An Improved Composite State Convergence Scheme with Disturbance Compensation for Multilateral Teleoperation Systems

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Abstract: Composite state convergence is a novel scheme applied for the bilateral control of a telerobotic system. The scheme offers an elegant design procedure and employs only three communication channels to establish synchronization between a single-master and a single-slave robotic system. This paper expands the capability of the composite state convergence scheme to accommodate any number of master and slave systems and proposes a disturbance observer-based composite state convergence architecture where *k*-master systems can cooperatively control *l*-slave systems in the presence of uncertainties. A systematic method is presented to compute the control gains while observer gains are determined in a standard way. To validate the proposed architecture, MATLAB simulations are performed on symmetric and asymmetric arrangements of single-degree-of-freedom teleoperation systems. Finally, experimental results are obtained using Quanser's Qube-Servo systems in QUARC/Simulink environment.

Keywords: State convergence, Composite variables, Teleoperation, MATLAB/Simulink, QUARC.

1. Introduction

In a teleoperation system, robots and humans interact in a defined fashion to accomplish a certain task in a remote environment. Due to several associated challenges such as delays and uncertainties, this research area is still fertile for scientists come up with more theoretical and practical contributions. Teleoperation systems have a wide range of applications ranging from nuclear plants (Ferre et al., 2007), military, surgical procedures (Iborra et al., 2000), underwater exploration (Yao et al., 2009), space missions (Yoon et al., 2004) to industrial tasks (Aracil et al., 2002). Raymond C. Geortz proposed the first teleoperator which enabled humans to control a robot remotely. Telepresence and stability are considered the two conflicting performance measures of a teleoperation system. The tradeoff between these two indices is at the heart of any bilateral control algorithm.

In a bilateral teleoperation system, the human operator manipulates the master robot, and motion and/or force commands are sent over the communication channel to the slave robot. The slave robot interacts with the remote environment and sends information back to the master side. Many control approaches have been discussed in the specialized literature to deal with teleoperation systems such as robust H_∞ controller (Gormus et al., 2021; Yan & Salcudean, 1996) adaptive control (Chen et al., 2014), stability analysis via generalized inequalities (Datta et al., 2019), scattering method (Anderson 1989), wave variable approach (Ye & Liu, 2009), passivity control (Sheng et al., 2019), model predictive control (Uddin & Ryu, 2016), state convergence (Azorin et al., 2004) and composite state convergence (Asad et al., 2019). Recent advancements in designing advanced controllers achieve better stability in the presence of parametric uncertainties. The adverse effects of parametric uncertainties (Motamedi et al., 2011) are addressed through the design of an adaptive sliding mode control for a single degree of freedom teleoperation system. In (Yang et al., 2017), to minimize the master-slave synchronization error, an observer-based output feedback controller is designed which ensures that the error converges to zero in a finite amount of time. A new control method is proposed by Zhang et al. (2018) to improve the synchronization performance of the master and slave using the non-singular terminal sliding mode and adaptive finite-time control to increase the tracking capabilities and robustness. In recent times, DOB-based control design is also applied to bilateral teleoperation systems. In (Chen et al., 2000), model approximation errors and

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parametric uncertainties are compensated by using a nonlinear disturbance observer (NDOB). The observer-supported TS-Fuzzy model-based control scheme is tested in MATLAB/Simulink/QUARC environment through simulations as well as in a semi-real environment. In another study (Aboutalebian et al., 2020), a nonlinear disturbance observer (NDOB) based adaptive control is proposed to deal with an uncertain dynamic model of master and slave and with environmental and operator forces. The analysis of the controller performance and stability shows the efficacy of observer-based controllers under time delays. In (Chen et al., 2014), to achieve excellent synchronization, a sliding mode controller (SMC) and disturbance observer with force compensation are integrated to deal with the manipulator model uncertainties. The control is verified and tested on two-degrees-of-freedom manipulators which demonstrate the effectiveness of the algorithm.

The use of multiple manipulators becomes essential in certain tasks due to the limitations of a bilateral teleoperation system. The authors Farooq et al. (2017) and Asad et al. (2021) have devised strategies to handle multiple systems which are required in applications such as industrial manufacturing processes, automotive assembling, rehabilitation (Culmer et al., 2010), surgical training, signal modification (Chebbi et al., 2005), and space missions. In the specialized literature, a vast amount of controller design techniques is reported for bilateral teleoperation systems. However, there is relatively little research done for the multilateral teleoperation system as there are some complex issues which need to be addressed such as coordinated control, complex nonlinear dynamics, external disturbances, time varying delays, and parametric uncertainties.

This paper is aimed to enhance the capability of the composite state convergence scheme for multilateral teleoperation systems in order to deal with uncertainties by integrating disturbance observers. Composite state convergence is proposed in (Asad et al., (2021), where *l*-slaves are synchronized to the reference motion of *k*-master systems through the method of state convergence. However, uncertainties are not treated in this work, which justifies the importance of conducting this present study. Based on single-degree-of-freedom master and slave systems, it is shown that the proposed enhancement compensates for the effect of disturbances by following the method of state convergence, thereby enhancing the performance of the existing composite state convergence scheme. The efficacy of the proposal is determined through simulations as well as experimentation in MATLAB/Simulink/QUARC environments.

The proposed scheme is presented in Section 2, while simulations and experimental results are presented in Sections 3 and 4, respectively. Conclusions are drawn in Section 5. The explanation for the abbreviations used in the paper can be found in Appendix, at the end of the paper.

2. Proposed Scheme

The proposed scheme offers an improvement over the existing composite state convergence scheme in that the lumped uncertainties can be estimated and compensated to improve the tracking performance. In addition, measurement of velocity signals is not required as disturbance-observers also estimate these signals. The proposed enhancement transmits composite variables constructed from the estimated position and velocity signals. However, measurement of the forces of the operators and environment forces is still required for the implementation of the controller. The block diagram of the proposed scheme is shown in Figure. 1.

Consider a single-degree-of-freedom master and slave system as:

$$\left.\begin{aligned} \dot{x}_{m1}^i &= x_{m2}^i \\ \dot{x}_{m2}^i &= a_{m1}^i x_{m1}^i + a_{m2}^i x_{m2}^i + b_m^i u_m^i + f_m^i \end{aligned}\right\}, i = 1,2,...,k \quad (1)$$

$$\left.\begin{aligned} \dot{x}_{s1}^i &= x_{s2}^i \\ \dot{x}_{s2}^i &= a_{s1}^i x_{s1}^i + a_{s2}^i x_{s2}^i + b_s^i u_s^i + f_s^i \end{aligned}\right\}, i = 1,2,...,l \quad (2)$$

where, subscript ‘z’ is used to denote either the master (z = m) or the slave (z = s) systems, and superscript ‘i’ is used to number the master (I = 1,2,…,k) and slave (I = 1,2,…,l) systems. The term f_z^i contains lumped uncertainty, i.e.

$$f_z^i = \left(a_{z1o}^i - a_{z1}^i\right) x_{z1}^i + \left(a_{z2o}^i - a_{z2}^i\right) x_{z2}^i + \left(b_{zo}^i - b_z^i\right) u_z^i$$

The objective of the proposed controller is to make the slave systems follow the combined motion of the master systems in the presence of uncertainties. Precisely, the position of l^{th} slave system will converge to the weighted position of *k*-master systems in the presence of uncertainties

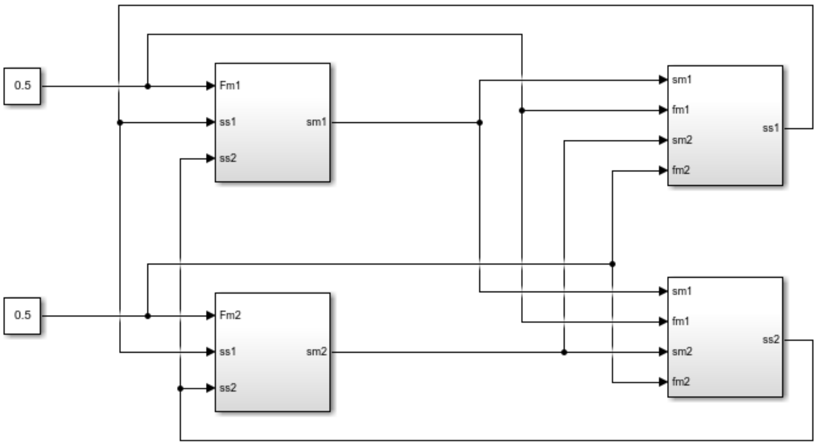

Figure 1(a). Wiring diagram of 2x2 composite state convergence architecture

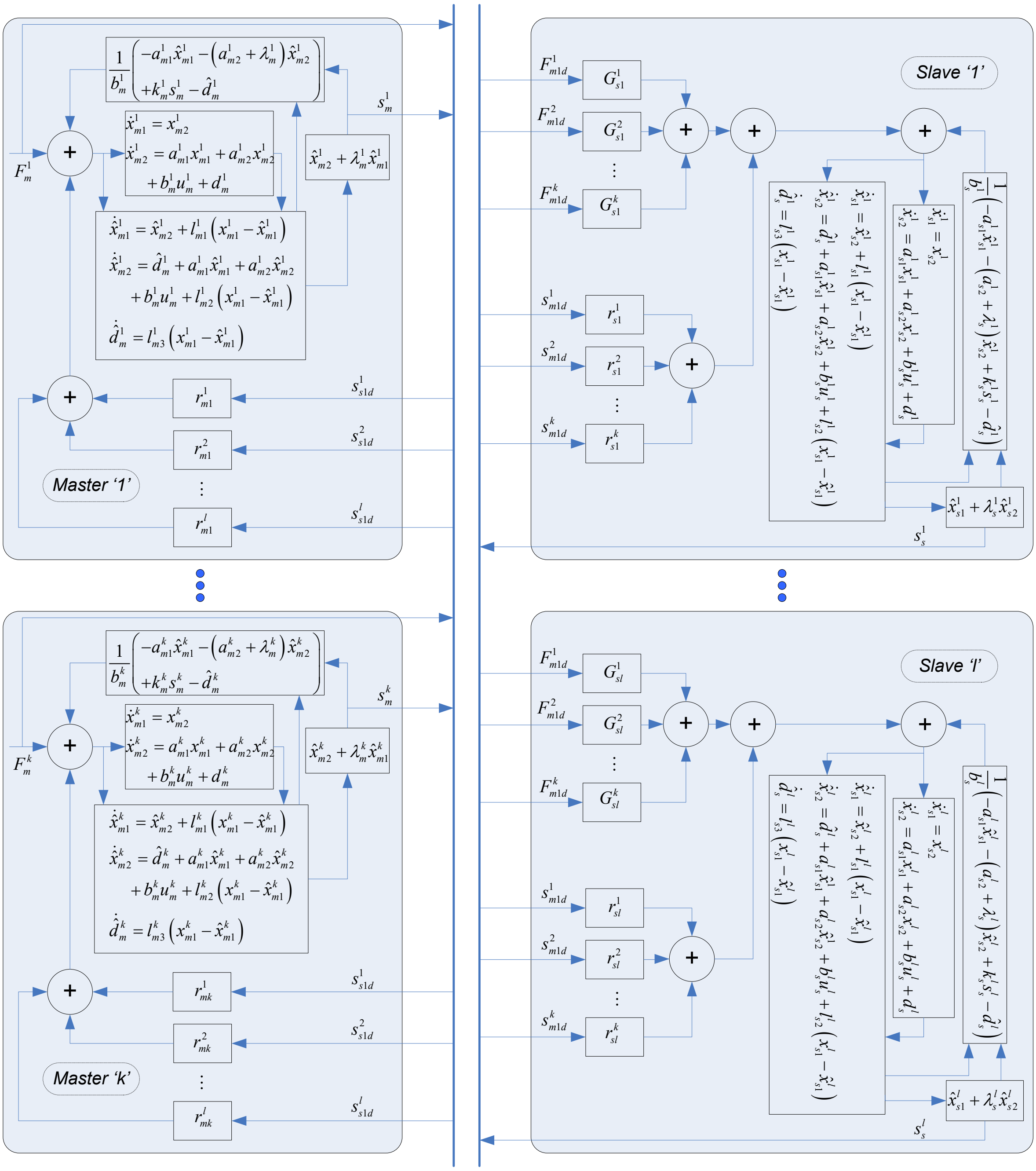

Figure 1(b). Detailed diagram of disturbance observer based composite state convergence architecture following the introduction of control inputs and disturbance observers (3), (4), and the application of the method of state convergence.

$$\left.\begin{aligned} u_m^i &= \frac{1}{b_m^i}\begin{pmatrix} -a_{m1}^i \hat{x}_{m1}^i - \left(a_{m2}^i + \lambda_m^i\right) \\ \hat{x}_{m2}^i + k_m^i s_m^i - \hat{f}_m^i \end{pmatrix} + \sum_{j=1}^{l} r_{mk}^j s_{sid}^j + F_m^i \\ \dot{\hat{x}}_{m1}^i &= \hat{x}_{m2}^i + l_{m1}^i\left(x_{m1}^i - \hat{x}_{m1}^i\right) \\ \dot{\hat{x}}_{m2}^i &= a_{m1}^i \hat{x}_{m1}^i + a_{m2}^i \hat{x}_{m2}^i + b_m^i u_m^i + l_{m2}^i\left(x_{m1}^i - \hat{x}_{m1}^i\right) + \hat{f}_m^i \\ \dot{\hat{f}}_m^i &= l_{m3}^i\left(x_{m1}^i - \hat{x}_{m1}^i\right) \end{aligned}\right\} \quad (3)$$

$$, i = 1, 2, \ldots, k$$

$$\left.\begin{aligned} u_s^i &= \frac{1}{b_s^i}\left(-a_{s1}^i \hat{x}_{s1}^i - \left(a_{s2}^i + \lambda_s^i\right)\hat{x}_{s2}^i + k_s^i s_s^i - \hat{f}_s^i\right) \\ &+ \sum_{j=1}^{k} r_{sl}^j s_{mid}^j + \sum_{j=1}^{k} G_{sl}^j F_{mid}^j \\ \dot{\hat{x}}_{s1}^i &= \hat{x}_{s2}^i + l_{s1}^i\left(x_{s1}^i - \hat{x}_{s1}^i\right) \\ \dot{\hat{x}}_{s2}^i &= a_{s1}^i \hat{x}_{s1}^i + a_{s2}^i \hat{x}_{s2}^i + b_s^i u_s^i + l_{s2}^i\left(x_{s1}^i - \hat{x}_{s1}^i\right) + \hat{f}_s^i \\ \dot{\hat{f}}_s^i &= l_{s3}^i\left(x_{s1}^i - \hat{x}_{s1}^i\right) \end{aligned}\right\} \quad (4)$$

$$i = 1, 2, \ldots, l$$

Now perform closed loop analysis to verify the claims. The composite variables for the master systems are firstly introduced as:

$$s_m^i = x_{m2}^i + \lambda_m^i x_{m1}^i, i = 1, 2, \ldots, k \quad (5)$$

Taking the time derivative of (5) and introducing the control inputs yields the following closed-loop composite master systems:

$$\dot{s}_m^i = k_m^i s_m^i + b_m^i \sum_{j=1}^{l} r_{mi}^j s_{sid}^j + b_m^i F_m^i + \xi_m^i e_m^i, i = 1, 2, \ldots, k \quad (6)$$

where

$$e_m^i = \begin{bmatrix} e_{m1}^i & e_{m2}^i & e_{m3}^i \end{bmatrix}^T = \begin{bmatrix} x_{m1}^i - \hat{x}_{m1}^i & x_{m2}^i - \hat{x}_{m2}^i & x_{m3}^i - \hat{x}_{m3}^i \end{bmatrix}^T$$

$i = 1, 2, \ldots, k$ constitute observation errors for the master systems with $\xi_m^i = \begin{bmatrix} a_{m1}^i & a_{m2}^i + \lambda_m^i & 1 \end{bmatrix}$. The observer error dynamics of master systems can be written as:

$$\left.\begin{aligned} \dot{e}_{m1}^i &= -l_{m1}^i e_{m1}^i + e_{m2}^i \\ \dot{e}_{m2}^i &= \left(a_{m1}^i - l_{m2}^i\right) e_{m1}^i + a_{m2}^i e_{m2}^i + e_{m3}^i \\ e_{m3}^i &= -l_{m3}^i e_{m1}^i + \dot{f}_m^i \end{aligned}\right\}, i = 1, 2, \ldots, k \quad (7)$$

By linearizing the time delayed terms in (6), one obtains:

$$\dot{s}_m^i = k_m^i s_m^i + b_m^i \sum_{j=1}^{l} r_{mi}^j s_s^j - b_m^i \sum_{j=1}^{l} r_{mi}^j T_{mi}^j \dot{s}_s^j + b_m^i F_m^i + \xi_m^i e_m^i \quad (8)$$

$$i = 1, 2, \ldots, k$$

By stacking composite master systems (8), one obtains:

$$\begin{bmatrix} \dot{s}_m^1 \\ \dot{s}_m^2 \\ \vdots \\ \dot{s}_m^k \end{bmatrix} = \begin{bmatrix} k_m^1 & 0 & \cdots & 0 \\ 0 & k_m^2 & \cdots & 0 \\ & \vdots & & \\ 0 & 0 & \cdots & k_m^k \end{bmatrix} \begin{bmatrix} s_m^1 \\ s_m^2 \\ \vdots \\ s_m^k \end{bmatrix} + \begin{bmatrix} b_m^1 r_{m1}^1 & b_m^1 r_{m1}^2 & \cdots & b_m^1 r_{m1}^l \\ b_m^2 r_{m2}^1 & b_m^2 r_{m2}^2 & \cdots & b_m^2 r_{m2}^l \\ & \vdots & & \\ b_m^k r_{mk}^1 & b_m^k r_{mk}^2 & \cdots & b_m^k r_{mk}^l \end{bmatrix}$$
$$\begin{bmatrix} s_s^1 \\ s_s^2 \\ \vdots \\ s_s^l \end{bmatrix} - \begin{bmatrix} b_m^1 r_{m1}^1 T_{m1}^1 & b_m^1 r_{m1}^2 T_{m1}^2 & \cdots & b_m^1 r_{m1}^l T_{m1}^l \\ b_m^2 r_{m2}^1 T_{m2}^1 & b_m^2 r_{m2}^2 T_{m2}^2 & \cdots & b_m^2 r_{m2}^l T_{m2}^l \\ & \vdots & & \\ b_m^k r_{mk}^1 T_{mk}^1 & b_m^k r_{mk}^2 T_{mk}^1 & \cdots & b_m^k r_{mk}^l T_{mk}^l \end{bmatrix} \begin{bmatrix} \dot{s}_s^1 \\ \dot{s}_s^2 \\ \vdots \\ \dot{s}_s^l \end{bmatrix} + \quad (9)$$
$$\begin{bmatrix} b_m^1 & 0 & \cdots & 0 \\ 0 & b_m^2 & \cdots & 0 \\ & \vdots & & \\ 0 & 0 & \cdots & b_m^k \end{bmatrix} \begin{bmatrix} F_m^1 \\ F_m^2 \\ \vdots \\ F_m^k \end{bmatrix} + \begin{bmatrix} \xi_m^1 & 0 & \cdots & 0 \\ 0 & \xi_m^2 & \cdots & 0 \\ & \vdots & & \\ 0 & 0 & \cdots & \xi_m^k \end{bmatrix} \begin{bmatrix} e_m^1 \\ e_m^2 \\ \vdots \\ e_m^k \end{bmatrix}$$

This can be conveniently written as:

$$\dot{s}_m = k_m s_m + b_{rm} s_s - b_{rmT} \dot{s}_s + b_m F_m + \xi_m e_m \quad (10)$$

By combining the closed loop composite master system (10) with observer error dynamics (7), one obtains:

$$\begin{bmatrix} \dot{s}_m \\ \dot{e}_m \end{bmatrix} = \begin{bmatrix} k_m & \xi_m \\ 0 & o_m \end{bmatrix} \begin{bmatrix} s_m \\ e_m \end{bmatrix} + \begin{bmatrix} b_{rm} & -b_{rmT} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} s_s \\ \dot{s}_s \end{bmatrix} + \begin{bmatrix} b_m \\ 0 \end{bmatrix} F_m + \begin{bmatrix} 0 \\ h_m \end{bmatrix} \dot{f}_m \quad (11)$$

It can be seen that observer design can be carried out separately from controller design. It is assumed that lumped disturbance is slowly varying and therefore convergence of observation error to origin is ensured by comparing the characteristic equation with the desired polynomial which yields the observer gains.

$$p_m(s) : s^3 + \left(l_{m1}^i - a_{m2}^i\right) s^2 + \left(l_{m2}^i - a_{m1}^i - a_{m2}^i l_{m1}^i\right) s + l_{m3}^i = 0, i = 1, 2, \ldots, k \quad (12)$$

Now, composite variables for the slave systems are defined as:

$$s_s^i = x_{s2}^i + \lambda_s^i x_{s1}^i, i = 1, 2, \ldots, l \quad (13)$$

Time-derivative of (13) along with control inputs yield closed loop composite slave systems as:

$$\dot{s}_s^i = k_s^i s_s^i + b_s^i \sum_{j=1}^{k} r_{si}^j s_{mid}^j + b_s^i \sum_{j=1}^{k} G_{si}^j F_{mid}^j + \xi_s^i e_s^i \quad (14)$$

$$i = 1, 2, \ldots, l$$

where

$$e_s^i = \begin{bmatrix} x_{s1}^i - \hat{x}_{s1}^i & x_{s2}^i - \hat{x}_{s2}^i & x_{s3}^i - \hat{x}_{s3}^i \end{bmatrix}^T, i = 1, 2, \ldots, l$$

represents the observation errors for the slave systems with $\xi_s^i = \begin{bmatrix} a_{s1}^i & a_{s2}^i + \lambda_s^i & 1 \end{bmatrix}$. The observer error dynamics of slave systems can be written as:

$$\left.\begin{aligned} \dot{e}_{s1}^{i} &= -l_{s1}^{i} e_{s1}^{i} + e_{s2}^{i} \\ \dot{e}_{s2}^{i} &= \left(a_{s1}^{i} - l_{s2}^{i}\right) e_{s1}^{i} + a_{s2}^{i} e_{s2}^{i} + e_{s3}^{i} \\ e_{s3}^{i} &= -l_{s3}^{i} e_{s1}^{i} + \dot{f}_{s}^{i} \end{aligned}\right\}, i = 1,2,...,l \tag{15}$$

The linearization of time-delay entities in equation (14) leads to:

$$\begin{aligned} \dot{s}_s^i &= k_s^i s_s^i + b_s^i \sum_{j=1}^{k} r_{si}^j s_m^j - b_s^i \sum_{j=1}^{k} r_{si}^j T_{si}^j s_m^j \\ &\quad + b_s^i \sum_{j=1}^{k} G_{si}^j F_{mi}^j + \xi_s^i e_s^i, i = 1,2,...,l \end{aligned} \tag{16}$$

By stacking the composite slave systems (16), one obtains:

$$\begin{aligned} \begin{bmatrix} \dot{s}_s^1 \\ \dot{s}_s^2 \\ \vdots \\ \dot{s}_s^l \end{bmatrix} &= \begin{bmatrix} k_s^1 & 0 & \cdots & 0 \\ 0 & k_s^2 & \cdots & 0 \\ & \vdots & & \\ 0 & 0 & \cdots & k_s^l \end{bmatrix} \begin{bmatrix} s_s^1 \\ s_s^2 \\ \vdots \\ s_s^l \end{bmatrix} + \begin{bmatrix} b_s^1 r_{s1}^1 & b_s^1 r_{s1}^2 & \cdots & b_s^1 r_{s1}^k \\ b_s^2 r_{s2}^1 & b_s^2 r_{s2}^2 & \cdots & b_s^2 r_{s2}^k \\ & \vdots & & \\ b_s^l r_{sl}^1 & b_s^l r_{sl}^2 & \cdots & b_s^l r_{sl}^l \end{bmatrix} \\ &\quad \begin{bmatrix} s_m^1 \\ s_m^2 \\ \vdots \\ s_m^k \end{bmatrix} - \begin{bmatrix} b_s^1 r_{s1}^1 T_{s1}^1 & b_s^1 r_{s1}^2 T_{s1}^2 & \cdots & b_s^1 r_{s1}^k T_{s1}^k \\ b_s^2 r_{s2}^1 T_{s2}^1 & b_s^2 r_{s2}^2 T_{s2}^2 & \cdots & b_s^2 r_{s2}^k T_{s2}^k \\ & \vdots & & \\ b_s^l r_{sl}^1 T_{sl}^1 & b_s^l r_{sl}^2 T_{sl}^2 & \cdots & b_s^l r_{sl}^l T_{sl}^k \end{bmatrix} \begin{bmatrix} \dot{s}_m^1 \\ \dot{s}_m^2 \\ \vdots \\ \dot{s}_m^k \end{bmatrix} \\ &\quad + \begin{bmatrix} b_s^1 G_{s1}^1 & b_s^1 G_{s1}^2 & \cdots & b_s^1 G_{s1}^k \\ b_s^2 G_{s2}^1 & b_s^2 G_{s2}^2 & \cdots & b_s^2 G_{s2}^k \\ & \vdots & & \\ b_s^l G_{sl}^1 & b_s^l G_{sl}^2 & \cdots & b_s^l G_{sl}^l \end{bmatrix} \begin{bmatrix} F_m^1 \\ F_m^2 \\ \vdots \\ F_m^k \end{bmatrix} + \begin{bmatrix} \xi_s^1 & 0 & \cdots & 0 \\ 0 & \xi_s^2 & \cdots & 0 \\ & \vdots & & \\ 0 & 0 & \cdots & \xi_s^k \end{bmatrix} \begin{bmatrix} e_s^1 \\ e_s^2 \\ \vdots \\ e_s^k \end{bmatrix} \end{aligned} \tag{17}$$

This can be written in compact form as:

$$\dot{s}_s = k_s s_s + b_{rs} s_m - b_{rsT} \dot{s}_m + b_{sG} F_m + \xi_s e_s \tag{18}$$

By augmenting closed loop slave composite systems (18) with observer error dynamics, one obtains:

$$\begin{aligned} \begin{bmatrix} \dot{s}_s \\ \dot{e}_s \end{bmatrix} &= \begin{bmatrix} k_s & \xi_s \\ 0 & o_s \end{bmatrix} \begin{bmatrix} s_s \\ e_s \end{bmatrix} + \begin{bmatrix} b_{rs} & -b_{rsT} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} s_m \\ \dot{s}_m \end{bmatrix} \\ &\quad + \begin{bmatrix} b_{sG} \\ 0 \end{bmatrix} F_m + \begin{bmatrix} 0 \\ h_s \end{bmatrix} \dot{f}_s \end{aligned} \tag{19}$$

The above system implies that observers for slave systems can be designed separately from controllers. To determine the observer gains for slave systems, the characteristic equation is compared with the desired polynomial. The convergence of observation error follows from the assumption of slowly varying lumped uncertainties.

$$\begin{aligned} p_s(s) : s^3 + \left(l_{s1}^i - a_{s2}^i\right) s^2 + \left(l_{s2}^i - a_{s1}^i - a_{s2}^i l_{s1}^i\right) s \\ + l_{s3}^i = 0, i = 1,2,...,l \end{aligned} \tag{20}$$

Now that observer design is performed separately for the master and slave systems, composite master and slave system can be manipulated without considering observation error terms, for the purpose of designing control gains. To this end, equation (18) is plugged in equation (10) and rearranged to obtain closed-loop composite master systems as:

$$\dot{s}_m = \left(I - b_{rmT} b_{rsT}\right)^{-1} \begin{bmatrix} \left(k_m - b_{rmT} b_{rs}\right) s_m \\ + \left(b_{rm} - b_{rmT} k_s\right) s_s \\ + \left(b_m - b_{rmT} b_{sG}\right) F_m \end{bmatrix} \tag{21}$$

Now, equation (10) is plugged in equation (18) and some algebraic manipulations are made in order to obtain:

$$\dot{s}_s = \left(I - b_{rmT} b_{rsT}\right)^{-1} \begin{bmatrix} \left(k_s - b_{rsT} b_{rm}\right) s_s \\ + \left(b_{rs} - b_{rsT} k_m\right) s_m \\ + \left(b_{sG} - b_{rsT} b_m\right) F_m \end{bmatrix} \tag{22}$$

Let α contain the authority factors for the slave systems. Now, composite state convergence error is introduced as:

$$s_e = s_s - \alpha s_m \tag{23}$$

By taking time-derivative of composite state convergence error and using equation (23), one obtains:

$$\begin{aligned} \dot{s}_e &= \begin{pmatrix} \left(I - b_{rmT} b_{rsT}\right)^{-1} \left(k_s - b_{rsT} b_{rm}\right) \\ -\alpha \left(I - b_{rmT} b_{rsT}\right)^{-1} \left(b_{rm} - b_{rmT} k_s\right) \end{pmatrix} s_e + \\ &\quad \begin{pmatrix} \left(I - b_{rmT} b_{rsT}\right)^{-1} \left(k_s - b_{rsT} b_{rm}\right) \alpha \\ -\alpha \left(I - b_{rmT} b_{rsT}\right)^{-1} \left(b_{rm} - b_{rmT} k_s\right) \alpha + \\ \left(I - b_{rmT} b_{rsT}\right)^{-1} \left(b_{rs} - b_{rsT} k_m\right) \\ -\alpha \left(I - b_{rmT} b_{rsT}\right)^{-1} \left(k_m - b_{rmT} b_{rs}\right) \end{pmatrix} s_m + \\ &\quad \begin{pmatrix} \left(I - b_{rmT} b_{rsT}\right)^{-1} \left(b_{sG} - b_{rsT} b_m\right) \\ -\alpha \left(I - b_{rmT} b_{rsT}\right)^{-1} \left(b_m - b_{rmT} b_{sG}\right) \end{pmatrix} F_m \end{aligned} \tag{24}$$

Now let the composite state convergence error behave as an autonomous system. This leads to the following conditions:

$$\begin{aligned} &\left(I - b_{rmT} b_{rsT}\right)^{-1} \left(k_s - b_{rsT} b_{rm}\right) \alpha \\ &-\alpha \left(I - b_{rmT} b_{rsT}\right)^{-1} \left(b_{rm} - b_{rmT} k_s\right) \alpha \\ &+\left(I - b_{rmT} b_{rsT}\right)^{-1} \left(b_{rs} - b_{rsT} k_m\right) - \\ &\alpha \left(I - b_{rmT} b_{rsT}\right)^{-1} \left(k_m - b_{rmT} b_{rs}\right) = 0 \end{aligned} \tag{25}$$

$$\begin{aligned} &\left(I - b_{rmT} b_{rsT}\right)^{-1} \left(b_{sG} - b_{rsT} b_m\right) \\ &-\alpha \left(I - b_{rmT} b_{rsT}\right)^{-1} \left(b_m - b_{rmT} b_{sG}\right) = 0 \end{aligned} \tag{26}$$

Now write the augmented system comprising of composite master and composite error systems as:

$$\begin{aligned}
&\begin{bmatrix} \dot{s}_m \\ \dot{s}_e \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} s_m \\ s_e \end{bmatrix} + \begin{bmatrix} \left(I - b_{rmT}b_{rsT}\right)^{-1}\left(b_m - b_{rmT}b_{sG}\right) \\ 0 \end{bmatrix} F_m \\
&a_{11} = \left(I - b_{rmT}b_{rsT}\right)^{-1}\left(k_m - b_{rmT}b_{rs} + b_{rm} - b_{rmT}k_s\right) \\
&a_{12} = -\left(I - b_{rmT}b_{rsT}\right)^{-1}\left(b_{rm} - b_{rmT}k_s\right) \\
&a_{21} = 0 \\
&a_{22} = \left(I - b_{rmT}b_{rsT}\right)^{-1}\left(k_s - b_{rsT}b_{rm}\right) \\
&\quad - \alpha\left(I - b_{rmT}b_{rsT}\right)^{-1}\left(b_{rm} - b_{rmT}k_s\right)
\end{aligned} \tag{27}$$

Now the desired dynamic behaviour is imposed onto this augmented system which results in the following additional design conditions:

$$\left(I - b_{rmT}b_{rsT}\right)^{-1}\left(k_m - b_{rmT}b_{rs} + b_{rm} - b_{rmT}k_s\right) = -p \tag{28}$$

$$\begin{aligned}
&\left(I - b_{rmT}b_{rsT}\right)^{-1}\left(k_s - b_{rsT}b_{rm}\right) \\
&-\alpha\left(I - b_{rmT}b_{rsT}\right)^{-1}\left(b_{rm} - b_{rmT}k_s\right) = -q
\end{aligned} \tag{29}$$

The design conditions lead to the conclusion that composite errors converge to zero which further implies that composite slave states converge to the weighted composite master states while their derivatives converge to zero. Based on this, it results that $x_{s2} + \lambda_s x_{s1} = \alpha\left(x_{m2} + \lambda_m x_{m1}\right)$. In addition, closed-loop analysis reveals that $x_{s2} + \lambda_s x_{s1} = \dot{s}_s, x_{m2} + \lambda_m x_{m1} = \dot{s}_m$. Therefore, velocity states converge to zero which implies $x_{s1} = \lambda_s^{-1}\alpha\lambda_m x_{m1}$. By assuming unity scaling factors, convergence of the slave positions to the weighted positions of master systems is ensured in the presence of uncertainties.

3. Simulation Results

The proposed enhanced composite state convergence scheme is simulated in MATLAB/Simulink environment on a 2x2 teleoperation system. The nominal parameters for the master and slave systems are given as:

$$\begin{aligned}
a_m^1 &= \begin{bmatrix} 0 & 1 \\ 0 & -7.8572 \end{bmatrix}, b_m^1 = \begin{bmatrix} 0 \\ 0.3187 \end{bmatrix} \\
a_m^2 &= \begin{bmatrix} 0 & 1 \\ 0 & -6.4286 \end{bmatrix}, b_m^2 = \begin{bmatrix} 0 \\ 0.2125 \end{bmatrix} \\
a_s^1 &= \begin{bmatrix} 0 & 1 \\ 0 & -7.500 \end{bmatrix}, b_s^1 = \begin{bmatrix} 0 \\ 0.3275 \end{bmatrix} \\
a_s^2 &= \begin{bmatrix} 0 & 1 \\ 0 & -8.1250 \end{bmatrix}, b_s^2 = \begin{bmatrix} 0 \\ 0.3275 \end{bmatrix}
\end{aligned} \tag{30}$$

In addition, slaves are interacting with environments having stiffness as $k_e^1 = k_e^1 = 20Nms/rad$. The closed loop poles of augmented system are placed at $p = diag(2,4), q = diag(2,10)$ and the design conditions (25), (26), (28), (29) are solved which yields the following controller and observer gains:

$$\begin{aligned}
&g_{s1}^1 = 0.6813, g_{s1}^2 = 0.1947, g_{s2}^1 = 0.5840, g_{s2}^2 = 0.2595 \\
&k_m^1 = -4.6967, k_m^2 = -2.4294, k_s^1 = -1.4661, k_s^2 = -9.4077 \\
&r_{s1}^1 = -4.6281, r_{s1}^2 = 0.3438, r_{s2}^1 = 10.8245, r_{s2}^2 = 9.7045
\end{aligned} \tag{31}$$

$$\begin{aligned}
L_{om}^1 &= \begin{bmatrix} 0.0082 & 0.2055 & 2.70 \end{bmatrix} \times 10^4, \\
L_{om}^2 &= \begin{bmatrix} 0.0084 & 0.2163 & 2.70 \end{bmatrix} \times 10^4 \\
L_{os}^1 &= \begin{bmatrix} 0.0083 & 0.2081 & 2.70 \end{bmatrix} \times 10^4, \\
L_{os}^2 &= \begin{bmatrix} 0.0082 & 0.2035 & 2.70 \end{bmatrix} \times 10^4
\end{aligned} \tag{32}$$

After computing the gains, the simulations are run with the following plant parameters:

$$\begin{aligned}
a_m^{1r} = a_m^{2r} &= \begin{bmatrix} 0 & 1 \\ 0 & -7.1429 \end{bmatrix}, b_m^{1r} = b_m^{2r} = \begin{bmatrix} 0 \\ 0.2656 \end{bmatrix} \\
a_s^{1r} = a_s^{2r} &= \begin{bmatrix} 0 & 1 \\ 0 & -6.25 \end{bmatrix}, b_s^{1r} = b_s^{2r} = \begin{bmatrix} 0 \\ 0.2729 \end{bmatrix}
\end{aligned} \tag{33}$$

In equation (33), superscript 'r' is added to denote the real parameters of the plant which differ by up to 30% from the nominal parameters. The simulation results with constant forces of the operators of 0.5N are depicted in Figures 2 and 3. It can be seen that composite reference is well-tracked by the composite slave systems and that the slave positions also converge to the composite reference which is inline with the theoretical results.

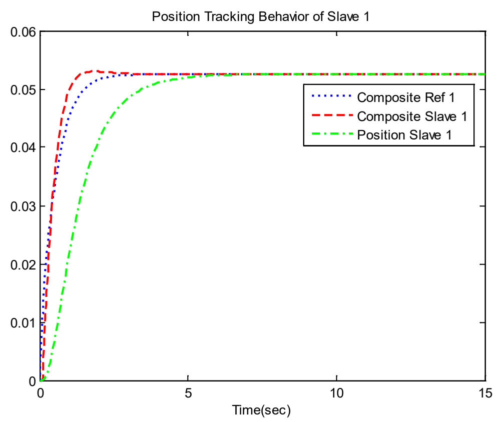

Figure 2. Reference tracking by first slave system

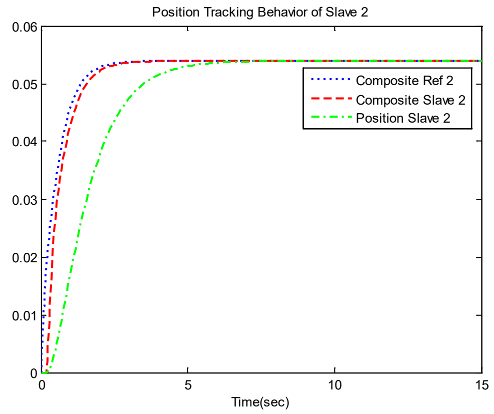

Figure 3. Reference tracking by second slave system

The proposed scheme is also compared with the existing composite state convergence scheme for the multilateral teleoperation system (Asad et al, 2021). The existing composite state convergence scheme does not utilize disturbance observers. The same control gains are used to simulate both the proposed and the already existing schemes as the former schemes also employ observer gains. The uncertainty levels in the control input coefficients of both slaves are increased, and simulations are performed. The position tracking errors of the slaves are recorded in Figures 4 and 5.

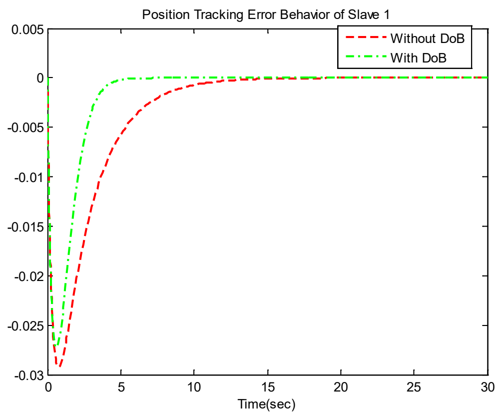

Figure 4. Tracking error for the first slave in the proposed and existing schemes

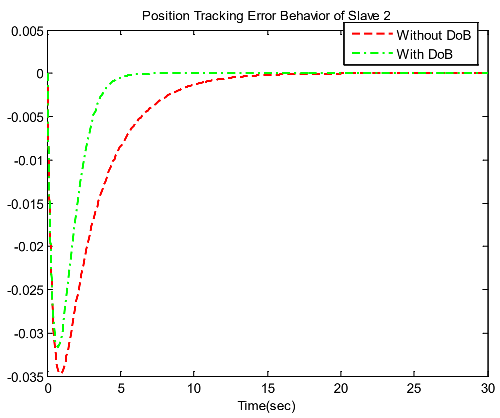

Figure 5. Tracking error for the second slave in the proposed and existing schemes

It can be seen that the proposed scheme offers a faster transient performance when compared to the existing schemes. In this way, the superiority of the proposed scheme over the existing ones is established.

4. Experimental Results

The proposed scheme is verified through experimentation on three QUBE Servo-2 platforms which are arranged to form a 1x2 teleoperation system as shown in Figure. 6. In order to determine controller and observer gains for the master and slave systems, the following nominal models are utilized:

$$a_z^i = \begin{bmatrix} 0 & 1 \\ 0 & -6.67 \end{bmatrix}, b_z^i = \begin{bmatrix} 0 \\ 149.34 \end{bmatrix} \tag{34}$$

It is assumed that slaves are interacting with a soft environment having a stiffness of 1Nms/rad. It is further assumed that the time delay between the master and the first slave is 0.1s while the time delay between the master and second slave is 0.2 s. The closed loop poles are selected as $p = 27.4, q_1 = 10, q_2 = 40$ and the design conditions are solved using MATLAB symbolic toolbox which yields the following control gains:

$$\begin{aligned} &k_m^1 = -180.025, k_s^1 = -8.445, k_s^2 = -17.938 \\ &G_{s1}^1 = 0.090, G_{s2}^1 = 0.1292 \\ &r_{s1}^1 = -0.0339, r_{s2}^1 = -0.0098 \end{aligned} \tag{35}$$

In order to compute the observer gains, the nominal models are utilised and all the observer poles are placed at 30, which yields the following master and slave observer gains:

$$L_{oz} = \begin{bmatrix} 0.0083 & 0.2144 & 2.70 \end{bmatrix} \times 10^4 \tag{36}$$

The teleoperation system is now setup in Simulink/QUARC environment such that the master system communicates its composite signal on the time delayed channels to the two slave systems via two stream serves having IDs ‘0’ (udp://localhost:18000?peer=’any’) and ‘1’ (udp://localhost:18001?peer=’any’). In addition to composite signal, master system also sends the force of the operator on the time delayed channels to the slave systems via two stream serves having IDs ‘2’ (udp://localhost:18002?peer=’any’) and ‘3’ (udp://localhost:18003?peer=’any’). In response, slave systems send their composite signals to the master system via stream clients having IDs ‘4’ (udp://localhost:18000) and ‘6’ (udp://localhost:18001). Slave systems also send force feedback to the master system using stream clients having IDs ‘5’ (udp://localhost:18002) and ‘7’ (udp://localhost:18003). The data received by stream servers is demultiplexed to read composite and force signals from the two slave systems. Two additional stream servers having IDs ‘10’ (udp://localhost:18004?peer=’any’) and ‘11’ (udp://localhost:18005?peer=’any’) are installed on the master side for the purpose of recording slave position signals. This also needs the deployment of two additional stream clients at the slave sides with IDs ‘8’ (udp://localhost:18004) and ‘9’ (udp://localhost:18005).

During the experiment, operator moves the servo-disk of the master while slave-servo disks interact with their virtual environments. The operator starts experiencing a greater environmental force as it continues to increase the rotational angle of the servo-disk. The experiment is run for 300 seconds, and the recorded results are displayed in Figures 7-9.

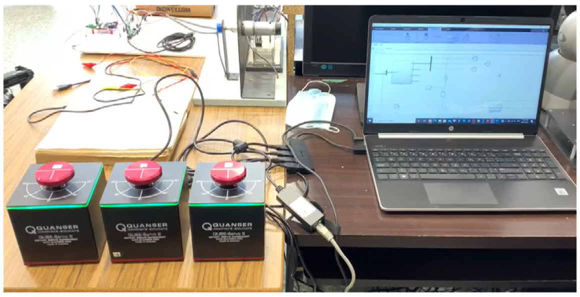

Figure 6. Experimental setup

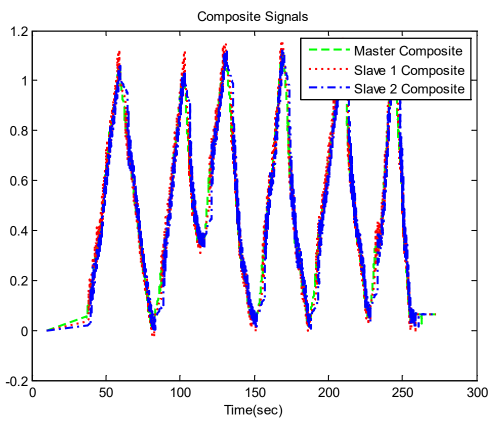

Figure 7. Composite states of 1x2 teleoperation system

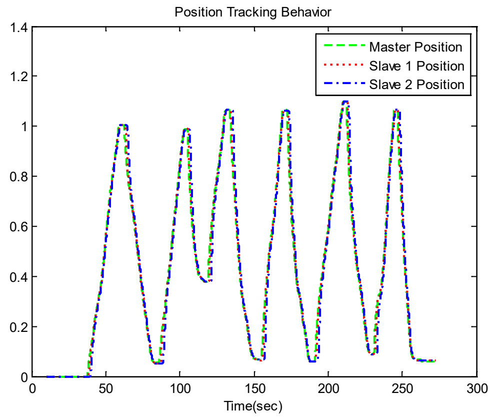

Figure 8. Position states of 1x2 teleoperation system

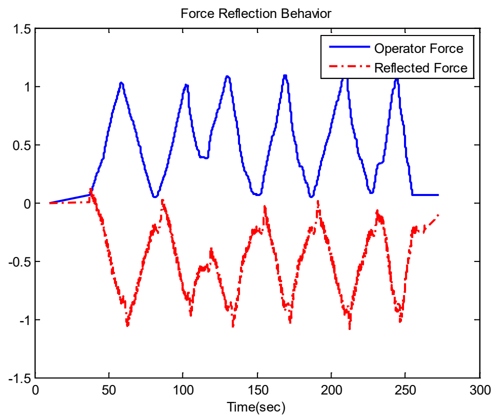

Figure 9. Force reflection behavior of 1x2 teleoperation system

From Figure 7, it can be seen that the composite signals of the slave systems follow the composite signal of the master. In theory, this should imply the convergence of position signals, which can be verified in Figure 8. In addition to position tracking, the results of the force reflection in Figure 9 suggest that proposed scheme can be used to design teleoperation systems.

5. Conclusion

This paper proposes a disturbance observer-based scheme for controlling a multi-master, multi-slave teleoperation system through a composite state convergence methodology. At first, the composite variables are constructed through estimated position and velocity states and the closed-loop composite master and the composite slave systems are found. By augmenting the composite master and composite error systems and by employing the method of state convergence, the control gains and the observer gains are determined. The stability of the scheme is guaranteed under fixed time delays. The proposed scheme is validated through simulations as well as experimentation in MATLAB/Simulink/QUARC environment by considering different arrangements of master and slave systems. Then, the comparison with the existing schemes shows that the proposed scheme can indeed counter the effect of disturbances, while ensuring the tracking of references in slave systems, which are set by the master systems. Future work will involve the designing of force observers to eliminate the dependence of design procedure on environmental parameters.

Acknowledgements

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Appendix Nomenclature:

F_m^j : Force applied by the jth operator onto the corresponding master system

T_{mi}^j : Time delay in the path from the jth slave system to the ith master system

T_{si}^j : Time delay in the path from the jth master system to the ith slave system

s_z^j : Composite variable of the jth master ($z = m$) or slave ($z = s$) system.

$s_{sid}^j = s_s^j\left(t - T_{mi}^j\right)$: Delayed composite variable of the jth slave system which affects the ith master system.

$s_{mid}^j = s_m^j\left(t - T_{si}^j\right)$: Delayed composite variable of the jth master system which affects the ith slave system.

k_z^j : Stabilizing gain for the jth composite-master ($z = m$) or composite-slave ($z = s$) system.

r_{mi}^j : Effect of the motion of the jth slave system onto the ith master system.

r_{si}^j : Effect of the motion of the jth master system onto the ith slave system.

G_{si}^j : Influence of the force exerted by the jth operator onto the ith slave system.

α_{si}^j : Contribution of the jth master system in setting the reference position for the ith slave system such that $\sum_{j=1}^{k} \alpha_{si}^j = 1$.

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