Robust Control Framework Based on Input-Output Models Enhanced with Uncertainty Estimation

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Abstract: A versatile and simple robust model-based control framework for regulating a class of (bio)-chemical processes is introduced. The proposed control framework departs from a simple low-order model which is enhanced by estimating model uncertainties due to model reduction, uncertain model parameters, and external disturbances. Three robust model-based control schemes are then formulated based on the enhanced simple input-output model. The proposed robust framework provides the robustness of two well-known model-based control approaches using a simple low-order model. A benchmark model of microalgae production is employed for illustrating the controller performance. Moreover, a classical PI controller and a nonlinear MPC controller are also designed and applied for comparison purposes. Numerical results show that the proposed simple robust model-based controllers are able to regulate the controlled variable to the desired reference despite external disturbances and set-point changes. Furthermore, the proposed controllers feature an acceptable performance in comparison with two of the most widely accepted controllers in the control engineering community for controlling (bio)-chemical processes.

Keywords: Robust control, Model-based control, Input-output models, State observers, (Bio)-chemical processes.

1. Introduction

(Bio)-chemical industrial processes are procedures involving chemical and biochemical reactions in the manufacturing of items, which are useful for everyday use or serve as input for other industrial processes, usually carried out on a huge scale. The chemical process industries broadly include organic and inorganic chemical industries, the petroleum industry, and the petrochemical industry (Turton et al., 2008). Biotechnological industries comprise the food and biopharmaceutical industries (Kondakci & Zhou, 2017; Hong et al., 2018). Moreover, biotechnological industries offer solutions to a broad spectrum of current challenges, including remediation of environmental pollution, sustainable food and energy production, and the supply of bulk chemicals (Gavrilescu & Chisti, 2005).

Batch and fed-batch processes are widely employed in order to produce fine chemicals, pharmaceutical products, polymers, and many other materials (Hong et al., 2018). Advantages of the batch and fed-batch operation include simplicity of equipment, the ability to clean completely between batches, and years of operational experience. Nevertheless, because of their higher long-term production and better quality control, continuous processes are preferable (Calabrese & Pissavini, 2011; Moulijn, Makkee, & Van Diepen, 2013).

The most common operation of continuous processes for achieving the desired product quality is steady-state. However, continuous processes are very challenging to operate. Indeed, process variables are usually unknown, slow attainment of the steady-state, potential instability of operation, and the complexity of the underlying (bio)-chemical reactions (Bequette, 2003; van Impe, Vanrolleghem & Iserentant, 2013). Moreover, even in normal operational conditions, several disturbances may occur which may degrade process performance. Hence, advanced control design is of utmost relevance for keeping the system performance as close as possible to the desired one.

Most (bio)-chemical processes display nonlinear behavior due to complex interactions between transport and reaction phenomena (Bequette, 2003; van Impe, Vanrolleghem & Iserentant, 2013). For control design purposes around a steady-state point, a linear model, obtained either by linearization or identification, is frequently used. Linear-based methods have been addressed using PID-type controllers, model predictive controllers (MPC), and H∞ control.

For a highly nonlinear behavior, as in the case of (bio)-chemical processes, a linear controller is inadequate for assuring stability and the closed-loop performance. Indeed, neglecting system

nonlinearities may lead to instability because of disturbances or setpoint variations (Bequette, 2003; Hangos et al., 2006).

Most advanced control algorithms rely on a process model to compute the control actions. Nonlinear model-based controllers applied in (bio)-chemical processes include Lyapunov-based methods, nonlinear H∞ control, sliding mode control approaches (SMC) (Bouyahia et al., 2020), nonlinear MPC (Lee & Lee, 2014), extremum seeking control (Dewasme & Vande Wouwer, 2020), and optimal approaches (Hangos et al., 2006; Iqbal et al., 2017). However, these approaches often rely on complicated design procedures and computationally demanding algorithms which are preventing their practical application (Saltık et al., 2018; Sariyildiz et al., 2019).

Model-based control of (bio)-chemical processes is also a challenging problem due to uncertainty related to the mathematical models of such systems. Although it is often possible to derive a phenomenological nonlinear model of the process, it can be imprecise due to uncertainties and lack of perfect knowledge (Mišković & Hatzimanikatis, 2011; Sharifian et al., 2019). Moreover, detailed nonlinear models are impractical for control design purposes due to their high order, nonlinearities, computational demand, and model uncertainties. Thus, low-order models have been commonly used (Bequette, 2003; Smith & Corripio, 2005).

Thus, despite industrial (bio)-chemical processes being nonlinear, many advanced control designs are based on linear models. Several authors have used linear step-response models. Internal model control (IMC) approaches use a first-order plus time delay model to design an optimal IMC (Grimholt & Skogestad, 2018). SMC design for chemical processes uses a first-order plus time delay transfer function (Camacho & Smith, 2000). Linear models in different regions have also been considered for an H∞ controller design (Galán et al., 2000). MPC uses linear identification of an input-output model and linearized approximations of the nonlinear model (Lee, 2011; Juneja & Murthy, 2018). Recent contributions addressing models with uncertainties are as follows. In (Letchindjio et al., 2019), block-oriented systems are introduced to approximate a class of non-linear systems and design an adaptable extremum seeking approach. Bhadra et al. (2019) present a non-inferential

type of nonlinear controller enhanced with an estimation scheme to compensate for the external disturbances and measurement noises. A linearized robust NMPC approach based on predicted outputs through a linear approximation of the non-linear model was proposed by Benattia, Tebbani & Dumur (2020) to reduce the computational time of min-max robust NMPC approaches.

Departing from modeling error compensation ideas (Alvarez-Ramirez, 1999), this paper introduces a simple, systematic, and novel robust model-based control framework for a class of (bio)-chemical processes. Based on an extended input-output model with uncertainty estimation, simple, robust feedback controllers are derived. The resulting control framework allows a systematic consideration of uncertainty providing high robustness capabilities against bounded and additive uncertainties in the input-output model approximation. Moreover, simple tuning rules are proposed. A benchmark case study of microalgae production, which has been used by the control engineering community in several control studies (Bernard, 2011; Telen et al., 2017), is used for illustrating the versatility, simple design, as well as the tuning of the proposed control framework. Furthermore, to compare the performance of the proposed controllers, two of the most widely accepted control designs are also applied: a classical PI controller tuned with IMC rules and a nonlinear MPC. Numerical simulations show an acceptable closed-loop performance of the proposed controller with high robustness capabilities despite the simplicity of the controller designs.

This paper is organized as follows. Section 2 presents the extended input-output model with uncertainty estimation. In Section 3, three robust model-based controllers are derived from the extended model. In Section 4, a case study is described, and numerical studies illustrate the performance of the proposed robust feedback control framework. In Section 5, concluding remarks are provided.

2. Simple Input-Output Model Enhanced with Model Error estimation

For model-based control design, simple models describing the dominant input-output dynamics are highly desirable. Due to its simplicity and the capability of representing the dominant process dynamics, a first-order input-output model

has been commonly used for controller design purposes (Bequette, 2003; Smith and Corripio, 2005). The first-order input-output model can be obtained using different identification methods. However, since uncertainties are not considered in the input-output identification process, the inaccurate model obtained can lead to a poorly designed controller, resulting in unsatisfactory control performance.

Consider a first-order input-output model for the nominal linear dynamics given by,

$$G_p(s) = \frac{Y(s)}{U(s)} = \frac{k_p}{\tau_0 s + 1} \qquad (1)$$

Where k_p is the steady-state process gain, and τ_0 is the process time constant. For control design purposes, the basic first-order model has proved to be the simplest one, but significant uncertainties are neglected in its formulation. Thus, the following assumption is introduced:

A1. The input-output model representation given by equation (1) is affected by unmodelled nonlinearities $\xi(y)$, with bounded variation, i.e., $|\xi(y)| \leq \iota_1 |y| + \iota_2$, where ι_1 and ι_2 are positive constants. Moreover, external disturbances $\pi(t)$ can be either constant or persistent with bounded variation.

Then, under A1, equation (1) in the time-domain can be represented as:

$$\frac{dy(t)}{dt} = -\tau_0^{-1} y(t) + k_p \tau_0^{-1} u(t) + \eta(t) \qquad (2)$$

where $\eta(t) = \xi(y(t)) + \pi(t)$, represents model uncertainties.

Thus, the uncertainty $\eta(t)$ is seen as an unknown state. Based on the model structure given by equation (2), the uncertain state can be reconstructed from the input $u(t)$ and output $y(t)$ signals.

Hence, to account for such uncertainties for control design purposes, the model given by equation (2), enhanced with a model uncertainty estimation is considered. To this end, the following reduced-order observer is considered,

$$\frac{d\tilde{\eta}(t)}{dt} = \tau_e^{-1}(\eta(t) - \tilde{\eta}(t)) \qquad (3)$$

where τ_e is an estimator paramerer, and $\tilde{\eta}(t)$ is the estimation of the real modeling error $\eta(t)$. The estimator in equation (3) provides an estimation of

real uncertain terms using only the available output and input. After simple algebraic manipulations, the estimator can be written as follows:

$$\frac{dw(t)}{dt} = \tau_0^{-1} y(t) - k_p \tau_0^{-1} u(t) - \tilde{\eta}(t) \qquad (4)$$

with,

$$\tilde{\eta}(t) = \tau_e^{-1}(w(t) + y(t)) \qquad (5)$$

The reduced observer in equation (4) can be considered as a signal estimator, where the modeling error signal is an extended state of the nominal model (Alvarez-Ramirez, 1999). Hence, the modeling error estimation endows the control system with robustness capabilities against model-matched uncertainties. The estimator parameter τ_e, which determines the convergence rate of the modeling error estimation, can be selected as $\tau_e < 0.5\tau_0$.

The practical enhanced input-output model with uncertainty estimation is written then as follows,

$$\frac{dy(t)}{dt} = -\tau_0^{-1} y(t) + k_p \tau_0^{-1} u(t) + \tilde{\eta}(t) \qquad (6)$$

which in conjunction with the estimator given by equations (4) and (5), leads to a practical and straightforward way of considering model uncertainties.

The following comments are appropriate:

- *A1* is a realistic assumption. Indeed, according to Mišković & Hatzimanikatis (2011) and Sharifian et al. (2019), uncertainties arise: (i) when the dynamics of actuators is not considered in the control design stage, (ii) in case of neglected high-order dynamics due to the approximation of input-output transfer function models, and (iii) in case of neglected small time-delays;

- Two main streams of robust process control for characterizing the model uncertainties are: (i) to quantify the uncertainty in terms of the bound of its gain (represented by using system norms) (Morari & Zafiriou, 1989); (ii) to accommodate uncertainty in the process model parameters using a set of uncertainty realizations based on deterministic or stochastic approaches (Lee & Lee, 2014). In this paper, the robustness properties are against lumped uncertain parameters in the approximated input-output model and neglected nonlinearities with bounded variations and of additive nature, which is

the case of most nonlinear functionalities modeling (bio)-chemical processes (Bequette, 2003; Hangos et al., 2006);

- For model-based robust control designs, the proposed uncertainty framework captures external perturbations and uncertainties in a simple way, and it is mathematically tractable.

3. Robust Control Designs Based on Enhanced Input-Output Models

In this section, three control designs departing from the extended input-output model with uncertainty estimation are derived. First, a general control problem is stated, and some assumptions for control design purposes are introduced.

3.1 Control Problem

Without loss of generality, the control objective is the regulation or tracking of a desired output variable $y(t)$ to the desired reference $y_{ref}(t)$, of the class of (bio)-chemical processes modeled by equation (6) manipulating the control variable $u(t)$, despite model uncertainties and external disturbances estimated with the estimator given by equations (4) and (5).

The following additional assumptions are made:

A2. The control input $u(t)$ is subjected to saturation, i.e., $u_{min} \leq u(t) \leq u_{max}$.

A3. The output variable $y(t)$ is measured continuously without measurement delay.

A4. The desired reference $y_{ref}(t)$ and its time derivative are available and bounded.

The following two observations can be made:

- For practical implementation purposes, the control input $u(t)$, which in several (bio)-chemical processes is restricted to different flow rates, is physically limited;

- Assumption A3 is realistic. Indeed, main on-line and instantaneously measurable variables in (bio)-chemical processes include temperature, pH, conductivity, flow rates, and chemical compositions. Moreover, even in the absence of an on-line measurement of the desired controlled variable $y(t)$, a state estimator design can be considered (Hangos et al., 2006).

3.2 Simple Robust Model-based Control

Consider a desired first-order asymptotically closed-loop behavior. The following inverse feedback function for the control input is proposed (Alvarez-Ramirez, 1999),

$$u(t) = \frac{\tau_0}{k_p}\left\{\left(\frac{1}{\tau_0} - \frac{1}{\tau_c}\right)e(t) + \frac{de(t)}{dt} - \tilde{\eta}(t)\right\} \qquad (7)$$

where $\tau_c > 0$ is a controller parameter. Under the proposed controller, a first-order dynamics of the regulation error is obtained, such that asymptotic convergence $e(t) \rightarrow 0$, and so $y(t) \rightarrow y_{ref}(t)$ is guaranteed.

Tuning of parameter τ_c can be made to attain a satisfactory closed-loop response following the simple rule (Alvarez-Ramirez, 1999): $\tau_e < \tau_c < \tau_0$.

3.3 Extended Sliding Mode Control

Sliding mode control is a powerful, robust control method suitable for dealing with model nonlinearity and parameter uncertainties. The control design is derived in two-steps (Slotine & Li, 1991):

1. Sliding phase: Design a sliding surface $s(t)$, such that $s(t) = 0$ and $ds(t)/dt = 0$, along which the controlled variable slides to its desired value. A continuous control action is obtained;

2. Reaching phase: Design a discontinuous control action that forces the controlled state to the sliding surface when $s(t) \neq 0$.

Several modifications have been introduced and applied in (bio)-chemical processes to improve the robustness capabilities against structured uncertainties and reduce chattering effects (Kanthalakshmi & Annal, 2018; Utkin et al., 2020).

Departing from the ideas introduced by Slotine and Li (1991), the sliding surface is proposed as,

$$s(t) = e(t) + \lambda \int_0^t e(\tau)d\tau \qquad (8)$$

Based on a first-order input-output model, Abu-Rmileh et al. (2010) introduce a simple controller, which is extended with the proposed control framework as follows,

$$u(t) = \frac{\tau_0}{k_p}\left(-\lambda e(t) + \frac{1}{\tau_0}y(t) - \tilde{\eta}(t)\right) + k_D sign(s(t)) \quad (9)$$

where k_D and λ are controller parameters, which, based on the input-output model, are proposed as $k_D = 0.1/k_p$ and $\lambda = 0.5/\tau_0$ (Abu-Rmileh et al., 2010).

3.4 Extended Generic Model Control

The standard Generic Model Control (GMC) consists in matching PI control actions with a nonlinear process model (Lee & Sullivan, 1988). Using concepts from differential geometry and a Luenberger-type nonlinear state estimator, an extended GMC controller was introduced by Jana (2013).

Consider the extended dynamic simple model given by equation (6) with the reference system given by,

$$\frac{dy_{ref}(t)}{dt} = K_1 e(t) + K_2 \int_0^t e(\tau) d\tau \qquad (10)$$

The extended GMC is formulated as the standard GMC, i.e., the dynamics of the process output given by equation (6), follows the desired reference system given by equation (10). Thus,

$$-\frac{1}{\tau_0} y(t) + \frac{k_p}{\tau_0} u(t) + \tilde{\eta}(t) = K_1 e(t) + K_2 \int_0^t e(\tau) d\tau \quad (11)$$

and solving $u(t)$ yields,

$$u(t) = \frac{\tau_0}{k_p} \left\{ \frac{1}{\tau_0} y(t) - \tilde{\eta}(t) + K_1 e(t) + K_2 \int_0^t e(\tau) d\tau \right\} \quad (12)$$

Two approaches for calculating controller parameters K_1 and K_2 are as follows (Lee & Sullivan, 1988).

The first approach is by using the following formulas,

$$K_1 = 2\tau_c^{-1}\xi, \quad K_2 = \tau_c^{-2} \qquad (13)$$

where the parameters ξ and τ_c can be set in two steps: (i) determine to give the desired shape of the closed-loop response, and (ii) choose τ_c to modulate the timing of the closed-loop response concerning the open-loop rate of response τ_0.

The second approach is to examine the characteristic equation of the closed-loop system, given by,

$$s^2 + K_1 s + K_2 \qquad (14)$$

and set the location of the two closed-loop poles to solve equation (14) for K_1 and K_2.

3.5 Extension to a Second-orden Input-Output Model

In the preceding sections, the simple robust control framework was formulated for a first-order input-output model. Consider a second-order system given as,

$$G_p(s) = \frac{Y(s)}{U(s)} = \frac{k_p}{\tau_0^2 s^2 + 2\varepsilon\tau_0 s + 1} \qquad (15)$$

where ε is a damping coefficient. Second-order input-output models can describe several (bio)-chemical processes, including first-order time-delayed input-output models with a first-order Taylor series approximation to the time-delay term (Camacho & Smith, 2000; Bequette, 2003).

In the time-domain, the second-order uncertain model can be written as,

$$\frac{dy(t)}{dt} = z(t) \qquad (16)$$

$$\frac{dz(t)}{dt} = -\frac{2\varepsilon}{\tau_0} z(t) - \frac{1}{\tau_0^2} y(t) + \frac{k_p}{\tau_0^2} u(t) + \eta(t) \quad (17)$$

In this case, the simultaneous estimation of $z(t)$ and the lumped uncertainties $\eta(t)$, from the available output $y(t)$ are given as,

$$\frac{dw_1(t)}{dt} = -\frac{2\varepsilon}{\tau_0} \tilde{z}(t) - \frac{1}{\tau_0^2} y(t) + \frac{k_p}{\tau_0^2} u(t) + \tilde{\eta}(t) - \frac{2}{\tau_e} \tilde{z}(t) \quad (18)$$

$$\frac{dw_2(t)}{dt} = -\frac{1}{\tau_e^2} \tilde{z}(t) \qquad (19)$$

with,

$$\tilde{\eta}(t) = w_2(t) + \tau_e^{-2} y(t)$$

$$\tilde{z}(t) = w_1(t) + 2\tau_e^{-1} y(t)$$

Hence, previous control designs can be formulated using a second-order model with uncertainty estimation.

3.6 Robustness and Stability Issues

Although formal robustness and stability analysis are beyond the scope of this paper, the closed-loop system can be analyzed using stability results from singular perturbation systems (Khalil, 1992), where the dynamics of the estimation error, $e_e(t) = \eta(t) - \tilde{\eta}(t)$, is the fast dynamics. Moreover, its reduced model is the closed-loop system resulting from the base version of the extended

control designs formulated in this paper. Finally, robustness properties of the closed-loop system are provided by the estimation and cancelation of uncertain terms in each control design.

3.7 Nonlinear MPC

For comparison purposes, the standard NMPC is considered. NMPC is a powerful control method for controlling nonlinear, constrained, and uncertain chemical and biological processes (Lee, 2011; Piceno-Díaz et al., 2020).

The objective of an NMPC formulation is to find the best control sequence (until a given control horizon, M) such that a given cost function is minimized. The corresponding optimization problem relies on predicting future behavior in a given prediction horizon (N).

The standard NMPC can be formulated as follows (Lee & Lee, 2014):

$$\min_{u_k} J = \gamma_1 \sum_{j=0}^{N} \left\| y_{k+j} - y_{ref} \right\|^2 + \gamma_2 \sum_{j=0}^{M} \left\| \Delta u_{k+j} \right\|^2 \quad (20)$$

subject to,

$$
\begin{aligned}
x_{k+1} &= f(x_k, u_k, \tilde{\theta}) \\
h(x_k, u_k, \tilde{\theta}) &\leq 0 \\
x_{\min} &\leq x_k \leq x_{\max} \\
u_{\min} &\leq u_k \leq u_{\max}
\end{aligned}
\quad (21)
$$

where y_{k+j} is the predicted value of the controlled variable using the non-linear process model, y_{ref} is the desired reference, u_k denotes the sequence of control inputs and Δu_{k+j} is the control move at instant $k+j$, i.e., $u_{k+j} - u_{k+j-1}$. γ_1 and γ_2 are weighting factors used to set the ratio of importance between costs due to deviation from desired states and costs due to control moves. Nominal parameter values are denoted as $\tilde{\theta}$. It can also be mentioned that function f in (21) represents the discretized nonlinear dynamical model of the system and h is the set of process system constraints. Moreover, k is the sampling instant, which relates to processing time t via $t = kT_s$, where T_s is the sampling time. Controller parameters are the prediction horizon N, the control horizon M, the sampling time T_s, and the weighting factors γ_1 and γ_2.

The standard NMPC algorithm has acceptable robustness capabilities. Indeed, the correction of the model´s future prediction and the actual system´s response in each iterative step of the

NMPC algorithm alleviates slight model mismatch providing some degree of robustness.

The simultaneous full-discretization approach is pursued to implement the NMPC algorithm, leading to a nonlinear programming (NLP) problem at each time interval (Biegler, 2007). To this end, both the state and control profiles are discretized by means of orthogonal collocation on finite elements using Legendre polynomials with three internal collocation points (Finlayson, 1980). The resulting NLP optimization problem was programmed in Matlab using the *fmincon* function with the interior point optimization method.

4. Numerical Example

In this section, the derived robust model-based control schemes in the above section are evaluated using a benchmark case study of microalgae production. First, the case study is described. Then, the optimal steady-state condition is computed. Finally, the robust model-based control schemes are applied and evaluated under external disturbances and set-point changes. Furthermore, a comparison with a classical PI controller and an NMPC controller is presented.

4.1 Microalage Growth

The recently renewed interest in microalgae production stems from its capacity to accumulate large amounts of carbohydrates and neutral lipids, which can be used as a feedstock for biorefineries (Li et al., 2008).

Consider a biological chemostat of volume V, made of a transparent material. The source of light for the microalgae growth is provided via external lamps surrounding the chemostat. The input stream with a volumetric flow rate (F) contains the input organic substrate (S_{in}). The output stream comprises the non-consumed substrate (S) and the produced microalgae biomass (X).

A simple non-structured, non-segregated model of three main species is considered for the continuous microalgae production in the chemostat (Bernard, 2011).

Model assumptions are (Droop, 1983; Darvehei, Bahri & Moheimani, 2018): (i) Homogeneous conditions. (ii) Constant volume, external light intensity, and temperature. (iii) The organic load is lumped in a single kind of substrate, S. (iv)

The microalgae biomass is lumped in a single state, X. (v) the kinetics of the microalgae growth is described with the Droop model, which considers an internal substrate (nutrients) pool within the cells, denoted as the internal quota Q_N.

The mathematical model is given by:

$$\frac{dX(t)}{dt} = -D(t)X(t) + \bar{\mu}\left(1 - \frac{K_Q}{Q_N(t)}\right)X(t)$$

$$\frac{dQ_N(t)}{dt} = \rho_m \frac{S(t)}{k_s + S(t)} - \bar{\mu}\left(Q_N(t) - K_Q\right) \quad (22)$$

$$\frac{dS(t)}{dt} = D(t)\left(S_{in} - S(t)\right) - \rho_m \frac{S(t)}{k_s + S(t)}X(t)$$

where $S(t)$ (μmol/L), $X(t)$ (μm³/L), and $Q_N(t)$ (μmol/ μm³) represent the concentrations of substrate, microalgae biomass, and the internal quota, respectively. The input substrate is S_{in} (μmol/L). The dilution rate is $D(t)=F(t)/V$ (day⁻¹). Droop kinetic parameters are the maximum kinetic growths ρ_m (μmol/μm³day) and $\bar{\mu}$ (day⁻¹) for the substrate consumption and the internal quota, respectively. K_Q (μmol/μm³) is the minimum growth constant for biomass growth. k_s (μmol/L) is the saturation constant for the substrate consumption.

The nominal or base operation is simulated with the parameter values given by S_{in} = 100 μmol/L, k_s = 0.105 μmol/L, K_Q = 1.8 μmol/μm³, μ =2 1/day, ρ_m = 9.3 μmol/μm³day, and D =1.3 day⁻¹. A single steady-state which is given by $[S, Q_N, X]^*$=[0.268 μmol/L, 5.142 μmol/μm³, 19.392 μm³/L] is obtained with the above parameter values. Moreover, standard linear stability analysis of the equilibrium point was performed to corroborate the stable behavior.

Figure 1 shows a step-change in the dilution rate to derive the input-output first-order model. Note that a smooth response is obtained. Based on the input-output response the first-order parameters k_p = -31.7 and τ_0 = 2 are obtained.
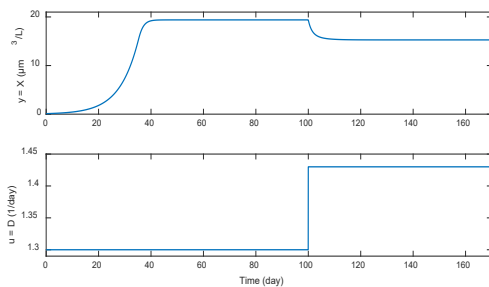

Figure 1. Step identification of the Droop model

4.2 Control Problem

The control problem is set as the regulation of the microalgae biomass concentration $y(t) = X(t)$, at the optimal productivity condition, using as the manipulated variable the dilution rate, $u(t)=D(t)$.

Linear and nonlinear MPC approaches were applied to similar control problems for optimal microalgae growth using the Droop model (Benattia, Tebbani & Dumur, 2020) and a simple three-state experimentally validated model (Juneja & Murthy, 2018).

The optimal conditions are obtained by solving the following nonlinear optimization problem:

$$\min J = -DX \quad (23)$$

subject to,

$$0 = -DX + \bar{\mu}\left(1 - \frac{K_Q}{Q_N}\right)X$$

$$0 = \rho_m \frac{S}{k_s + S} - \bar{\mu}\left(Q_N - K_Q\right) \quad (24)$$

$$0 = D\left(S_{in} - S\right) - \rho_m \frac{S}{k_s + S}X$$

$$S \le S_{in} \quad (25)$$

$$D_{min} \le D \le D_{max} \quad (26)$$

Constraints of the optimization problem correspond to the steady-state model equations (24), the upper limit of the input substrate given by equation (25), and minimum and maximum constraints on the decision variable given by equation (26), with D_{min} = 0.1 day⁻¹, and D_{max} = 5 day⁻¹.

The resulting optimization problem is nonlinear and constrained. In this case, the interior-point algorithm available in Matlab for solving the NLP optimization problem was applied. The resulting optimal steady-state is given by $[S, Q_N, X, D]_{op}$=[3.59 μmol/L, 0.066 μmol/μm³, 27.8 μm³/L, 1 1/day], which correspond to a 10 % increase in the nominal biomass microalgae productivity.

The control action is activated at t = 100 days. An external perturbation on the input substrate is applied at t = 200 days. Finally, a set-point change is applied at t = 300 days.

4.3 Simple Robuts Control Design

Figure 2 shows the closed-loop behavior using the simple robust model-based control design and three sets of controller parameters chosen according to the tuning guidelines described in the section above. The comparison with a PI controller with IMC-PI tuning parameters is also illustrated.

Figure 2 shows that the proposed robust model-based controller can regulate the output despite the presence of the input substrate disturbance. It can also be seen that the change of set point is achieved with an acceptable control effort. The best closed-loop performance is obtained for the first set of controller parameters, $\tau_e = 1.0$, $\tau_c = 1.5$.
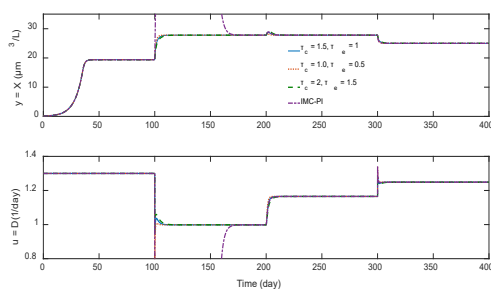

Figure 2. Closed-loop performance of the simple robust model-based controller for the Droop model

Lower values of the controller parameters lead to a more aggressive control action. On the other hand, higher values for controller parameters lead to a smoother but also slower closed-loop response. However, as it is shown in Figure 2, the proposed simple robust model-based controllers display a superior performance to that of the PI for the control input saturation at its lower value when the controller is activated at 100 days.

4.4 Extended Sliding Mode Control

The closed-loop performance of the extended SMC approach is shown in Figure 3.
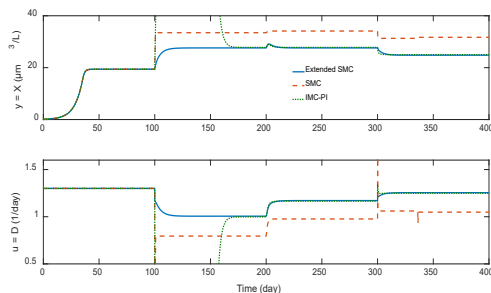

Figure 3. Closed-loop performance of the extended SMC for the Droop model

It can be noticed that the proposed extended SMC is able to regulate the microalgae biomass concentration with acceptable closed-loop behavior. Moreover, the original SMC approach is unable to achieve the desired reference. It is to be noted that for the original SMC approach, the tuning parameters were performed using a trial-and-error procedure instead of the tuning rules described in the above section. However, for the extended SMC approach that rules were applied leading to the illustrated closed-loop behavior.

4.5 Extended Generic Model Control

The extended GMC was implemented using three sets of parameter values of K_1 and K_2 with the pole placement approach.

Figure 4 illustrates the numerical simulations of the closed-loop behavior of the extended GMC. It is to be noted that the closed-loop performance is similar to that of PI controller.
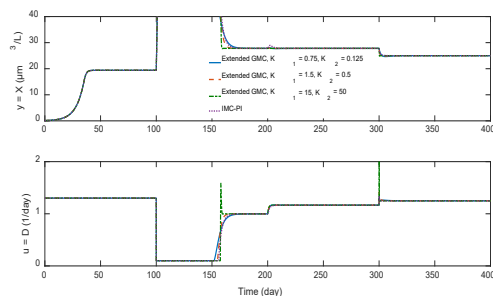

Figure 4. Closed-loop performance of the extended GMC for the Droop model

The poor performance of both controller approaches can be related to the effect of the integral action producing an initial aggressive control action that saturates the control input to its lower limit. However, after this transitory dynamics, and when the disturbances are applied along with the set-point change, the closed-loop performance displays an acceptable behavior.

4.6 NMPC

Figure 5 shows the numerical results of the NMPC implementation and the comparison with the simple robust model-based controller and the extended SMC. NMPC parameters are $[\Delta t, T_s, N, M, \gamma_1, \gamma_2] = [1, 0.5, 2, 1, 10, 1]$.

It can be noticed that the NMPC approach displays the best closed-loop behavior with more pronounced peaks at the activation of the control action and the setpoint change. The NMPC approach is also able to reject the external perturbation on the input substrate. However, despite the simplicity of its

implementation the proposed robust model-based controller and the extended SMC show similar closed-loop performances. Moreover, it is to be noted that the NMPC formulation uses the nonlinear model for the control design and requires the full state measurement for implementation purposes in the current applied formulation.
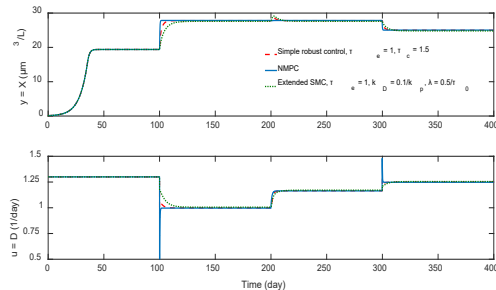

Figure 5. Closed-loop performance of the NMPC for the Droop model

5. Conclusion

This paper presents a simple robust control framework for regulating a class of (bio)-chemical processes. The robust control framework is formulated in three steps: (i) step identification of a low first-order model, (ii) derivation of an enhanced first-order model, with the estimation of model uncertainties associated with the model reduction and external disturbances, and (iii) the design of robust control schemes based on the enhanced model. The resulting controllers have a simple structure with simple tuning rules based on the process time constant. The proposed robust control framework is applied to a benchmark case study of microalgae biomass production. The effectiveness of the proposed, approach is demonstrated with the derivation of three robust control designs with a good closed-loop performance despite external perturbations. Moreover, the comparison with a NMPC design shows similar closed-loop performance without the requirements of a full state measurement and a bigger computational effort.

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