions and do not consider actuator saturation.

This paper presents a new class of NPID controller with three nonlinear gains and a first-order filter added to the derivative term. The three gains of the NPID controller are realized by a simple nonlinear function based on the error or the error rate. Its parameters are tuned by an evolutionary algorithm (EA) to improve the set-point tracking performance of the overall control system which consists of the NPID controller, a saturator, and a process. In tuning the nonlinear PID controller gains, the integral of time-weighted absolute error (ITAE) criterion is used for the performance evaluation of the overall control system. The performance of the proposed method was compared to that of the existing methods through simulation on two processes to verify its effectiveness.

This paper consists of 5 sections in total, and contents of each section are as follows. Section 2 presents a brief overview of the linear PID controller. Section 3 describes the proposed NPID controller and discusses how to optimize the parameters of the NPID controller. Section 4 applies the proposed NPID controller to control the two processes and its performance is compared to the existing linear PID controller. Section 5 highlights the conclusion of this paper.

2. Linear PID Controller

Consider the closed-loop control system depicted in Figure 1. It consists of a PID controller \( C(s) \), a controlled object \( P(s) \) and a saturator.

If the standard form is used as the PID controller in Figure 1, the transfer function yields

\[
C(s) = k_p + \frac{k_i}{s} + k_ds
\]

where \( k_p \), \( k_i \), and \( k_d \) denote proportional, integral and derivative gains, respectively.

The LPID controller in formula (1) consists of a parallel combination of three terms acting on the error: proportional action (\( u_p \)), integral action (\( u_i \)), and derivative action (\( u_d \)). \( u_p \) acts on the magnitude of the error, \( u_i \) on the cumulative error from the initial time to the present and \( u_d \) on the error rate. Finally the control input is calculated by adding all these values.

The performance of the closed-loop system is affected directly by the selection of \( k_p \), \( k_i \), and \( k_d \). \( k_p \) contributes to accelerating system response and reduces the rise time; however, it also increases the oscillation. \( k_i \) contributes to reducing the steady-state error but has a poor transient response; \( k_d \) plays the role of reducing the overshoot and the settling time. As the three controls are interconnected, if one of the three gains is changed, it can be affected by the two other controls at the same time.

When the controller operates within the linear part of the saturator and the noise is disregarded, the input/output relationship of the overall control system is expressed as follows:

\[
Y(s) = G_{p}(s)R(s) + G_{d}(s)D(s)
\]

\[
G_{p}(s) = \frac{P(s)C(s)}{1+P(s)C(s)}
\]

\[
G_{d}(s) = \frac{P(s)}{1+P(s)C(s)}
\]

From formula (2), as long as the following conditions under a step change in the set-point and/or a step change in the disturbance are satisfied, the steady-state error goes to zero:

\[
\lim_{s \to 0} P(s) \neq 0, \lim_{s \to \infty} C(s) \to \infty
\]

Meanwhile, the nonlinear saturator is defined by the following formula:

\[
u_{\text{sat}} = \begin{cases} 
\min_{\text{max}} & \text{if } u > u_{\text{max}} \\
u & \text{if } u_{\text{max}} \leq u \leq u_{\text{min}} \\
\min_{\text{min}} & \text{if } u < u_{\text{min}}
\end{cases}
\]

where \( u_{\text{min}} \) and \( u_{\text{max}} \) are the minimum and maximum values of the saturator, respectively; and \( u_{\text{sat}} \) is the output of the saturator.

In Figure 1, \( y_r, y, d, n \) denote the set-point, measurement output, disturbance, and measurement noise, respectively; the error \( e \) is defined as \( e = y_r - y \); \( u \) and \( u_{\text{sat}} \) are the control input and the saturator output, respectively. It is assumed that \( d \) is unmeasurable.
3. Proposed Nonlinear PID Controller

As the ideal derivative term in formula (1) results in an undesirable phenomenon known as *derivative kick* which may occur when measurement noise is big or the set-point is abruptly changed, this study proposes an NPID controller with a first-order filter added to the derivative term.

\[
C(s) = K_p(e) + \frac{K_i(e)}{s} + \frac{K_d(e)\dot{e}}{1+T_f(e,\dot{e})s} \quad (5a)
\]

\[
T_f(e,\dot{e}) = \frac{K_i(e,\dot{e})}{NK_f(e)} \quad (5b)
\]

where \(K_p(e)\), \(K_i(e)\), and \(K_f(e,\dot{e})\) are time-varying gains, which are nonlinear functions of error \(e\) and/or error rate \(\dot{e}\). \(T_f(e,\dot{e})\) is filter equation, and \(N\) is a fixed value that is empirically determined between 8 and 20 (Åström & Hägglund, 2006; O’dwyer, 2009; Rathinam, Maria & Ramaveerapathiran, 2017). As many studies in the literature have generally used \(N = 10\), this value is adopted here. These functions can be expressed in various forms according to the purpose of use or the control environment. But if possible, they should be simple and allow for easy hardware implementation.

3.1 Nonlinear Proportional Gain

The proportional action increases in proportion to the proportional gain or the error; if too large, overshoot and oscillation may occur due to excessive control.

To increase the response speed, the proportional gain should be sufficiently large when the error is a major one, but if this big value is maintained even when the error is small after the response reaches steady-state, the effect of the error is amplified, which may cause oscillation or instability in some cases.

In this study, based on this knowledge, the magnitude of the proportional gain is adjusted in tune with that of error \(e\), and the proposed \(K_p(e)\) is a smooth function as follows:

\[
K_p(e) = k_p g_p(e) \quad (6a)
\]

\[
g_p(e) = 1 - \frac{1}{a_p + (c_p e)^c} \quad (6b)
\]

where \(k_p\) is a positive constant, and \(g_p(e)\) is a nonlinear function with two user-defined parameters, \(a_p(\geq 1)\) and \(c_p(>0)\). \(g_p(e)\) is upper-bounded by 1 when \(e \to \infty\) and lower-bounded by \((1-1/a_p)\) when \(e = 0\), but its magnitude depends on the value of \(a_p\).

Figure 2 depicts typical variations of \(a_p\) and \(c_p\). Meanwhile, the depth of the point where \(g_p(e)\) becomes smaller is determined by \(a_p\), and the smaller the \(c_p\) value, the greater the width.

3.2 Nonlinear Integral Gain

The greater the absolute value of the cumulative error or the shorter the integral time, the greater the integral action. When the error is high, if the integral gain is also large, overshoot will occur; if the control input is saturated, *integrator windup* may occur.

Considering this fact, it is necessary to prepare for the occurrence of overshoot by reducing the integral gain value when the absolute value of error \(e\) is high, and to reduce the steady-state error by increasing the integral gain value when the absolute value of error \(e\) is low. For this, the following equation is used:

\[
K_i(e) = k_i g_i(e) \quad (7a)
\]

\[
g_i(e) = \frac{1}{1 + (c_i e)^c} \quad (7b)
\]

where \(k_i\) is the positive gain, and \(g_i(e)\) is a nonlinear function with parameter \(c_i(>0)\) which has a value between 0 and 1. Figure 3 shows \(g_i(e)\).
3.3 Nonlinear Derivative Gain

PID controllers are prominently featured in general process control, as it is possible to cope with high frequency noise partly by using a first-order filter added to the derivative term. The derivative action \( u_d \) increases in proportion to the error rate or the derivative gain and damps down by predicting in advance that if \( u_e \) and \( u_d \) increase, the output will increase. If damping is more than necessary during the overall control cycle, the response speed will be slow; but if damping is high during a specific cycle only, it can use \( u_e \) and \( u_d \) more aggressively and reduce overshoot. Thus, the size of the derivative gain is changed so that big damping can be applied when the response is in the control cycle of the red (solid) area (i.e. \( e\ddot{e}>0 \)), as shown in Figure 4.

For this, the time-varying derivative gain is used as follows:

\[
K_d(e, \dot{e}) = k_d g_d(e, \dot{e})
\]

\[
g_d(e, \dot{e}) = \begin{cases} 
1 - \frac{1}{a_d} + (e, \dot{e})^2, & \dot{e} > 0 \\
1 - \frac{1}{a_d}, & \text{elsewhere}
\end{cases}
\]

where \( k_d \) is the positive gain, and \( g_d(e, \dot{e}) \) is a nonlinear function of two parameters \( a_d(\geq 1) \) and \( c_d(>0) \) and also has a value between 0 and 1.

If the absolute value of error \( e \) is high on the \( e\dot{e}>0 \) plane, \( g_d(e, \dot{e}) \) will converge to 1; on the contrary, if the absolute value of error \( e \) is low, it will converge to \((1-1/a_d)\), but the size depends on the value of \( a_d \).

3.4 Optimal Tuning of the NPID Controller Gains

As seen before, the proposed NPID controller has three time-varying gains \( K_p(e) \), \( K_i(e) \), and \( K_d(e, \dot{e}) \) and there exist a total of eight adjusting parameters \( \{k_p, k_i, k_d, a_d, c_d, c_i, a_p, c_p\} \). In this study, the parameters are tuned in the overall control system including the nonlinear saturator to obtain the optimal tracking response.

Evolutionary computation (EC) has become a common approach to solving difficult, real world problems such as optimization of functions, design of neural networks and fuzzy controllers, system identification, etc. Typical examples of EC are Genetic algorithms (GAs), Evolutionary algorithms (EAs), Simulated annealing (SA), and Taboo search (Gu, Zhang & Gao, 2009). These are able to find a global solution without any other information except the objective function.

Recently, a new class of evolutionary algorithms which utilizes both a nature-inspired operator – namely, attractor – and the dynamic mutation operator was proposed by Jin & Tran (2010). The Attractor emulating the behavior of spiral movements is implemented using a dynamic system model. The new operator in combination with the dynamic mutation is applied to a population of solution candidates to iteratively evolve these into better and better solutions. This EA operates on a hierarchical basis as follows:

1. Initialize population \( P(0) \) of size \( M \) randomly;
2. Evaluate fitness \( f_i(0)(1 \leq i \leq M) \) and select the best \( \{x_i(0), f_i(0)\} \);
3. for \( k = 1 \) to max_generation do
   1. Assign a new vector \( x_i(k) \) to \( x_i(k-1)(1 \leq i \leq M) \) using the attractor;
   2. Apply non-uniform mutation;
   3. Apply elitism if necessary;
   4. Evaluate fitness \( f(k)(1 \leq M) \) and select the best \( \{x_i(k), f_i(k)\} \);
4. end
5. Output \( x_b \).

In the problem of tuning the NPID controller gains, the integral of time-weighted absolute error (ITAE) was used for the performance evaluation of the overall control system:

\[
J_1(\phi) = \int_0^t |e(t)| dt
\]

where \( \phi = [k_p, k_i, k_d, a_p, c_p, c_i, a_d, c_d]^T \in \mathbb{R}^8 \) is a vector composed of NPID controller parameters; \( e(t) \) is the error between the set-point and the output; and the integral time \( t_f \) is large enough to render the integral after it becomes negligible. \( \phi \) is tuned using an EA so that the evaluation of formula (9) is minimized.

For the EA, there have been used \( P_{size} = 40 \) as the size of group; \( \omega = 0.618 \) (golden ratio), \( \lambda = -0.3 \),
and \( m = 20 \) as attractor parameters and \( P = 0.05 \) and \( b = 4 \) as mutation parameters.

### 4. Simulation and Review

To verify the effectiveness of the proposed NPID controller, a set of simulation works was performed through two virtual processes. For the first process, the performance of the EA-NPID controller was compared to that of the conventional fixed-gain PID controller tuned by the IMC method (Garcia & Morari, 1982) and Cvejn’s method (Cvejn, 2009). For the second process, the performance of the EA-NPID controller was compared with that of the NPID controller demonstrated by Korkmaz’s method.

#### 4.1 Process I

The first process to be controlled is an FOPTD (First order Plus Time Delay) system in formula (10) and its parameters are \( K = 1 \), \( \tau = 1 \) [sec], \( L = 0.5 \) [sec]:

\[
P(s) = \frac{Ke^{-\tau s}}{1 + \tau s}
\]

(10)

The minimum and maximum values of the saturator were assumed to be \( u_{\text{min}} = -3 \) and \( u_{\text{max}} = 3 \), respectively.

Each parameter of the EA-NPID controller was tuned optimally in the controlled system which combines the model in formula (10) and the saturator, and the parameters were searched within the range \( 0 < k_p, k_i, k_d \leq 10 \), \( 0.1 \leq a_p, c_p, c_i, a_d, c_d \leq 10 \).

Table 1 shows the summary of the parameters tuned by the proposed method and the existing methods.

<table>
<thead>
<tr>
<th>Method</th>
<th>Parameters</th>
<th>( k_p )</th>
<th>( k_i )</th>
<th>( k_d )</th>
<th>( a_p )</th>
<th>( c_p )</th>
<th>( c_i )</th>
<th>( a_d )</th>
<th>( c_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed</td>
<td></td>
<td>1.74</td>
<td>1.62</td>
<td>0.97</td>
<td>3.12</td>
<td>9.54</td>
<td>0.88</td>
<td>1.45</td>
<td>5.50</td>
</tr>
<tr>
<td>IMC</td>
<td></td>
<td>2.78</td>
<td>2.22</td>
<td>0.56</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Cvejn</td>
<td></td>
<td>1.75</td>
<td>1.5</td>
<td>0.25</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

#### 4.1.1 Response to Set-point (SP) Change

To check the SP tracking performance of the proposed controller, unit step response simulation was conducted and the results were compared with those of the other methods.

As Figure 5 shows, all the controllers are able to track the step-wise variation without steady-state error, but the proposed NPID controller reaches it faster and with smaller overshoot than the other controllers. The response of the IMC method is the poorest, whereas Cvejn’s method shows a long settling time.

![Set-point tracking responses](image1)

![Saturator outputs](image2)

For the quantitative performance comparison, overshoot \( M_p \), rise time \( t_r \), 2% settling time \( t_s \), and the integral of the absolute error (IAE) were calculated and listed in Table 2. Here, \( t_{10} \) and \( t_{90} \) refer to the times required for the output to reach 10% and 90% of the set point, respectively.

<table>
<thead>
<tr>
<th>Method</th>
<th>Performances</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( M_p ) [%]</td>
</tr>
<tr>
<td>Proposed</td>
<td>0.78</td>
</tr>
<tr>
<td>IMC</td>
<td>28.9</td>
</tr>
<tr>
<td>Cvejn</td>
<td>12.77</td>
</tr>
</tbody>
</table>

#### 4.1.2 Response to Noise Rejection

A simulation was carried out to verify the performance of the proposed controller in the presence of noise. The output was perturbed by an additive white Gaussian noise component \( N(0,0.002^2) \).
Figure 6 shows that the proposed methods have little change in the responses, but the responses of the other methods are severely distorted due to the ideal derivative action. The quantitative performance comparisons were listed in Table 3.

Comparing the saturator outputs in Figure 6, it can easily be observed that the severe fluctuations of $u_{sat}$ in the IMC method and Cvejn’s method were greatly reduced in the EA-NPID controller response.

### 4.1.3 Response to Parameter Change

Next, the sensitivity of the system to parameter changes was verified. It was assumed that the gain $K$ and time constant $\tau$ of process I change most severely. A simulation which increases these values with 10% of the nominal value has been performed. As Figure 7 shows, the proposed method is less sensitive to the parameter changes than the other methods.

Comparing the saturator outputs in Figure 6, it can easily be observed that the severe fluctuations of $u_{sat}$ in the IMC method and Cvejn’s method were greatly reduced in the EA-NPID controller response.

### 4.2 Process II

#### 4.2.1 Response to Set-point (SP) Change

The second process to be controlled is a third-order system given by Korkmaz’s method and its poles are $p_1 = 2$, $p_2 = 4$.

$$P(s) = \frac{1}{s(s + p_1)(s + p_2)}$$

The minimum and maximum values of the saturator were assumed to be $u_{\text{min}} = -10$ and $u_{\text{max}} = 40$, respectively.

---

**Table 3. Noise rejection performances**

<table>
<thead>
<tr>
<th>Method</th>
<th>$M_p$ [%]</th>
<th>$t_r$ [sec]</th>
<th>$t_s$ [sec]</th>
<th>IAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed</td>
<td>1.59</td>
<td>0.48</td>
<td>2.21</td>
<td>0.34</td>
</tr>
<tr>
<td>IMC</td>
<td>20.3</td>
<td>0.40</td>
<td>4.18</td>
<td>0.47</td>
</tr>
<tr>
<td>Cvejn</td>
<td>12.3</td>
<td>0.50</td>
<td>3.17</td>
<td>0.44</td>
</tr>
</tbody>
</table>

**Table 4. Parameter changing performances**

<table>
<thead>
<tr>
<th>Method</th>
<th>$M_p$ [%]</th>
<th>$t_r$ [sec]</th>
<th>$t_s$ [sec]</th>
<th>IAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed</td>
<td>3.60</td>
<td>0.50</td>
<td>2.81</td>
<td>0.32</td>
</tr>
<tr>
<td>IMC</td>
<td>32.77</td>
<td>0.32</td>
<td>4.14</td>
<td>0.48</td>
</tr>
<tr>
<td>Cvejn</td>
<td>16.75</td>
<td>0.47</td>
<td>3.31</td>
<td>0.47</td>
</tr>
</tbody>
</table>

---

https://www.sic.ici.ro
As already mentioned, each parameter of the EA-NPID controller was optimally tuned in the controlled system which combines the model in formula (11) and the saturator, and the parameters were searched within the range $0 < k_p, k_i, k_d \leq 100$, $1 \leq a_p, c_p, c_i, a_d, c_d \leq 50$.

The gains of the NPID controller proposed by Korkmaz’s method are as follows:

\begin{align}
K_p(e) &= a_1 + a_2 f(e) \quad (12a) \\
K_i(e) &= b_1 - b_2 f(e) \quad (12b) \\
K_d(e) &= c_1 + c_2 f(e) \quad (12c) \\
f(e) &= \frac{2}{\sqrt{\pi}} \int_0^t \exp(-\tau^2) d\tau \quad (12d)
\end{align}

Table 5 shows the summary of the parameters tuned by the proposed method and the parameters demonstrated by Korkmaz’s method.

**Table 5.** Tuned parameters for Process II

<table>
<thead>
<tr>
<th>Method</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed</td>
<td>$k_p$</td>
</tr>
<tr>
<td>55.4</td>
<td>6.23</td>
</tr>
<tr>
<td>Korkmaz</td>
<td>$a_1$</td>
</tr>
<tr>
<td>28.8</td>
<td>7.2</td>
</tr>
</tbody>
</table>

As in the previous case, Figure 8 shows the responses of the unit step input by employing the proposed method and Korkmaz’s method, and Table 6 compares the quantitative performance values.

**Table 6.** Set-point tracking performances

<table>
<thead>
<tr>
<th>Method</th>
<th>$M_p$</th>
<th>$t_r$</th>
<th>$t_s$</th>
<th>IAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed</td>
<td>6.41</td>
<td>0.58</td>
<td>2.03</td>
<td>0.62</td>
</tr>
<tr>
<td>Korkmaz</td>
<td>16.33</td>
<td>0.66</td>
<td>4.53</td>
<td>0.84</td>
</tr>
</tbody>
</table>

As shown in the figure, both methods approach the set-point as oscillations decrease with time, but the proposed method has better performance than Korkmaz’s method.

### 4.2.2 Response to Noise Rejection

As in the previous case, the output was also perturbed by an additive white Gaussian noise $N(0,0.01^2)$.

As it can be seen in Figures 8 and 9, the Korkmaz’s response has been distorted, but the difference between the responses of the proposed method is hardly noticeable. Comparing the two saturator outputs in Figure 9, it can easily be noticed that...
severe fluctuations of $u_{sat}$ in Korkmaz’s response have been greatly reduced in the EA-NPID controller response.

As shown in Table 7, Korkmaz’s method shows poor performance due to higher overshoot as well as longer settling time.

<table>
<thead>
<tr>
<th>Method</th>
<th>$M_i$ [%]</th>
<th>$t_r$ [sec]</th>
<th>$t_s$ [sec]</th>
<th>IAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed</td>
<td>5.66</td>
<td>0.59</td>
<td>2.05</td>
<td>0.63</td>
</tr>
<tr>
<td>Korkmaz</td>
<td>26.4</td>
<td>0.68</td>
<td>-</td>
<td>1.17</td>
</tr>
</tbody>
</table>

### 4.2.3 Responses to Parameter Change

It was assumed that the pole $p_i$ of the open loop transfer function in formula (11) for process II changes. A simulation which increases this value with 10% of the nominal value has been performed. It is shown in Figure 10 that the proposed method is less sensitive to the parameter change than Korkmaz’s method.

![Figure 10. Set-point tracking responses to parameter change](image)

The quantitative performance comparisons were listed in Table 8. It can be clearly seen that the proposed methods exhibit smaller overshoot and a reduced settling time compared to Korkmaz’s method.

### 5. Conclusion

In the design of controllers, there is a Catch-22 relationship between fast response characteristics and low overshoot. A wide range of research studies has been performed in order to address this situation.

This paper presents an EA-based nonlinear PID controller to achieve the desirable quick response with low overshoot. In addition, each nonlinear time-varying gain has been achieved by multiplying the fixed gain and the scaled error by a nonlinear function. The parameters of the proposed controller have been tuned in the overall control system including the nonlinear saturator using an evolutionary algorithm. In the tuning of the parameters, the ITAE is used for the performance evaluation of the overall control system. The results of simulations performed through the two processes have confirmed that the performance of the proposed method is superior to that of other methods.

### REFERENCES


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