Event-Triggered Piecewise Continuous Tracking Control of Networked Control Systems Using Linear Perturbed System Models with Time Delays

Chengcheng SONG[1], Zhixin SUN[2], Haoping WANG[3]*, Yang TIAN[3]

[1] Industrial Center/School of Innovation and Entrepreneurship, Nanjing Institute of Technology, 211167, Nanjing, China
Schengcheng@126.com

[2] NARI Group Corporation (State Grid Electric Power Research Institute), 211106, Nanjing, China
sunzhixin51@163.com

[3] School of Automation, Nanjing University of Science & Technology, 210094, Nanjing, China
hp.wang@njust.edu.cn (**Corresponding author*), tianyang@njust.edu.cn

Abstract: In this paper, an event-triggered piecewise continuous trajectory tracking controller is proposed for linear perturbed systems. A network-induced delay which is time-varying but bounded is considered. The new event-triggered mechanism (ETM) presented in this paper is designed based on the predicted system state estimation. A reconstruction calculation module is implemented on the controller node of the control system in order to compensate for the impact of the delay on the state estimation transmitted by the sensor node of the system. By using the proposed controller with the related new ETM, the tracking error can be guaranteed to be norm-bounded, and also the data communication is reduced, while the controller tracking performance remains satisfactory. Finally, in order to show the effectiveness of the proposed theoretical method and the superiority of the new ETM, the proposed controller was applied to a networked mobile cart system and some comparative simulations were carried out.

Keywords: Piecewise continuous, Trajectory tracking, Controller, Time delays, Event-triggered mechanism.

1. Introduction

In networked control systems, the sampled data is usually transmitted from sensor side to controller side periodically. For avoiding the redundant data, the event-triggered control method is proposed, which has the feature of aperiodic data sampling. It was mentioned in some earlier works, such as (Sánchez, Guarnes & Dormido, 2009) and (Heemels et al., 1999). The purpose of event-triggered control is to obtain a better control performance with fewer data transmissions, by seeking the optimal trigger instants for transmission. Besides saving bandwidth, less transmitted information in an event-triggered control system decreases the energy consumption of the sensor, further prolonging the service life of the sensor, which is convenient especially for the cases when it is hard to replace sensor batteries. Also, information security is guaranteed since less data is transmitted through a network with event-triggered control.

The time-varying trajectory tracking control is one of the essential problems in control system theory. However, the trajectory tracking control with an event-triggered rule has not drawn enough attention. Even though papers of Choi & Yoo (2018), Su, Liu & Lai (2018) and Wang, Wen & Li (2023) study the event-triggered trajectory tracking control, all of them consider the known system state. Besides, in the papers of Su, Liu & Lai (2018) and Wang, Wen & Li (2023), the external perturbation is not considered, and in (Choi & Yoo, 2018; Su, Liu & Lai, 2018) the network-induced time delay is negligible. Thus, in this work, the event-triggered trajectory tracking control for linear perturbed systems with time delays will be studied, which is based on system output when an observer is designed for unknown state information. The detection manner is important for the design of trigger rule. The majority of the above papers about event-triggered tracking control, such as those of Choi & Yoo (2018), Su, Liu & Lai (2018) and Wang, Wen & Li (2023), focus on the continuous detection of trigger condition. In the paper of Xing & Deng (2019), the continuous event-triggered mechanism is combined with the time-triggered mechanism, which can effectively avoid Zeno behavior. Besides the continuous detection manner, periodic detection is presented in the papers of Song et al. (2020), Song et al. (2022) and Sun et al. (2022), which not only avoids the Zeno behavior but also brings more convenience for application, thus it will be used in this paper.

It is worth mentioning that the problem of time delays has been widely studied in networked control systems. In the paper of Van De Wouw et

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al. (2007), the network-induced time delay which is lower than the sampling interval is studied. Also, the article by Wu et al. (2015) considers the network-induced time delay which is lower than event detection period which is equal to the basic sampling interval in event-triggered control. However, these kinds of time delays are limited by sampling interval (or event detection period), which cannot be applied to systems with large delays. In the work of Del, Vazquez Guerra & Marquez-Rubio (2020), the large time-delay is considered but without involving any other network constraints. Consequently, in this paper, the delay which is equal to a multiple of the sampling interval (or event detection period) together with network constraints related to sampling and communication limitations will be considered, thus it is available for systems with both small and large time delays. In the papers of Hu et al. (2012) and Peng & Yang (2013), a type of event-triggered control method is proposed on the basis of Lyapunov function approach when considering delays, however, it actually only considers the presence of time delays, but it does not propose the compensation method, which will be studied in this paper.

Different kinds of ETMs have been proposed in event-triggered control areas. In the works of Pawlowski et al. (2009) and Lehmann, Lunze & Johansson (2012), ETM in the form of absolute deadband is studied. Besides, ETM in the form of relative deadband is studied in the papers of Peng & Yang (2013), Hu & Yue (2012), Kolarijani & Mazo (2018). With regard to the threshold of deadband, the relative deadband based ETM can also be classified into two forms, namely the deadband with the current data based threshold as in the papers of Hu & Yue (2012) and Kolarijani & Mazo (2018), and the deadband with the latest transmitted data based threshold as in the work of Peng & Yang (2013). Also, in the work of Hu, Zhang & Du (2012) for tracking control, the ETM which introduces the reference data exists, and it is designed based on tracking error. In contrast to the methods presented above, this work proposes a new ETM based on the predicted state estimation, which brings about advantages for systems with time delays.

In this work, an event-triggered trajectory tracking controller for a class of perturbed linear systems with unknown state information subject to network limitations including aperiodic sampling and time delays is proposed. Due to the network-induced time delays, the ETM in this work was designed according to the predicted state estimation. Piecewise Continuous Systems (PCS) were used for designing the controller presented in this paper. PCS is actually a particular hybrid system which was firstly proposed in (Koncar & Vasseur, 2003) as inspired by (Koncar & Vasseur, 2002) and then developed in (Mohammed, Wang & Tian, 2018; Wang, Tian & Vasseur, 2015; Mohammed, Wang & Tian, 2019). PCS controller is of great benefit to the tracking of time-varying trajectories because it brings the state response of systems to set points for all sampling time instants. The new proposed event-triggered piecewise continuous controller is actually an extension of the papers of Koncar & Vasseur (2003) in which only periodic time-triggered transmission is considered and of Song et al. (2017) in which only the small time delay which is lower than detection period is considered. In contrast to the two papers above, this paper actually considers aperiodic event-triggered transmission and the time delay which is bigger and equal to a multiple of the detection period. Specifically, the proposed controller uses the event-triggered sampled and delayed data received from the sensor system, and a reconstruction calculation module is designed in the framework of controller system to deal with the delayed state estimation received from the sensor by using the previous data.

The remainder of this paper is structured as follows: The problem formulation part is given in Section 2. The event-triggered piecewise continuous trajectory tracking control system subject to time delays is presented and analyzed in Section 3. The comparative simulation results based on a mobile cart are presented in Section 4, which is followed by the conclusion in Section 5.

2. Problem Formulation

In this paper, the analysis is based on the networked control system configuration shown in Figure 1, which includes: the considered plant, the sensor system, and the controller system. The sensor system and controller system communicate through the network. Studies considering the communications for both the sensor/controller and the controller/actuator network channel are presented in (Zhang et al., 2023) and (Du et al., 2017).

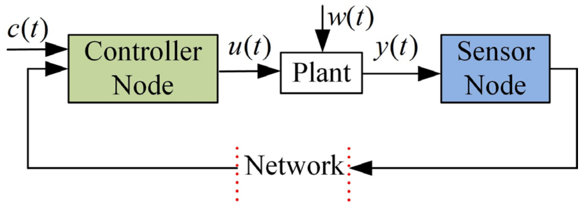

Figure 1. The networked control system configuration

The following linear perturbed system shall be considered as plant:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + w(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$, $u(t) \in \mathbb{R}^{n_u}$, $y(t) \in \mathbb{R}^{n_y}$ and $w(t) \in \mathbb{R}^{n_w}$ represent the state of the system, the control input, the system output and the external perturbation, respectively. The parameters A, B and C are constant matrices with similar dimensions. It is assumed that the pairs (A, B) and (A, C) satisfy the conditions for controllability and observability, respectively.

Hypothesis 1: It is assumed that the external perturbation $w(t)$ is restricted by:

$$\left\| \int_{t_a}^{t_b} w(t)dt \right\| < \bar{\mu}, \forall t_a, t_b \tag{2}$$

where $\bar{\mu} \in R$ is a positive constant.

$c(t) \in \mathbb{R}^{n_x}$ was defined as the reference for tracking, and $e_c(t) = x(t) - c(t - T_e)$ is the tracking error where $T_e \in \mathbb{R}^+$ is the event detection period. For the above system, the purpose of this work is to present a controller for tracking with few data transmissions through the communication network.

3. Event-triggered Piecewise Continuous Trajectory Tracking Controller Design

This section presents an event-triggered tracking controller which was innovatively designed for the system expressed by equation (1). The time delay is set as $\tau_e \in (0, qT_e]$ where $q \in \mathbb{N}^+$.

3.1 The Design Process

At the sensor node, a Luenberger observer was included to estimate the system state as follows:

$$\begin{cases} \dot{\hat{x}}(t) = A_o x(t) + B_o u(t) + L(y(t) - y(t)) \\ \hat{y}(t) = C_o x(t) \end{cases} \tag{3}$$

where $\hat{x}(t)$ represents the system state estimation, the matrix L represents the observer gain, and $\hat{y}(t)$ represents the observer output. The parameters can be set as $A_0 = A$, $B_0 = B$, and $C_0 = C$.

Then, an ETM was designed at the sensor node to decide the transmission time instants for state estimation data, so that the aperiodically sampled state estimation data be transmitted from sensor node to controller node. This new kind of ETM is designed as follows:

ETM: $\hat{x}((k-q)T_e)$ is transmitted $\Leftrightarrow$

$$\begin{aligned} &\left\| e^{AT_e}(\bar{x}_q(kT_e) - \bar{x}_q(h((k-q)T_e) + qT_e)) \right. \\ &\left. + c(h((k-q)T_e) + qT_e) - c(kT_e) \right\| > \bar{\sigma} \end{aligned} \tag{4}$$

where $h((k-q)T_e)$ denotes the latest trigger instant at time instant $(k-q)T_e$ which naturally satisfies $h((k-q)T_e) < (k-q)T_e$, $\bar{\sigma} > 0$ is a positive constant to be designed, and $\bar{x}_q(kT_e)$ is defined as the predicted state estimation which is calculated as:

$$\begin{cases} \bar{x}_{k-q+1}^{(1)} = e^{AT_e}\hat{x}((k-q)T_e) + M\lambda^+((k-q)T_e) \\ \bar{x}_{k-q+2}^{(2)} = e^{AT_e}\bar{x}_{k-q+1}^{(1)} + M\lambda^+((k-q+1)T_e) \\ \cdots \\ \bar{x}_{k}^{(q)} = e^{AT_e}\bar{x}_{k-1}^{(q-1)} + M\lambda^+((k-1)T_e) \\ \bar{x}_q(kT_e) = \bar{x}_k^{(q)} \end{cases} \tag{5}$$

The term λ^+ in (5) can be calculated as:

$$\begin{aligned} &\lambda^+((k-q+n)T_e) \\ &= M^{-1}\big(c(h((k-2q+n+1)T_e) + qT_e) \\ &- e^{AT_e}\bar{x}_q(h((k-2q+n+1)T_e) + qT_e)\big), \quad 1 \le n \le q-1 \end{aligned} \tag{6}$$

where $M \in \mathbb{R}^{n_x \times n_x}$ is a parameter of the controller which will be calculated later, and the value of $\lambda^+((k-q)T_e)$ is provided by the copy of the controller model set at sensor node. Also, the copy of controller model set at sensor node will provide the equivalent control input $u(t)$ to the Luenberger observer.

Due to the existence of time delays, the sending of current data would affect the future tracking performance, so the prediction method should be used for the design of the ETM. The advantage of the prediction method lies in the prior knowledge of future system performance, and then in taking action in time to limit it within the expected range.

At the controller node, an event-triggered piecewise-continuous controller is set in order to carry out the discrete-time trajectory tracking. $E = \{t_k\}$ is defined as the set of event-triggered time instants, which means the data at time instants will be transmitted. Thus, the controller can be designed as follows:

$$\begin{cases} \psi(kT_e) = \begin{cases} c(h((k-q)T_e)+qT_e) - e^{AT_e}\bar{x}_q(h((k-q)T_e)+qT_e), & (k-q)T_e \notin E \\ c(kT_e) - e^{AT_e}\bar{x}_q(kT_e), & (k-q)T_e \in E \end{cases} \\ \psi(t) = \psi(kT_e), \forall t \in [kT_e, (k+1)T_e) \\ \beta^d = M^{-1} \\ \lambda_c^+(t) = \beta^d \psi(t), \forall t \in S \\ \dot{\lambda}_c(t) = \alpha\lambda_c(t), \forall t \notin S \\ u(t) = \gamma\lambda_c(t) \end{cases} \tag{7}$$

where $\bar{x}_q(kT_e)$ represents the reconstructed state estimation, and it is calculated in the same way as the predicted state estimation in equation (5), thus the same symbol $\bar{x}_q(t)$ was used. The parameter α is a stable constant matrix with an appropriate dimension. The item $S = \{kT_e\}$ is the set of event detection time instants when $k \in \mathbb{N}$, and the parameter $M = e^{AT_e}\int_0^{T_e} e^{-A\tau}B\gamma e^{\alpha\tau}d\tau$. The work of Koncar & Vasseur (2003) provides details about the existence of matrix M^{-1}. The gain γ is a constant matrix with an appropriate dimension.

The reconstruction calculation module is proposed to deal with the state estimation for the delay transmitted by sensor node, and the reconstruction calculation process is just like the calculation of predicted state estimation. It is assumed that the event-triggered condition is checked at the current time instant $(k-q)T_e$. Logically, if the current state estimation $\hat{x}((k-q)T_e)$ is transmitted, then after a period of time $\tau_e \in (0, qT_e]$, it will be received by controller node and then through a buffer for obtaining a delay, after which the reconstruction will be carried out based on $\hat{x}((k-q)T_e)$ at the controller node to obtain $\bar{x}_q(kT_e)$ at the time instant kT_e. Conversely, if the current state estimation is not transmitted, the former reconstructed state estimation $\bar{x}_q(h((k-q)T_e)+qT_e)$ will still be used by the controller.

3.2 Performance Analysis

In this subsection, the performance of the designed discrete tracking controller was analyzed. The error of the observer can be defined as:

$$e(t) = x(t) - \hat{x}(t) \tag{8}$$

Thus, the following dynamics can be achieved:

$$\dot{e}(t) = (A - LC)e(t) + w(t) \tag{9}$$

With the appropriate gain L of the observer, the matrix A-LC can be set as stable.

Hypothesis 2: It is assumed that the initial state of the system expressed in equation (1) is norm-bounded as:

$$\|x_0\| < g \quad (10)$$

where $g \in R$ is a positive constant.

Thus, $\|e_0\| < g$ is obtained since $e_0 = x(t_0) - \hat{x}(t_0)$ and $\hat{x}(t_0)$ is normally set as zero.

Theorem 1: Based on the observer in equation (3), the designed ETM in equation (4), and the proposed controller in equation (7), the discrete tracking error of the system in equation (1) is norm-bounded as:

$$\|x((k+1)T_e) - c(kT_e)\| < \varepsilon_r \quad (11)$$

where

$$\varepsilon_r = \bar{\sigma} + \sum_{n=0}^{q}\left(\left\|e^{AnT_e}\right\| g\,\theta\big|_{\tau_\theta=(k-n)T_e}\eta + \left\|e^{AnT_e}\right\|\bar{f}(n)\right) + g\,\theta\big|_{\tau_\theta=(k+1)T_e} + \mu_2$$

with

$$\mu_2 \triangleq \left\|\int_0^{(k+1)T_e} e^{(A-LC)\tau} w((k+1)T_e - \tau)d\tau\right\| < \bar{\mu},$$

$$\bar{f}(n) \triangleq \int_{(k-n)T_e}^{(k-n+1)T_e}\left\|e^{A((k-n+1)T_e-\tau)}LC\right\|\mu_1(\tau)d\tau < \eta\bar{\mu},$$

$$\mu_1(\tau) \triangleq \left\|\int_0^{\tau} e^{(A-LC)\tau_1} w(\tau-\tau_1)d\tau_1\right\|,$$

$$\theta\big|_{\tau_\theta} = \left\|e^{(A-LC)\tau_\theta}\right\|, \text{ and } \int_0^{T_e}\left\|e^{A\tau}LC\right\|d\tau \triangleq \eta,$$

and when $k \to \infty$, the discrete-time tracking error satisfies $\lim_{k\to\infty}\|x((k+1)T_e) - c(kT_e)\| < \bar{\sigma} + \sum_{n=0}^{q}\left\|e^{AnT_e}\right\|\bar{f}(n) + \mu_2$ with $\mu_2 \ll \bar{\mu}$ and $\bar{f}(n) \ll \eta\bar{\mu}$.

Proof: The discrete tracking error is calculated as:

$$\|x((k+1)T_e) - c(kT_e)\| = \|\hat{x}((k+1)T_e) + e((k+1)T_e) - c(kT_e)\| \quad (12)$$

Based on the systems in equations (3) and (7), the state of the observer $\hat{x}(t)$ can be calculated as:

$$\hat{x}((k+1)T_e) = e^{AT_e}x(kT_e) + M\beta^d\psi(kT_e) + \int_{kT_e}^{(k+1)T_e} e^{A((k+1)T_e-\tau)}LCe(\tau)d\tau \quad (13)$$

According to equation (9), the item $e(t)$ can be calculated as:

$$e((k+1)T_e) = e^{(A-LC)((k+1)T_e)}e_0 + \int_0^{(k+1)T_e} e^{(A-LC)((k+1)T_e-\tau)}w(\tau)d\tau \quad (14)$$

Based on equations (12), (13) and (14), the following is obtained:

$$\begin{aligned} &\|x((k+1)T_e) - c(kT_e)\| \\ &= \Big\| e^{AT_e}\hat{x}(kT_e) + M\beta^d\psi(kT_e) + \int_{kT_e}^{(k\ \ 1)T_e} e^{A((k+1)T_e-\tau)}LCe(\tau)d\tau \\ &+ e^{(A-LC)((k+1)T_e)}e_0 + \int_0^{(k+1)T_e} e^{(A-LC)((k+1)T_e-\tau)}w(\tau)d\tau - c(kT_e)\Big\| \end{aligned} \quad (15)$$

Further on, two cases are presented for the proof above.

Case 1: The condition in equation (4) is satisfied so that the data $\hat{x}((k-q)T_e)$ is transmitted.

According to equation (7), one obtains:

$$\begin{cases} \beta^d = M^{-1} \\ \psi(kT_e) = c(kT_e) - e^{AT_e}\bar{x}_q(kT_e) \end{cases} \tag{16}$$

Based on condition (16) and equation (15), one obtains:

$$\begin{aligned} &\|x((k+1)T_e) - c(kT_e)\| \\ &= \Big\| e^{AT_e}\hat{x}(kT_e) + MM^{-1}(c(kT_e) - e^{AT_e}\bar{x}_q(kT_e)) \\ &+ \int_{kT_e}^{(k+1)T_e} e^{A((k+1)T_e-\tau)} LCe(\tau)d\tau + e^{(A-LC)((k+1)T_e)}e_0 \\ &+ \int_0^{(k+1)T_e} e^{(A-LC)((k+1)T_e-\tau)} w(\tau)d\tau - c(kT_e)\Big\| \\ &= \Big\| e^{AT_e}\left(\hat{x}(kT_e) - \bar{x}_q(kT_e)\right) + \int_{kT_e}^{(k+1)T_e} e^{A((k+1)T_e-\tau)} LCe(\tau)d\tau \\ &+ e^{(A-LC)((k+1)T_e)}e_0 + \int_0^{(k+1)T_e} e^{(A-LC)((k+1)T_e-\tau)} w(\tau)d\tau \Big\| \end{aligned} \tag{17}$$

Based on the discrete-time mode of observer state estimation $\hat{x}(t)$ in equation (13) and the reconstructed state estimation which actually can be calculated by equation (5), one can get:

$$\hat{x}(kT_e) - \bar{x}_q(kT_e) = \sum_{n=1}^{q} e^{A(n-1)T_e} \int_{(k-n)T_e}^{(k-n+1)T_e} e^{A((k-n+1)T_e-\tau)} LCe(\tau)d\tau \tag{18}$$

With equations (17) and (18), one obtains:

$$\begin{aligned} &\|x((k+1)T_e) - c(kT_e)\| \\ &= \Big\| \sum_{n=1}^{q} e^{AnT_e} \int_{(k-n)T_e}^{(k-n+1)T_e} e^{A((k-n+1)T_e-\tau)} LCe(\tau)d\tau \\ &+ \int_{kT_e}^{(k+1)T_e} e^{A((k+1)T_e-\tau)} LCe(\tau)d\tau \\ &+ e^{(A-LC)((k+1)T_e)}e_0 + \int_0^{(k+1)T_e} e^{(A-LC)((k+1)T_e-\tau)} w(\tau)d\tau \Big\| \\ &\triangleq \bar{E} \end{aligned} \tag{19}$$

The item $\bar{E}$ here can be further deduced as:

$$\begin{aligned} \bar{E} \le & \sum_{n=0}^{q} \left\| e^{AnT_e} \right\| \int_{(k-n)T_e}^{(k-n+1)T_e} \left\| e^{A((k-n+1)T_e-\tau)} LCe(\tau) \right\| d\tau \\ & + \left\| e^{(A-LC)((k+1)T_e)}e_0 + \int_0^{(k+1)T_e} e^{(A-LC)((k+1)T_e-\tau)} w(\tau)d\tau \right\| \end{aligned} \tag{20}$$

With condition (10) and equation (14), the first item in inequality (20) can be further deduced as:

$$\begin{aligned} &\sum_{n=0}^{q} \left\| e^{AnT_e} \right\| \int_{(k-n)T_e}^{(k-n+1)T_e} \left\| e^{A((k-n+1)T_e-\tau)} LCe(\tau) \right\| d\tau \\ &\le \sum_{n=0}^{q} \left\| e^{AnT_e} \right\| \int_{(k-n)T_e}^{(k-n+1)T_e} \left\| e^{A((k-n+1)T_e-\tau)} LC(e^{(A-LC)\tau}e_0 + \int_0^{\tau} e^{(A-LC)(\tau-\tau_0)} w(\tau_0)d\tau_0) \right\| d\tau \\ &< \sum_{n=0}^{q} \Big(\left\| e^{AnT_e} \right\| g \int_{(k-n)T_e}^{(k-n+1)T_e} \left\| e^{A((k-n+1)T_e-\tau)} LC \right\| \left\| e^{(A-LC)\tau} \right\| d\tau \\ &+ \left\| e^{AnT_e} \right\| \int_{(k-n)T_e}^{(k-n+1)T_e} \left\| e^{A((k-n+1)T_e-\tau)} LC \right\| \left\| \int_0^{\tau} e^{(A-LC)\tau_1} w(\tau-\tau_1)d\tau_1 \right\| d\tau \Big) \end{aligned} \tag{21}$$

Because the item *A-LC* is a stable matrix, the item $\left\| e^{(A-LC)\tau} \right\|$ can be bounded as:

$$\max_{any\,value \ge \tau \ge \tau_\theta \ge 0} \left\| e^{(A-LC)\tau} \right\| = \left\| e^{(A-LC)\tau_\theta} \right\| \triangleq \theta|_{\tau_\theta} \tag{22}$$

Besides, there exists:

$$\lim_{\tau_\theta \to \infty} \theta|_{\tau_\theta} = 0 \tag{23}$$

According to the condition (23), the inequality (21) can be calculated as:

$$\begin{aligned} &\sum_{n=0}^{q} \left\| e^{AnT_e} \right\| \int_{(k-n)T_e}^{(k-n+1)T_e} \left\| e^{A((k-n+1)T_e-\tau)} LCe(\tau) \right\| d\tau \\ &< \sum_{n=0}^{q} \Big(\left\| e^{AnT_e} \right\| g\, \theta|_{\tau_\theta=(k-n)T_e} \int_{(k-n)T_e}^{(k-n+1)T_e} \left\| e^{A((k-n+1)T_e-\tau)} LC \right\| d\tau \\ &+ \left\| e^{AnT_e} \right\| \int_{(k-n)T_e}^{(k-n+1)T_e} \left\| e^{A((k-n+1)T_e-\tau)} LC \right\| \mu_1(\tau)d\tau \Big) \end{aligned} \tag{24}$$

where $\mu_1(\tau) \triangleq \left\| \int_0^{\tau} e^{(A-LC)\tau_1} w(\tau-\tau_1)d\tau_1 \right\|$. Because *A-LC* is a stable matrix, with condition (2), one can get the inequality $\mu_1(\tau) < \left\| \int_0^{\tau} w(\tau-\tau_1)d\tau_1 \right\| < \bar{\mu}$ for $\forall \tau > 0$.

With $\bar{f}(n) \triangleq \int_{(k-n)T_e}^{(k-n+1)T_e} \left\| e^{A((k-n+1)T_e-\tau)} LC \right\| \mu_1(\tau)d\tau$, $\int_0^{T_e} \left\| e^{A\tau} LC \right\| d\tau \triangleq \eta$, and $\mu_1(\tau) < \bar{\mu}$ for $\forall \tau > 0$, $\bar{f}(n) < \eta\bar{\mu}$ can be obtained. Thus, the inequality (24) can be calculated as:

$$\begin{aligned} &\sum_{n=0}^{q} \left\| e^{AnT_e} \right\| \int_{(k-n)T_e}^{(k-n+1)T_e} \left\| e^{A((k-n+1)T_e-\tau)} LCe(\tau) \right\| d\tau \\ &< \sum_{n=0}^{q} \left(\left\| e^{AnT_e} \right\| g\, \theta|_{\tau_\theta=(k-n)T_e} \eta + \left\| e^{AnT_e} \right\| \bar{f}(n) \right) \end{aligned} \tag{25}$$

With inequalities (20) and (25), one can get:

$$\begin{aligned} \bar{E} < & \sum_{n=0}^{q} \left(\left\| e^{AnT_e} \right\| g\, \theta|_{\tau_\theta=(k-n)T_e} \eta + \left\| e^{AnT_e} \right\| \bar{f}(n) \right) \\ & + \left\| e^{(A-LC)((k+1)T_e)}e_0 + \int_0^{(k+1)T_e} e^{(A-LC)((k+1)T_e-\tau)} w(\tau)d\tau \right\| \end{aligned} \tag{26}$$

Because *A-LC* is a stable matrix, then with condition (2) and $\mu_2 \triangleq \left\| \int_0^{(k+1)T_e} e^{(A-LC)\tau} w((k+1)T_e-\tau)d\tau \right\|$, one can get $\mu_2 < \left\| \int_0^{(k+1)T_e} w((k+1)T_e-\tau)d\tau \right\| < \bar{\mu}$. Thus, the inequality (26) can be further expressed as

$$\begin{aligned} \bar{E} < & \sum_{n=0}^{q} \left(\left\| e^{AnT_e} \right\| g\, \theta|_{\tau_\theta=(k-n)T_e} \eta + \left\| e^{AnT_e} \right\| \bar{f}(n) \right) \\ & + g\, \theta|_{\tau_\theta=(k+1)T_e} + \mu_2 \end{aligned} \tag{27}$$

where $\mu_2 < \bar{\mu}$ and $\bar{f}(n) < \eta\bar{\mu}$.

When $k \to \infty$, one obtains:

$$\lim_{k \to \infty} \bar{E} < \sum_{n=0}^{q} \left\| e^{AnT_e} \right\| \bar{f}(n) + \mu_2 \tag{28}$$

where $\mu_2 \ll \bar{\mu}$ and $\bar{f}(n) \ll \eta\bar{\mu}$.

With the positive parameter $\bar{\sigma}$, and based on the inequalities (27) and (28), $\|x((k+1)T_e) - c(kT_e)\| < \varepsilon_r$ and $\lim_{k\to\infty} \|x((k+1)T_e) - c(kT_e)\| < \bar{\sigma} + \sum_{n=0}^{q} \left\| e^{AnT_e} \right\| \bar{f}(n) + \mu_2$ are obtained. Thus, the condition (11) is satisfied in Case 1.

Case 2: The condition in equation (4) is not satisfied so that the data $\hat{x}((k-q)T_e)$ is not transmitted.

With (7), one obtains:

$$\begin{cases} \beta^d = M^{-1} \\ \psi(kT_e) = c(h((k-q)T_e)+qT_e) - e^{AT_e}\overline{x}_q(h((k-q)T_e)+qT_e) \end{cases} \tag{29}$$

With condition (29) and equation (15), the following is obtained:

$$\begin{aligned} &\left\|x((k+1)T_e)-c(kT_e)\right\| \\ &= \Big\| e^{AT_e}\hat{x}(kT_e) + MM^{-1}(c(h((k-q)T_e)+qT_e) - e^{AT_e}\overline{x}_q(h((k-q)T_e)+qT_e)) \\ &+ \int_{kT_e}^{(k+1)T_e} e^{A((k+1)T_e-\tau)}LCe(\tau)d\tau + e^{(A-LC)((k+1)T_e)}e_0 \\ &+ \int_0^{(k+1)T_e} e^{(A-LC)((k+1)T_e-\tau)}w(\tau)d\tau - c(kT_e)\Big\| \\ &\le \left\| e^{AT_e}(\overline{x}_q(kT_e) - \overline{x}_q(h((k-q)T_e)+qT_e)) + c(h((k-q)T_e)+qT_e) - c(kT_e)\right\| \\ &+ \Big\| e^{AT_e}\left(\hat{x}(kT_e) - \overline{x}_q(kT_e)\right) + \int_{kT_e}^{(k+1)T_e} e^{A((k+1)T_e-\tau)}LCe(\tau)d\tau \\ &+ e^{(A-LC)((k+1)T_e)}e_0 + \int_0^{(k+1)T_e} e^{(A-LC)((k+1)T_e-\tau)}w(\tau)d\tau \Big\| \end{aligned} \tag{30}$$

With equation (18) and inequality (30), one can get:

$$\left\|x((k+1)T_e)-c(kT_e)\right\| \le \overline{\sigma} + \overline{E} \tag{31}$$

and the conditions (27) and (28) are also satisfied here. The inequality (31) can be further expressed as:

$$\begin{aligned} &\left\|x((k+1)T_e)-c(kT_e)\right\| < \overline{\sigma} \\ &+ \sum_{n=0}^{q}\left(\left\|e^{AnT_e}\right\| g\,\theta\big|_{\tau_\theta=(k-n)T_e}\eta + \left\|e^{AnT_e}\right\|\overline{f}(n)\right) + g\,\theta\big|_{\tau_\theta=(k+1)T_e} + \mu_2 \end{aligned} \tag{32}$$

where $\mu_2 < \overline{\mu}$ and $\overline{f}(n) < \eta\overline{\mu}$.

When $k \to \infty$, one obtains:

$$\lim_{k\to\infty}\left\|x((k+1)T_e)-c(kT_e)\right\| < \overline{\sigma} + \sum_{n=0}^{q}\left\|e^{AnT_e}\right\|\overline{f}(n) + \mu_2 \tag{33}$$

where $\mu_2 \ll \overline{\mu}$ and $\overline{f}(n) \ll \eta\overline{\mu}$.

Thus, according to the inequalities (32) and (33), the condition (11) is satisfied in Case 2. The proof is valid.

Remark 1. The negative poles of matrix A-LC can be set far away from the origin, so that the item $\theta|_{\tau_\theta}$ can be very small, and the conditions $\mu_2 \ll \overline{\mu}$, $\overline{f}(n) \ll \eta\overline{\mu}$ can be satisfied even if k is not infinite, in which case the tracking error will be very small. Besides, the gain L can be better selected so as to satisfy the condition $\eta < 1$. The regulation of the controller tracking performance can be accomplished by selecting the parameter $\overline{\sigma}$, thus a trade-off can be established between the tracking performance and data communications.

Remark 2. When the condition $w(t) \to 0$ is satisfied, one gets $\lim_{k\to\infty}\left\|x((k+1)T_e)-c(kT_e)\right\| < \overline{\sigma}$, which is the case without external perturbation. When both of the conditions $w(t) \to 0$ and $\overline{\sigma} \to 0$ are satisfied, one gets $\lim_{k\to\infty}\left\|x((k+1)T_e)-c(kT_e)\right\| = 0$, which is the case of the time-triggered mechanism without external perturbation.

4. Simulation Results

A mobile cart system is considered, which is actually a networked visual servoing system, as it is shown in Figure 2. The cart moves horizontally and in a straight line through a notched belt powered by a servo motor. With a dSpace I/O card and a power amplifier, the servo motor can be driven by the control input calculated by the Host PC on the controller node. On the sensor node, a camera is used to capture the position data for the cart, which will be processed by the Sensor PC subsequently. The data communication between the Sensor PC (or the sensor node) and the Host PC (or the controller node) takes place through the network.

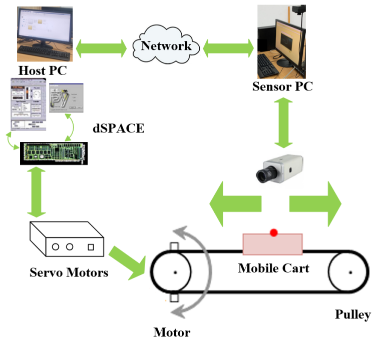

Figure 2. The configuration of the networked mobile cart visual servoing system

The mobile cart system model can be expressed as follows:

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -\dfrac{1}{\tau_0} \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \dfrac{k_0}{\tau_0} \end{bmatrix} u(t) + w(t) \\ y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) \end{cases} \tag{34}$$

where $x(t) = [x_1(t) \;\; x_2(t)]^T$ represents the system state. The component $x_1(t) \in R$ is the position of the cart and the component $x_2(t) \in R$ is the speed of the cart. $y(t)$ is the output of the cart system. The parameters $\tau_0 = 0.332\text{s}$ and $k_0 = 2.9\text{m•s/V}$ represent the time constant and overall gain respectively. The perturbation $w(t) = [w_1(t) \;\; w_2(t)]^T$ is set as $\mathrm{w}_1(t) = \mathrm{w}_2(t) = 0.6\sin(4t + 0.5\pi) + 0.5\sin(15t)$ in this paper.

In this section, the controller was applied to the aforementioned mobile cart system. For the considered plant, the initial state value was set at $x_0 = \begin{bmatrix} 0.5 & 0 \end{bmatrix}^T$. Besides, the reference was set at $c(t) = \begin{bmatrix} -0.17t^3 + 2.5t^2 - 10t & -0.51t^2 + 5t - 10 \end{bmatrix}^T$. The poles of *A-LC* were selected as $-4.0060 \pm 2.6480\mathrm{i}$, thus the gain of Luenberger observer was $L = \begin{bmatrix} 5 & 8 \end{bmatrix}$. The parameters of the controller were set as $\alpha = \mathrm{diag}\{-10,-20\}$ and $\gamma = \begin{bmatrix} 10 & 10 \end{bmatrix}$.

Since the time delay of the network is denoted by $\tau_e \in (0, qT_e]$, two cases, namely $q = 2$ and $q = 4$ were taken into account to verify the efficiency of the new proposed controller. The detection period for the event-triggered condition was set as $T_e = 0.3$ and $T_e = 0.2$, respectively for the two cases. Also, the constant parameter of ETM was set as $\bar{\sigma} = 0.9$ and $\bar{\sigma} = 1$, respectively for the two cases.

Figures 3 to 5 show the tracking results, the control input, and trigger signals for the case of $q = 2$, while Figures 7 to 9 show the tracking results, the control input, and trigger signals for the case of $q = 4$. From these figures, one can see that the tracking is effective in the two cases. For showing the accuracy of the reconstructed state estimation calculated by equation (5), the following shall be defined:

$$x_p(t) = \bar{x}_q(h((k-q)T_e) + qT_e), t \in [(k-1)T_e, kT_e) \tag{35}$$

Then, Figure 6 and Figure 10 show the system state and the reconstructed state estimation for the first case and the second case, respectively, and one can see that the reconstructed state estimation falls below the real system state at almost every reconstructed time instant.

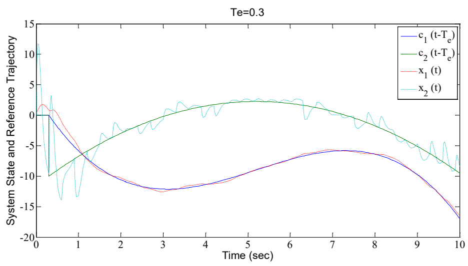

Figure 3. Tracking under double-period size of time delays ($\tau_e \in (0, 2T_e]$)

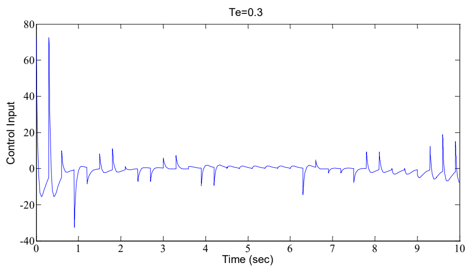

Figure 4. Control input under double-period size of time delays ($\tau_e \in (0, 2T_e]$)

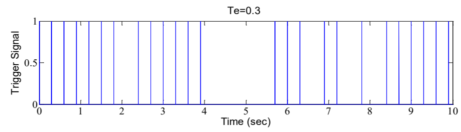

Figure 5. Trigger signals under double-period size of time delays ($\tau_e \in (0, 2T_e]$) (The number of transmissions is 25)

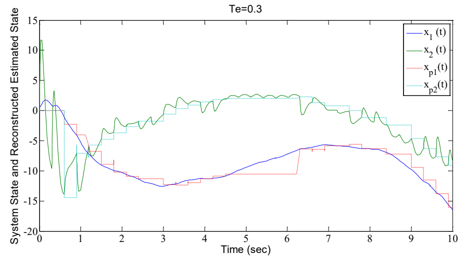

Figure 6. Reconstructed state estimation under double-period size of time delays ($\tau_e \in (0, 2T_e]$)

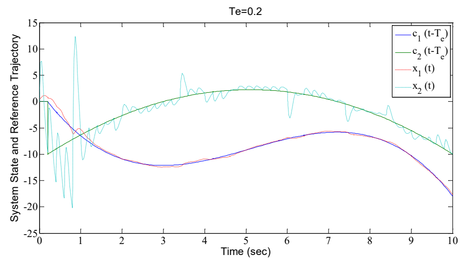

Figure 7. Tracking under fourfold-period size of time delays ($\tau_e \in (0, 4T_e]$)

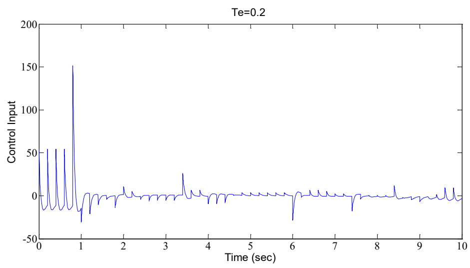

Figure 8. Control input under fourfold-period size of time delays ($\tau_e \in (0, 4T_e]$)

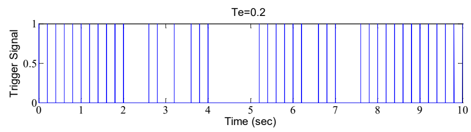

Figure 9. Trigger signals under fourfold-period size of time delays ($\tau_e \in (0, 4T_e]$) (The number of transmissions is 37)

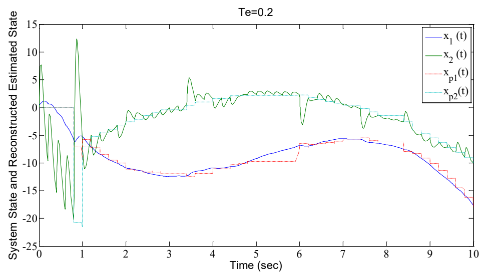

Figure 10. Reconstructed state estimation under fourfold-period size of time delays ($\tau_e \in (0, 4T_e]$)

Besides the above simulation results, three other ETMs in literature were applied for comparison purposes to demonstrate the effectiveness of the new proposed ETM in this work. All of the selected ETMs were applied on the basis of the state estimation since the system state is unknown.

The first one is abbreviated as RD-ETM-C (Relative deadband ETM with the boundary based on the sampled data of current time instants) as in (Hu & Yue, 2012; Sun, Yang & Zeng, 2022), in which the sampled data $\hat{x}(kT_e)$ is transmitted through the network if the condition $(\hat{x}(kT_e)-x(h(kT_e)))^T V(x(kT_e)-x(h(kT_e)))>\sigma_1^2 x(kT_e)^T Vx(kT_e)$ is satisfied.

The second one is abbreviated as RD-ETM-L (Relative deadband ETM with the boundary based on the latest transmitted data) as in (Peng & Yang, 2013; Li, Niu & Song, 2021), in which the sampled data $\hat{x}(kT_e)$ is transmitted through the network if the condition $(\hat{x}(kT_e)-x(h(kT_e)))^T V(x(kT_e)-x(h(kT_e)))>\sigma_2^2 x(h(kT_e))^T Vx(h(kT_e))$ is satisfied.

The third one is abbreviated as R-ETM (Reference based ETM) such as in (Hu, Zhang & Du, 2012), in which the sampled data $\hat{x}(kT_e)$ is transmitted through network if the condition $[e_c(kT_e)-e_c(h(kT_e))]^T V[e_c(kT_e)-e_c(h(kT_e))]>\sigma_3^2 e_c(kT_e)^T Ve_c(kT_e)$ is satisfied, where $e_c(t)=\hat{x}(t)-c(t-T_e)$, and $c(t)$ is the reference of control systems.

The parameter v was chosen as a unit matrix, and the parameters of the event rule were chosen as $\bar{\sigma}=0.82, \sigma_1=0.087, \sigma_2=0.08$ and $\sigma_3=0.7$. The simulation was carried out based on the case of $q=2$ and $T_e=0.3$.

Figures 11 and 12 show the tracking results and trigger signals under four kinds of ETMs. Table 1 shows the corresponding data results. Based on Table 1, it can be concluded that the trajectory tracking error under the new ETM proposed in this paper is lower than that obtained under the other three selected ETMs, considering that the number of transmissions for the new proposed ETM is equal to that of the R-ETM and a little lower than that of the other two ETMs.

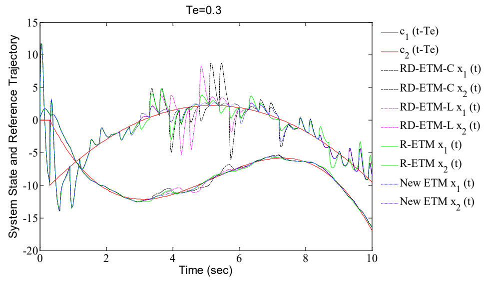

Figure 11. Tracking under four ETMs with double-period size of time delays ($\tau_e\in(0,2T_e]$)

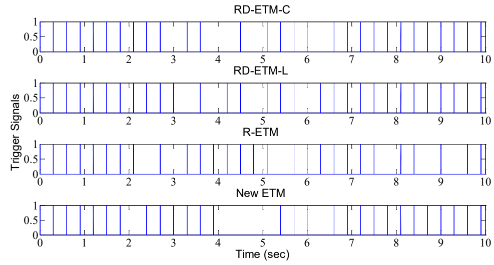

Figure 12. Trigger signals under four ETMs with double-period size of time delays ($\tau_e\in(0,2T_e]$)

5. Conclusion

Based on its theoretical design and on the simulation results, the proposed event-triggered piecewise continuous tracking controller proved to be effective even with large time delays. Besides, a better tracking performance can be obtained based on the new proposed ETM in comparison with other existing ETMs which have been widely used when the bandwidth occupancy is almost identical for all of them. Nevertheless, the performance index is not considered in the design process. Based on the performance index, the optimal value of controller parameters could be achieved, which will be studied in a future work.

Table 1. Comparison for four ETMs

Mechanism	Average Tracking Error	Number of Transmissions in 10 seconds	Average Period (seconds)
RD-ETM-C	[0.3613;0.9155]	28	0.357
RD-ETM-L	[0.3139;0.7045]	29	0.345
R-ETM	[0.3067;0.6621]	27	0.370
New ETM	[0.2698;0.5724]	27	0.370

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