On a Practical Algorithm of P-IMC Type

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Abstract: As a consequence of the strategy of choosing the control parameters (that is, tuning gain, process feedback gain and three model parameters – steady-state gain, settling time and time delay), the proposed P-IMC algorithm enjoys a quasi-universal practical applicability. The main purpose of this paper is to analyse how the weighting coefficient \( \alpha \) of the proportional component \( P \) of the P-IMC algorithm may affect the control quality, thereby providing useful information to the manufacturer and human control operator. Moreover, this paper presents certain useful results regarding the influence and effect of the parameters of the proposed model upon the control performance of a control system.

Keywords: Practical control algorithm, Internal model, Robustness, Tuning gain, Model settling time, Model time delay, Process feedback gain.

1. Introduction

The proportional-internal model control (P-IMC) is a simple and practical control algorithm, which is easily accessible to a controller user and achieves a robust and efficient control (Cirtoaje, 2017; Cirtoaje & Baiesu, 2018; Cirtoaje, 2020). The IMC concept is based on the idea that a robust and accurate control can be obtained by inserting a suitable process model in the controller structure (Francis & Wonham, 1976; Garcia & Morari, 1982; Morari & Zafiriou, 1989). In the last years, many improvements of the IMC structure and method have been made for proportional or integrating stable processes (with or without time delay), and also for some unstable processes (Tan et al., 2003; Arun & Prakash, 2018; Touati et al., 2018). Many adaptive and/or optimal IMC solutions including one or two degree-of-freedom controller were proposed for compensating the trade-off between setpoint tracking and disturbance (Zazueta & Alvarez, 2000; Rupp & Guzzella, 2010; Wang et al., 2018). In addition, many PID tuning techniques have been proposed by applying IMC strategy for processes and plants of low order plus time delay (Rivera et al., 1986; Leva, 2006; Santosh Kumar & Padma Sree, 2016).

The block diagram of a control system with \( \alpha \)-IMC algorithm (or, simpler, P-IMC) is illustrated in Figure 1 (Cirtoaje, 2020).

![Figure 1. Closed-loop control system with P-IMC controller](image_url)

The transfer functions of the compensated process model (of second order and with equal time constants) and the internal controller (with the tuning gain \( K \)) can be expressed as follows:

\[
G_M(s) = \frac{K_M e^{-\tau_M s}}{(T_M s + 1)^2},
\]

\[
G_i(\alpha, s) = \frac{(T_M s + 1)^2}{K_M (T_i s + 1)^2},
\]

where \( \alpha \in [0,1] \) is the weighting coefficient of the proportional component \( P \) of the P-IMC algorithm, and

\[
T_i = \frac{T_M}{K_i}, \quad K_i = K^{\frac{1-\alpha}{2}}.
\]

The time constant \( T_M \) of the model (1) can be calculated with

\[
T_M = \frac{T_{M95} - \tau_M}{4.74},
\]

where \( T_{M95} \) (denoted by \( T_{sM95} \) from now on) is the settling time of the compensated process model (when its response to a step input reaches 95% of the final value). It should be noted that a first-order model can’t describe with sufficient accuracy the process dynamics, while a second-order model with distinct time constants or a third-order model would complicate the modeling operation without significantly improving the control performances.

The feedback path around the process (with the process feedback gain \( K_r \)) is used only if the process is an integrating or unstable one, in order to turn it into a stable compensated proportional-
type process. For a stable proportional process, the feedback gain $K_f$ is set to zero. The model parameters $K_M$, $T_M$ and $\tau_M$ are chosen to be approximately equal to the compensated process parameters $P_M$, $T_P$ and $\tau_P$ (usually, experimentally determined).

If the response $y(t)$ of the compensated process $P$ to a unit step input $c$ has an overshoot at $t_1$, then the model parameters are determined as follows: the compensated process $P$ is approximated with a process $P_1$ having the response $y_1(t)$ to a unit step input $c$ such that $y_1(t) = y(t)$ for $t \leq t_1$ and $y_1(t) = y(t_1)$ for $t \geq t_1$, which means that $K_M = y(t_1)$, $T_M = t_1$ and $\tau_M = \tau_P$ (Cirtoaje, 2020).

If the compensated process is of non-minimum phase and the sign of its step response $y(t)$ to a step input $c$ is for $0 \leq t \leq t_0$ opposite to the sign of $y(\infty)$, then $\tau_M$ must be set to $t_0$.

If the process is unstable or integrating-stable, the feedback gain $K_f$ is experimentally determined so as to achieve a stable compensated proportional-type process. It is recommended to choose a high value for $K_f$, but not so high as to cause an overshoot response of the compensated process to a step input $c$.

The control system in Figure 1 satisfies the following properties (Cirtoaje, 2020):

- The steady-state error for a step reference (setpoint) or load disturbance is zero (if the control system is asymptotically stable);
- For $K_M = K_P$, the initial value $c(0_+)$ of the control response $c(t)$ to a step setpoint is $K$ times its final value $c(\infty)$;
- If $K = 1$ and the compensated process model is perfect, then the control response $c(t)$ to a unit step setpoint is a step with the magnitude $1/K_M$ (whatever the value of $\alpha$).

The third property is an opportunity for the human control operator to verify the model accuracy and suitably improve the value of the model parameters, while the second property offers a simple and practical interpretation of the tuning parameter $K$, highlighting its role in the control action. It should be noted that the human control operator can adjust the tuning parameter $K$ with the purpose of obtaining a stronger or weaker control action.

The discrete-time equivalent (with the sampling period $T$) of the continuous model (1) of the compensated process (with input $c$ and output $w$) has the transfer function:

$$G_c(z) \approx \frac{K M (1 - p_M)^2 z^{-1 - jM}}{(1 - p_M z^{-1})^2}, \quad (5)$$

where

$$p_M = e^{-T/T_M}, \quad T_M = \frac{T_M \tau_M}{4.74}, \quad l_M = \left[ \frac{\tau_M}{T} \right], \quad (6)$$

and the difference equation

$$w_k - 2p_M w_{k-1} + p_M^2 w_{k-2} = K_M (1 - p_M)^2 c_{k-1-jM} \quad (7)$$

According to (2), the discrete-time internal controller (with input $f$ and output $c$) has the approximated transfer function

$$G_f(\alpha, z) = \frac{K_1^2}{K_M} \cdot \frac{(1 - q_1 z^{-1})^2}{(1 - p_1 z^{-1})^2}, \quad (8)$$

where

$$p_1 = e^{-\tau_1/T_1}, \quad q_1 = 1 - \frac{1 - p_1}{K_1}, \quad T_1 = \frac{T_M}{K_1}, \quad (9)$$

and the difference equation

$$c_k - 2p_1 c_{k-1} + p_1^2 c_{k-2} = \frac{K_1^2}{K_M} (f_k - 2q_1 f_{k-1} + q_1^2 f_{k-2}). \quad (10)$$

From the bloc diagram of the control system in Figure 1, and the difference equations (7) and (10), the following numerical control algorithm of P-IMC type is obtained (Cirtoaje, 2020):

$$\varepsilon_k = r_k - y_k,$$
$$w_k = 2p_M w_{k-1} - p_M^2 w_{k-2} + K_M (1 - p_M)^2 (c_{k-1-jM} - u_0),$$
$$f_k = K_M (\varepsilon_k - \varepsilon_0) + w_k,$$
$$c_k = u_0 + 2p_1 (c_{k-1} - u_0) - p_1^2 (c_{k-2} - u_0) + \frac{K_1^2}{K_M} (f_k - 2q_1 f_{k-1} + q_1^2 f_{k-2}),$$
$$u_k = c_k - K_f (y_k - y_0). \quad (11)$$
where $\varepsilon_0$, $u_0$ and $y_0$ are the values of $\varepsilon$, $u$ and $y$, respectively, before switching the controller to AUTOMATIC mode.

To have a bumpless transfer when switching the controller from MANUAL to AUTOMATIC mode, the following settings must be applied before switching:

\[
\begin{align*}
    c_{k-1} &= c_{k-2} = \cdots = c_{k-\lambda} = u_0, \\
    w_{k-1} &= w_{k-2} = 0, \\
    f_{k-1} &= f_{k-2} = 0.
\end{align*}
\]

The paper, which is meant to determine how the weighting coefficient of the proportional component $P$ (a priori fixed between 0 and 1) and the model parameters of the P-IMC algorithm influence the control performance of a control system, is organized as follows. Section 2 presents a simulation-based study of the role of the weighting coefficient $\alpha$ upon the control action and control quality. Several simulation experiments with three suitable values of the weighting coefficient are presented for three processes: an overdamped process, an underdamped proportional process and an integrating process. Section 3 analyzes the control robustness of the P-IMC algorithm with respect to the time parameters of the proposed model. It highlights how these parameters influence the control performance of a control system and provides certain useful recommendations on their choice. Finally, Section 4 sets forth the conclusions of this paper and recommendations on the possible future practical applications of the proposed control algorithm.

2. Choice of the Weighting Coefficient

The influence of the tuning parameter $K$ upon the control action is a bit aggressive and weak overall for $\alpha = 0$ (P0-IMC algorithm – Cirtoaje, 2017), and smoother and stronger for $\alpha = 1$ (P1-IMC algorithm – Cirtoaje & Baiesu, 2018). It should be noticed that for $\alpha = 0$, the P-IMC algorithm turns into the classical variant of the IMC algorithm. Because all control simulations based on the P-IMC algorithm were performed so far in the Matlab/Simulink environment for $\alpha = 0$, $\alpha = 1$ and $\alpha = 0.2$ (Cirtoaje, 2020), further on some comparative simulations will be made for $\alpha = 0.2$, $\alpha = 0.4$ and $\alpha = 0.6$.

Application 1. Consider an overdamped process with the transfer function

\[
G_p(s) = \frac{(4s + 1)e^{-4s}}{(2s + 1)(6s + 1)(10s + 1)}.
\]

Based on the process response $y$ to a unit step input $u$ (Figure 2), the following model parameters may be determined: $K_M = 1$, $T_{SM} = 36$ and $\tau_M = 4$.

For these model parameters, and for $K_f = 0$ and $K = 1$, Figure 3 illustrates the control system responses $c$ and $y$ to a unit step setpoint $r$.

Since the shape of the control response $c$ is close to step shape, it follows that the values of the model parameters are suitable. As a consequence, the shape of the control system response $y$ to a unit step setpoint $r$ is close to that of the process response $y$ to a step input $u$.
parameter was chosen to be $K = 3.2$, $K = 1.8$ and $K = 1.5$, respectively, so that all responses $y$ have almost the same overshoot (around 2%) – Figures 4, 5 and 6.

$\alpha = 0.2$, and $c(\alpha) = 1.5$ for $\alpha = 0.6$ (generally, $c(\alpha) = K$ and $c(\alpha) = 1$).

Figures 7, 8, and 9 show the control system responses $y$ to a unit step load disturbance $v$ for the same values of $\alpha$ and $K$ as in Figures 4, 5 and 6, respectively. One could make the same remarks as for the system responses to a unit step setpoint: the response for $\alpha = 0.2$ is a little better than the response for $\alpha = 0.4$, which is a little better than the response for $\alpha = 0.6$.

It can be seen that the three responses $y$ are close to each other. However, the response for $\alpha = 0.2$ is a little better than the response for $\alpha = 0.4$, which is a little better than the response for $\alpha = 0.6$. This result can be justified by the shape of the control response $c(t)$, whose initial value is equal to its final value multiplied by $K$, more precisely, $c(\alpha) = 3.2$ for $\alpha = 0.2$, $c(\alpha) = 1.8$ for...
**Application 2.** Consider an underdamped proportional process with

\[ G_p(s) = \frac{(4s + 1)e^{-4s}}{(2s + 1)(6s + 1)(9s^2 + 2s + 1)}. \]  

(13)

Based on the process response \( y \) to a unit step input \( u \) (Figure 10), the following model parameters may be determined: \( K_M = y(t_I) = 1.12 \), \( T_{m} = t_I = 18 \) and \( \tau_M = 4 \).

**Figure 10.** Process response \( y \) to a unit step input \( u \)

Figures 11, 12 and 13 show the responses \( c \) and \( y \) to a unit step setpoint for \( \alpha = 0.2 \) and \( K = 0.53 \), for \( \alpha = 0.4 \) and \( K = 0.63 \), for \( \alpha = 0.6 \) and \( K = 0.69 \), respectively.

**Application 3.** Consider an integrating process with the transfer function

\[ G_p(s) = \frac{(4s + 1)e^{-4s}}{60s(2s + 1)(6s + 1)}. \]  

(14)

By selecting \( K_f = 2.5 \), the compensated process becomes of proportional type, with the unit step input represented in Figure 14. Based on this response, the following model parameters may be determined: \( K_M = 0.4 \), \( T_{m} = 51 \) and \( \tau_M = 4 \).

**Figure 11.** Responses \( c \) and \( y \) to a unit step setpoint for \( \alpha = 0.2 \) and \( K = 0.53 \)

**Figure 12.** Responses \( c \) and \( y \) to a unit step setpoint for \( \alpha = 0.4 \) and \( K = 0.63 \)

**Figure 13.** Responses \( c \) and \( y \) to a unit step setpoint for \( \alpha = 0.6 \) and \( K = 0.69 \)

It should be noted that the values of the tuning gain \( K \) were selected such that the first maximum of the oscillatory closed-loop responses \( y \) to a unit step setpoint be equal to 1. The three responses \( y \) are very close to each other.

To compare the control system responses \( y \) to a unit step setpoint for \( \alpha = 0.2 \), \( \alpha = 0.4 \) and \( \alpha = 0.6 \), the tuning parameter was chosen to be \( K_f = 2.5 \), \( K = 1.6 \) and \( K = 1.4 \), respectively, so that all responses \( y \) have almost the same overshoot.
(around 2%) – Figures 15, 16 and 17. The three control system responses \( y \) are close to each other.

To illustrate this statement, consider the overdamped process (12) with \( K_p = 1 \), \( T_s = 36 \) and \( \tau = 4 \), and choose \( k_i = k_v = 1.1 \), which means \( K_M = K_p = 1 \), \( T_M = k_i T_s = 39.6 \) and \( \tau_M = k_v \tau = 4.4 \). The responses \( y \) of the control system to a unit step setpoint are illustrated in Figures 18, 19 and 20 for \( \alpha = 0.2 \), \( \alpha = 0.4 \) and \( \alpha = 0.6 \), respectively, and for suitable values of the tuning gain \( K \) (\( K = 6.4 \), \( K = 3.7 \) and \( K = 2.7 \), respectively) so that all responses have almost the same small overshoot.

The time parameters of the compensated process model are the settling time \( T_{sM} \) and the time delay \( \tau_M \). As a general recommendation, it is better to choose \( T_{sM} > T_s \) than \( T_{sM} < T_s \), and \( \tau_M > \tau \) than \( \tau_M < \tau \). After many simulations on overdamped compensated processes (without overshoot), it is recommendable that one chooses \( T_{sM} = k_i T_{sP} \) and \( \tau_M = k_v \tau_P \), where \( k_i, k_v \leq 1.15 \).

3. The Time Parameters for the Proposed Model

The time parameters of the compensated process model are the settling time \( T_{sM} \) and the time delay \( \tau_M \). As a general recommendation, it is better to choose \( T_{sM} > T_s \) than \( T_{sM} < T_s \), and \( \tau_M > \tau \) than \( \tau_M < \tau \). After many simulations on overdamped compensated processes (without overshoot), it is recommendable that one chooses \( T_{sM} = k_i T_{sP} \) and \( \tau_M = k_v \tau_P \), where \( k_i, k_v \leq 1.15 \).
The quality of the three responses is comparable, but it is better than that of the previous similar responses in Figures 4, 5 and 6. This can be justified by the approximately double value of the tuning parameter \( K \), which ensures responses with the same overshoot (6.4 versus 3.2, 3.7 versus 1.8, and 2.7 versus 1.5, respectively).

For the underdamped proportional process (13) with \( K_p = 1.12 \), \( T_{sp} = 18 \) and \( \tau_p = 4 \), by choosing \( k_1 = k_2 = 1.1 \), one obtains \( K_M = 1.12 \), \( T_{SM} = k_1 T_{sp} = 20 \) and \( \tau_M = k_2 \tau_p = 4.4 \). Figures 21, 22 and 23 illustrate the control system responses \( y \) to a unit step setpoint for \( \alpha = 0.2 \), \( \alpha = 0.4 \) and \( \alpha = 0.6 \) and for \( K = 0.86 \), \( K = 0.9 \) and \( K = 0.92 \), respectively (selected so that the first maximum of the responses is equal to 1). Since these responses are comparable to the previous ones or a little worse than them, it is not recommended to use a higher \( T_{SM} \) and a higher \( \tau_M \) for a proportional process with overshoot.

For the integrating process (14), with the compensated process characterized by \( K_p = 0.4 \), \( T_{sp} = 51 \) and \( \tau_p = 4 \) (Figure 14), by choosing \( k_1 = k_2 = 1.1 \), one obtains \( K_M = 0.4 \), \( T_{SM} = k_1 T_{sp} = 56 \) and \( \tau_M = k_2 \tau_p = 4.4 \). Figures 24, 25 and 26 illustrate the control system responses \( y \) to a unit step setpoint for \( \alpha = 0.2 \), \( \alpha = 0.4 \) and \( \alpha = 0.6 \), and for suitable values of \( K \) (8, 3.3 and 2.2, respectively, so that all responses have the same small overshoot).
These responses are better than the responses for $k_i = k_c = 1$ (Figures 15, 16 and 17), but in this context the response for $\alpha = 0.2$ is better than the response for $\alpha = 0.4$, which is better than the response for $\alpha = 0.6$.

To test the robustness of the control algorithm with respect to the model parameters for the integrating process (14), it shall now be considered that all three parameters of the model are 33.3% higher than the parameters of the compensated process, which means $K_M = 4K_p / 3 \approx 0.53$, $T_{sM} = 4T_{sP} / 3 = 68$ and $\tau_M = 4\tau_P / 3 \approx 5.3$. The control system responses $y$ to a step setpoint for $\alpha = 0.2$, $\alpha = 0.4$ and $\alpha = 0.6$, and for suitable values of the tuning gain $K$ (namely 16, 7.7 and 5, respectively, so that all responses have the same small overshoot) are shown in Figures 27, 28 and 29. As it can be noticed, the control performance of the control system remains acceptable for these higher values of the model parameters.

If the model parameters of the compensated integrating process (14) take on their nominal values, then the control system becomes unstable for $\alpha = 0.2$, $\alpha = 0.4$ and $\alpha = 0.6$ when $K > 92.4$, $K > 28.4$ and $K > 15.8$, respectively. If the time parameters $T_{sM}$ and $\tau_M$ are 10% higher, then the control system becomes unstable for $\alpha = 0.2$, $\alpha = 0.4$ and $\alpha = 0.6$ when $K > 62.4$, $K > 24.4$ and $K > 14.7$, respectively. If all model parameters are 33.3% higher, then the control system becomes unstable for $\alpha = 0.2$, $\alpha = 0.4$ and $\alpha = 0.6$ when $K > 55$, $K > 27.5$ and $K > 17.7$, respectively.

4. Conclusion

The practical and quasi-universal character of the P-IMC algorithm results from the fact that the compensated process model is built based on the following three process parameters (which are experimentally determined): steady-state gain, settling time and time delay. Due to its simplicity, robustness and control performance, the P-IMC algorithm is easily accessible to a human operator with no high ability in the control field. Once the
model parameters have been set, the operator can only increase/decrease the tuning gain $K$ in order to obtain a stronger/weaker control action. Usually, the recommendation is to increase $K$ until the response of the controlled variable to a step setpoint will have an overshoot in the range of 1 to 10%.

With regard to the weighting coefficient $\alpha$ (which characterizes the impact of the component $P$ of the P-IMC algorithm upon the control action), all control simulations for proportional and integral processes lead to the recommendation to choose $\alpha$ in the interval $[0.2, 0.4]$. For $\alpha = 0.2$, the influence of the tuning gain $K$ on the control action is more aggressive but overall weaker than for $\alpha = 0.4$ As a consequence, in the case of an overdamped compensated process, it is necessary to use different values of $K$ for $\alpha = 0.2$ and $\alpha = 0.4$ in order to obtain a similar control performance (a higher value of $K$ in the case $\alpha = 0.2$ than in the case $\alpha = 0.4$).

Also, for an overdamped compensated process (the most common case in practice), it is recommended to settle the time parameters of the proposed model (settling time and time delay) to values which are about 5-10% higher than the values of the time parameters of the compensated process. This way, the tuning gain $K$ which achieves the best response to a step setpoint is greater and the control performance would be better.

It should be noted that the control performance remains acceptable even if the parameters of the proposed model (including the steady-state gain) have values which exceed those of the process parameters by up to 33.3%. This proves that the P-IMC algorithm is robust with respect to the model parameters.

In the authors’ opinion, the practical implementation of the P-IMC algorithm (for an a priori fixed $\alpha = 0.2$ or $\alpha = 0.4$) in various industrial processes, which will highlight the simplicity, robustness and control performance of the P-IMC algorithm, could be a beneficial future research direction.

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REFERENCES


Santosh Kumar, D. B. & Padma Sree, R. (2016). Tuning of IMC based PID controllers for integrating
systems with time delay, *ISA Transactions*, 63, 242-255. DOI: 10.1016/j.isatra.2016.03.020


