# Robust Model Predictive Control for Systems Affected by Constant and Norm 2 Bounded Disturbances

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**Abstract:** This paper proposes a robust model predictive control approach for systems affected by two types of disturbances: a constant one and a norm 2 bounded one. Through several computations, the norm 2 bounded disturbance is removed from the control optimization problem which consists in minimizing a robust performance objective. The problem is then reduced to a convex one with constraints on the input. The simulation results indicate that the proposed method successfully manages to reject the system disturbances.

**Keywords**: Robust model predictive control, Robust performance objective, Linear matrix inequality, Bounded disturbance, Input constraint.

## 1. Introduction

Methods on model predictive control (MPC) (Camacho & Bordons, 2013; Löfberg, 2003) have received a lot of interest from the scientific community in the last years. The method is used both in academia and industry, respectively in chemical plants, oil refineries, power electronics, for managing energy in buildings etc (Morari & Lee, 1999; Sousa, Leite & Rubio Scola, 2018). Classical MPC does not take into consideration model uncertainties and disturbances and therefore robust MPC emerged (Kothare, Balakrishnan & Morari, 1996; Jouirou, Boukadida & Benamor, 2023).

In (Tahir & Jaimoukha, 2011), the authors present two controllers, one that steers the system state to a robust positively invariant set and another one that maintains the state in this set when disturbances appear.

Smith (2004) proposes a control method with feedback and feedforward. The feedback assures that the state is in an ellipse when bounded disturbances and system perturbations are present in the system. The feedforward component makes these ellipses to be at a desired reference state.

In (Ojaghi, Bigdeli & Rahmani, 2016) the nonlinear system has a linear and a nonlinear term with a bounded additive uncertainty. The nonlinear and uncertain terms are bounded by a quadratic function from a sum of squares optimization problem.

Yu et al. (2010) considers the case of unknown bounded disturbances. The solution of nominal MPC problem is computed and the nominal trajectory is defined. The trajectories of the error system are kept in a disturbance invariant set.

In (Yang, Cai & Ding, 2019), the authors present a robust MPC method with less computational burden. The estimation error set is formulated and recursive feasibility is kept.

The authors Liu, Chen & Knoll (2020) present robust MPC for a system with bounded uncertainties, norm-bounded external disturbances and bounded time-varying delay. The system state is in a robust positively invariant set through a Lyapunov-Razumikhin function. The robust MPC controller in (Tahir & Jaimoukha, 2013) minimizes an upper bound on an $H_2 / H_\infty$ based cost function. The robust positively invariant set is computed as solution of a linear matrix inequality (LMI) optimization problem.

In (Esfahani & Pieper, 2019) robust MPC is presented for switched linear systems when model uncertainty and norm bounded disturbances are present. The control problem is expressed in Riccati-Metzler inequalities. This problem is then converted to one with LMIs.

Model uncertainty and unknown disturbances are considered in (González & Odloak, 2010). The control problem includes cost contracting constraints and the output feedback case is with a non-minimal state-space model that takes into consideration past output measurements and past input increments.

Yang et al. (2016) present robust MPC for parameter varying systems and norm-bounded external disturbances. A minimal ellipsoidal robust positively invariant set and observer gain are determined. A $H_\infty$ cost function is used in order to optimize performance.

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Robust MPC for nonlinear systems that have bounded disturbances with unknown upper bound is studied in (Huofa & Marrani, 2020). The worst-case objective function is considered and with LMI the calculation time and complexity are improved.

The main contribution of this article is to propose robust MPC for systems under two disturbances: a constant and a time varying norm 2 bounded one. The method is a modified version of the one from (Kothare, Balakrishnan & Morari, 1996), that takes into consideration these disturbances. The steps are to define the robust performance objective that needs to be minimized and to determine an upper bound for it used for defining the control optimization problem. The time varying disturbance does not appear anymore in the control computations being replaced by its norm 2 bound. The proposed approach is compared in a MATLAB simulation with the robust MPC method in (Kothare, Balakrishnan & Morari, 1996). This initial method served as a base for many papers in the scientific literature. Note that disturbances are not taken into consideration in (Kothare, Balakrishnan & Morari, 1996), while the presented method studies the behaviour under disturbances. A subject of investigation of the present paper is if, in the presence of large disturbances, the proposed approach is innovative in the sense that it succeeds to have a smaller steady state error than the one resulted from the method proposed in (Kothare, Balakrishnan & Morari, 1996).

This article is organized as follows. Section 2 contains mathematical preliminaries and the proposed robust MPC approach. Section 3 presents the simulation results and Section 4 is devoted to the conclusions.

# 2. Robust Model Predictive Control for Systems Affected by Disturbances

## 2.1 Mathematical Preliminaries

Lemma 1. Let $\Upsilon_1$, $\Upsilon_2$ be real and constant matrices and $\Upsilon_0$ a positive matrix. Then, for any $\varepsilon > 0$ the following inequality holds (Poursafar, Taghirad & Haeri, 2010):

$$\Upsilon_1^T \Upsilon_0 \Upsilon_2 + \Upsilon_2^T \Upsilon_0 \Upsilon_1 \le \varepsilon \Upsilon_1^T \Upsilon_0 \Upsilon_1 + \varepsilon^{-1} \Upsilon_2^T \Upsilon_0 \Upsilon_2 \quad (1)$$

## 2.2 The Proposed Approach

Consider the linear discrete time system affected by a constant disturbance $\rho$ and a norm 2 bounded disturbance $w(k)$.

$$\begin{aligned} & x(k+1) = \rho + A_d(k)x(k) + B_d(k)u(k) + w(k) \\ & \left[A_d(k) \quad B_d(k)\right] \in \Omega, k \ge 1 \end{aligned} \quad (2)$$

where $x(k) \in \mathbb{R}^{n\times 1}$, $u(k) \in \mathbb{R}^{m\times 1}$, $\rho \in \mathbb{R}^{n\times 1}$, $w(k) \in \mathbb{R}^{n\times 1}$, $A_d(k) \in \mathbb{R}^{n\times n}$, $B_d(k) \in \mathbb{R}^{n\times m}$.

The polytope $\Omega$ where Co devotes to the convex hull is:

$$\Omega = Co\left\{\left[A_{d1} \quad B_{d1}\right], \left[A_{d2} \quad B_{d2}\right], \ldots, \left[A_{d\sigma} \quad B_{d\sigma}\right]\right\} \quad (3)$$

if $\left[A_d(k) \quad B_d(k)\right] \in \Omega$, then $\left[A_d(k) \quad B_d(k)\right] = \sum_{l=1}^{\sigma} \lambda_l \left[A_{dl} \quad B_{dl}\right]$ for $\sum_{l=1}^{\sigma} \lambda_l = 1$, $\lambda_l \ge 0$, $l = \overline{1,\sigma}$.

The system is disturbed by constant $\rho$ and $w(k)$ which is in the following set $w(k) \in W$, $W = \left\{w(k) \mid \left\|w(k)\right\|_2 \le \alpha, k \ge 1\right\}$, $\alpha \in \mathbb{R}_+^*$.

The objective is to find $u(k) = L_k x(k)$, $u(k) \in U$ $U = \left\{u_c(k) \mid \left\|u_c(k)\right\|_2 \le \bar{u}, k \ge 1\right\}$, $\bar{u} \in \mathbb{R}_+^*$ using the proposed robust MPC such that the system is stable. A min-max problem, where a robust performance objective is minimized is considered for all $\left[A_d(k) \quad B_d(k)\right] \in \Omega$ and $w(k) \in W$:

$$\min_{u(k+i|k)\in U, i\ge 0} \ \max_{\substack{\left[A_d(k+i) \quad B_d(k+i)\right]\in\Omega \\ w(k+i|k)\in W, i\ge 0}} J_\infty(k) \quad (4)$$

where $J_\infty(k) = \sum_{i=0}^{\infty} \big( x^T(k+i\mid k) Q x(k+i\mid k) + u^T(k+i\mid k) R u(k+i\mid k) \big)$ with $Q \in \mathbb{R}^{n\times n}, R \in \mathbb{R}^{m\times m}$, $Q > 0$, $R > 0$ positive definite and $x(k+i\mid k)$, $u(k+i\mid k)$ the state and input predicted at $k + i$ from time $k$. An upper bound is derived on the robust performance objective. Let $V(x(k+i\mid k)) = x^T(k+i\mid k) P_k x(k+i\mid k)$, $P_k > 0$ and the following inequality to hold:

$$\begin{aligned} & V(x(k+i+1\mid k)) - V(x(k+i\mid k)) \le \\ & \le -\big( x^T(k+i\mid k) Q x(k+i\mid k) + \\ & + u^T(k+i\mid k) R u(k+i\mid k) \big) \end{aligned} \quad (5)$$

If (5) is summed from $i = 0$ to $i = \infty$, then the following is obtained:

$$\max_{\substack{\left[A_d(k+i) \quad B_d(k+i)\right]\in\Omega \\ w(k+i|k)\in W, i\ge 0}} J_\infty(k) \le V\big(x(k\mid k)\big) \quad (6)$$

Thus, the problem is reduced to minimizing a variable $\gamma_k$ under $V\big(x(k\mid k)\big) \le \gamma_k$ and (5).

**Theorem**

Let $x(k) = x(k|k)$ be the state of the uncertain system (2) at $k$ and $\theta \in \mathbb{R}_+^*$, $\phi \in \mathbb{R}_+^*$, $\psi \in \mathbb{R}_+^*$. A robust stabilizing control law $u(k+i|k) = L_k x(k+i|k)$, $\|u(k+i|k)\|_2 \le \bar{u}$, $i \ge 0$, $\bar{u} \in \mathbb{R}_+^*$ is given by $L_k = Y_k X_k^{-1}$, where $X_k > 0$ and $Y_k$ are the solutions of optimization problem:

$$\min_{\gamma_k, X_k, Y_k} \gamma_k$$
$$s.t.$$
$$\begin{bmatrix} 1 & x^T(k|k) \\ x(k|k) & X_k \end{bmatrix} \ge 0$$
$$\begin{bmatrix} X_k & 0 & \Xi_l^T & \left(Q^{\frac{1}{2}} X_k\right)^T & \left(R^{\frac{1}{2}} Y_k\right)^T \\ 0 & -\gamma_k \Lambda & 0 & 0 & 0 \\ \Xi_l & 0 & \frac{1}{1+\psi^{-1}} X_k & 0 & 0 \\ Q^{\frac{1}{2}} X_k & 0 & 0 & \gamma_k I_n & 0 \\ R^{\frac{1}{2}} Y_k & 0 & 0 & 0 & \gamma_k I_m \end{bmatrix} \ge 0$$
$$l = \overline{1, \sigma}$$
$$X_k - \frac{\gamma_k}{\theta} I_n \ge 0$$
$$\begin{bmatrix} X_k & Y_k^T \\ Y_k & \bar{u}^2 I_m \end{bmatrix} \ge 0 \tag{7}$$

with

$$\Lambda = \theta(1+\psi)(1+\phi)\rho^T\rho + \theta(1+\psi)\left(1+\phi^{-1}\right)\alpha^2$$
$$\Xi_l = A_{dl} X_k + B_{dl} Y_k$$

**Proof**

If $V(x(k|k)) = x^T(k|k) P_k x(k|k)$ with $P_k = \gamma_k X_k^{-1}$ is substituted in $V(x(k|k)) \le \gamma_k$ then:

$$\begin{aligned} & x^T(k|k)\gamma_k X_k^{-1} x(k|k) \le \gamma_k \\ & 1 - x^T(k|k) X_k^{-1} x(k|k) \ge 0 \end{aligned} \tag{8}$$

Then, using Schur complement, the following constraint is obtained:

$$\begin{bmatrix} 1 & x^T(k|k) \\ x(k|k) & X_k \end{bmatrix} \ge 0 \tag{9}$$

If $x(k+i+1|k)$ using (2) and $u(k+i|k) = L_k x(k+i|k)$ are substituted in $V(x(k+i+1|k))$, then, by using Lemma 1 with $\Gamma_{cl} = A_d(k+i) + B_d(k+i) L_k$ and $\psi \in \mathbb{R}_+^*$, it results:

$$\begin{aligned} & V(x(k+i+1|k)) - V(x(k+i|k)) = \\ & = \left(\rho + w(k+i|k) + \Gamma_{cl} x(k+i|k)\right)^T P_k \times \\ & \left(\rho + w(k+i|k) + \Gamma_{cl} x(k+i|k)\right) - \\ & - x^T(k+i|k) P_k x(k+i|k) = \\ & = \left(\rho + w(k+i|k)\right)^T P_k \left(\rho + w(k+i|k)\right) + \\ & + \left(\rho + w(k+i|k)\right)^T P_k \Gamma_{cl} x(k+i|k) + \\ & + \left(\Gamma_{cl} x(k+i|k)\right)^T P_k \left(\rho + w(k+i|k)\right) + \\ & + \left(\Gamma_{cl} x(k+i|k)\right)^T P_k \Gamma_{cl} x(k+i|k) - \\ & - x^T(k+i|k) P_k x(k+i|k) \le \\ & \le (1+\psi)\left(\rho + w(k+i|k)\right)^T P_k \left(\rho + w(k+i|k)\right) + \\ & + (1+\psi^{-1})\left(\Gamma_{cl} x(k+i|k)\right)^T P_k \Gamma_{cl} x(k+i|k) - \\ & - x^T(k+i|k) P_k x(k+i|k) \end{aligned} \tag{10}$$

Set $P_k \le \theta I_n$, $\theta \in \mathbb{R}_+^*$ with $P_k = \gamma_k X_k^{-1}$ and add $X_k^{-1} \le \frac{\theta}{\gamma_k} I_n$, $X_k - \frac{\gamma_k}{\theta} I_n \ge 0$ as constraint such that the following is obtained:

$$\begin{aligned} & \left(\rho + w(k+i|k)\right)^T P_k \left(\rho + w(k+i|k)\right) \le \\ & \le \theta\left(\rho + w(k+i|k)\right)^T \left(\rho + w(k+i|k)\right) \end{aligned} \tag{11}$$

Thus, by using Lemma 1 with $\phi \in \mathbb{R}_+^*$, (10) becomes the following:

$$\begin{aligned} & V(x(k+i+1|k)) - V(x(k+i|k)) \le \\ & \le \theta(1+\psi)\left(\rho + w(k+i|k)\right)^T \left(\rho + w(k+i|k)\right) + \\ & + \left(1+\psi^{-1}\right)\left(\Gamma_{cl} x(k+i|k)\right)^T P_k \Gamma_{cl} x(k+i|k) - \\ & - x^T(k+i|k) P_k x(k+i|k) \le \\ & \le \theta(1+\psi)(1+\phi)\rho^T\rho + \theta(1+\psi)\left(1+\phi^{-1}\right) \times \\ & \times w^T(k+i|k) w(k+i|k) + \left(1+\psi^{-1}\right) \times \\ & \times \left(\Gamma_{cl} x(k+i|k)\right)^T P_k \Gamma_{cl} x(k+i|k) - \\ & - x^T(k+i|k) P_k x(k+i|k) \le \\ & \le \theta(1+\psi)(1+\phi)\rho^T\rho + \theta(1+\psi)\left(1+\phi^{-1}\right)\alpha^2 + \\ & + \left(1+\psi^{-1}\right)\left(\Gamma_{cl} x(k+i|k)\right)^T P_k \Gamma_{cl} x(k+i|k) - \\ & - x^T(k+i|k) P_k x(k+i|k) \end{aligned} \tag{12}$$

From (12) with $\Lambda = \theta(1+\psi)(1+\phi)\rho^T\rho + \theta(1+\psi)\left(1+\phi^{-1}\right)\alpha^2$ and (5) it results that:

$$\begin{aligned} & \Lambda + \left(1+\psi^{-1}\right) x^T(k+i|k) \Gamma_{cl}^T P_k \Gamma_{cl} x(k+i|k) - \\ & - x^T(k+i|k) P_k x(k+i|k) \le \\ & \le -\Big(x^T(k+i|k) Q x(k+i|k) + \\ & + x^T(k+i|k) L_k^T R L_k x(k+i|k)\Big) \end{aligned} \tag{13}$$

$$\begin{aligned} & x^T(k+i|k)\Big[\left(1+\psi^{-1}\right)\Gamma_{cl}^T P_k \Gamma_{cl} - P_k + Q + \\ & + L_k^T R L_k\Big] x(k+i|k) + \Lambda \le 0 \end{aligned} \tag{14}$$

$$\begin{bmatrix} x(k+i\,|\,k) \\ 1 \end{bmatrix}^T \begin{bmatrix} \Delta_{11} & 0 \\ 0 & \Delta_{22} \end{bmatrix} \begin{bmatrix} x(k+i\,|\,k) \\ 1 \end{bmatrix} \le 0 \quad (15)$$

With:

$$\Delta_{11} = \left(1+\psi^{-1}\right)\Gamma_{cl}^T P_k \Gamma_{cl} - P_k + Q + L_k^T R L_k$$
$$\Delta_{22} = \Lambda \quad (16)$$

If the diagonal matrix in (15) denoted by $\Delta \in \mathbb{R}^{(n+1)\times(n+1)}$, $\Delta = diag(\Delta_{11}, \Delta_{22})$ is multiplied on the left with $diag\left(X_k^T, \gamma_k\right)$, on the right with $diag\left(X_k, \gamma_k\right)$ and then $Y_k = L_k X_k$, $P_k = \gamma_k X_k^{-1}$:

$$X_k^T \Delta_{11} X_k = \left(1+\psi^{-1}\right)\Xi^T(k+i)\gamma_k X_k^{-1}\Xi(k+i) -$$
$$-\gamma_k X_k + \left(Q^{\frac{1}{2}} X_k\right)^T Q^{\frac{1}{2}} X_k + \left(R^{\frac{1}{2}} Y_k\right)^T R^{\frac{1}{2}} Y_k \quad (17)$$

$$\gamma_k^2 \Delta_{22} = \gamma_k^2 \Lambda$$
$$\Xi(k+i) = A_d(k+i)X_k + B_d(k+i)Y_k$$

$$-\frac{1}{\gamma_k} X_k^T \Delta_{11} X_k =$$
$$= -\left(1+\psi^{-1}\right)\Xi^T(k+i)X_k^{-1}\Xi(k+i) + X_k -$$
$$-\frac{1}{\gamma_k}\left(Q^{\frac{1}{2}} X_k\right)^T Q^{\frac{1}{2}} X_k - \frac{1}{\gamma_k}\left(R^{\frac{1}{2}} Y_k\right)^T R^{\frac{1}{2}} Y_k \quad (18)$$
$$-\frac{1}{\gamma_k}\gamma_k^2 \Delta_{22} = -\gamma_k \Lambda$$

Thus:

$$\begin{bmatrix} X_k & 0 \\ 0 & -\gamma_k \Lambda \end{bmatrix} - \begin{bmatrix} \Xi(k+i) & 0 \\ Q^{\frac{1}{2}} X_k & 0 \\ R^{\frac{1}{2}} Y_k & 0 \end{bmatrix}^T \begin{bmatrix} \frac{1}{1+\psi^{-1}} X_k & 0 & 0 \\ 0 & \gamma_k I_n & 0 \\ 0 & 0 & \gamma_k I_m \end{bmatrix}^{-1} \times \begin{bmatrix} \Xi(k+i) & 0 \\ Q^{\frac{1}{2}} X_k & 0 \\ R^{\frac{1}{2}} Y_k & 0 \end{bmatrix} \ge 0 \quad (19)$$

$$\begin{bmatrix} X_k & 0 & \Xi^T(k+i) & \left(Q^{\frac{1}{2}} X_k\right)^T & \left(R^{\frac{1}{2}} Y_k\right)^T \\ 0 & -\gamma_k \Lambda & 0 & 0 & 0 \\ \Xi(k+i) & 0 & \frac{1}{1+\psi^{-1}} X_k & 0 & 0 \\ Q^{\frac{1}{2}} X_k & 0 & 0 & \gamma_k I_n & 0 \\ R^{\frac{1}{2}} Y_k & 0 & 0 & 0 & \gamma_k I_m \end{bmatrix} \ge 0 \quad (20)$$

Next, (20) holds if the following hold:

$$\begin{bmatrix} X_k & 0 & \Xi_l^T & \left(Q^{\frac{1}{2}} X_k\right)^T & \left(R^{\frac{1}{2}} Y_k\right)^T \\ 0 & -\gamma_k \Lambda & 0 & 0 & 0 \\ \Xi_l & 0 & \frac{1}{1+\psi^{-1}} X_k & 0 & 0 \\ Q^{\frac{1}{2}} X_k & 0 & 0 & \gamma_k I_n & 0 \\ R^{\frac{1}{2}} Y_k & 0 & 0 & 0 & \gamma_k I_m \end{bmatrix} \ge 0 \quad (21)$$

$\Xi_l = A_{dl} X_k + B_{dl} Y_k, l = \overline{1,\sigma}$

Next, the input constraint is considered after defining the invariant ellipsoid for the predicted states of the system. Knowing $Q > 0$ and $R > 0$, from (5) it results that:

$$V(x(k+i+1\,|\,k)) - V(x(k+i\,|\,k)) < 0 \quad (22)$$

Thus $x^T(k+i+1\,|\,k)P_k x(k+i+1\,|\,k) <$
$< x^T(k+i\,|\,k)P_k x(k+i\,|\,k)$

and because $x^T(k\,|\,k)P_k x(k\,|\,k) \le \gamma_k$, then it follows that $x^T(k+1\,|\,k)X_k^{-1}x(k+1\,|\,k) < 1$. Similar, using (22) it is obtained that $x^T(k+i\,|\,k)X_k^{-1}x(k+i\,|\,k) < 1$, $i > 0$. The invariant ellipsoid for the predicted states of the system is defined:

$$IE = \left\{e \,|\, e^T X_k^{-1} e \le 1\right\} \quad (23)$$

The input constraint is considered next.

$$\max_{i\ge 0}\left\|u(k+i\,|\,k)\right\|_2^2 = \max_{i\ge 0}\left\|Y_k X_k^{-1} x(k+i\,|\,k)\right\|_2^2 \le$$
$$\le \max_{e\in IE}\left\|Y_k X_k^{-\frac{1}{2}}\right\|_2^2 e^T X_k^{-1} e == \lambda_{\max}\left(X_k^{-\frac{1}{2}} Y_k^T Y_k X_k^{-\frac{1}{2}}\right) \quad (24)$$

Imposing $\left\|u(k+i\,|\,k)\right\|_2^2 \le \bar{u}^2$ leads to the following constraint.

$$\begin{bmatrix} X_k & Y_k^T \\ Y_k & \bar{u}^2 I_m \end{bmatrix} \ge 0 \quad (25)$$

## 3. Simulation Results

The proposed robust min-max MPC approach is implemented using MATLAB with YALMIP toolbox (Löfberg, 2004).

**Example 1:**

The system model is represented by:

$$A_{d1} = \begin{bmatrix} -0.6890 & -0.0993 & 0.1767 \\ -0.0993 & -1.1274 & -0.1724 \\ 0.1767 & -0.1724 & -0.8237 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} -0.6890 & -0.0993 & 0.2 \\ -0.0993 & -1.1274 & -0.1724 \\ 0.2 & -0.1724 & -0.8237 \end{bmatrix},$$

$$B_{d1} = \begin{bmatrix} 0.4900 & 0 \\ 0.7394 & -2.1384 \\ 1.7119 & 0 \end{bmatrix}, B_{d2} = B_{d1},$$

$$x(k) = \begin{bmatrix} x_1(k) & x_2(k) & x_3(k) \end{bmatrix}^T$$

The disturbance $\rho$ that affects the system:

$$\rho = 10^{-3} * \begin{bmatrix} 11 & 12 & 15 \end{bmatrix}^T$$

The other simulation parameters:

$x(1) = \begin{bmatrix} 0.5\pi & 0.75\pi & -5 \end{bmatrix}^T$, $\overline{u} = 10$, $Q = 10 * I_3$, $R = 10 * I_2$, $\theta = 10^2$, $\psi = 10^2$, $\phi = 10^2$, $\alpha = 5$

*Case 1:* Small disturbances:

$$w(k) = 10^{-1} * \begin{bmatrix} 0 & 0 & \sin(x_2(k)) \end{bmatrix}^T \text{ for every } k \geq 1$$

Figures (1)-(3) indicate the simulation results for the proposed approach and for the robust predictive control method in (Kothare, Balakrishnan & Morari, 1996) when small disturbances are present. For both methods, all the states of the system are stabilized and they reach 0 in steady state. The oscillations are damped in a longer period of time for the proposed method, but the advantage comes when large disturbances are present.

*Case 2:* Large disturbances:

$$w(k) = 5 * \begin{bmatrix} 0 & 0 & \sin(x_2(k)) \end{bmatrix}^T \text{ for every } k \geq 1$$

Figures (4)-(6) indicate the simulation results for the proposed approach and for the robust predictive control method in (Kothare, Balakrishnan & Morari, 1996) when large disturbances are present. For both methods, the states of the system are stabilized, but not all of

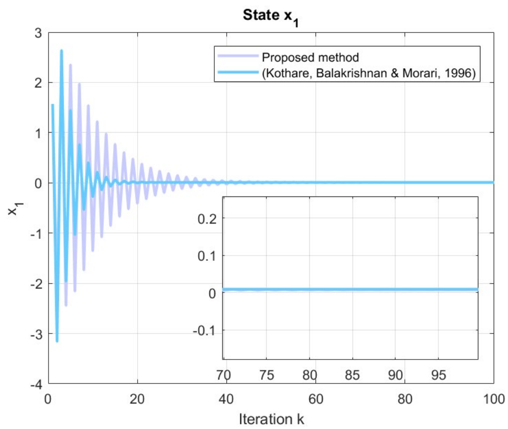


**Figure 1.** Stabilized $x_1$- small disturbance

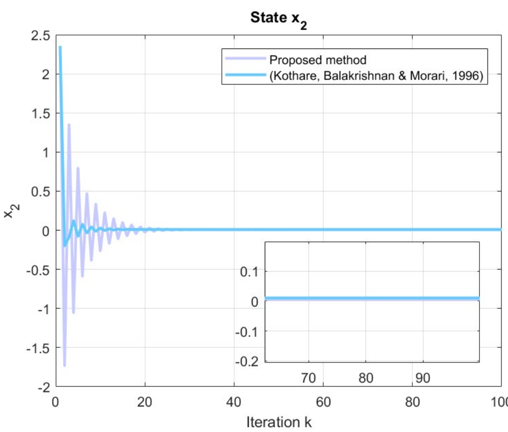


**Figure 2.** Stabilized $x_2$- small disturbance

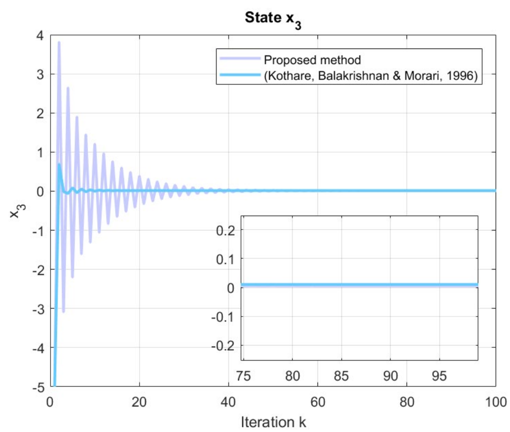


**Figure 3.** Stabilized $x_3$- small disturbance

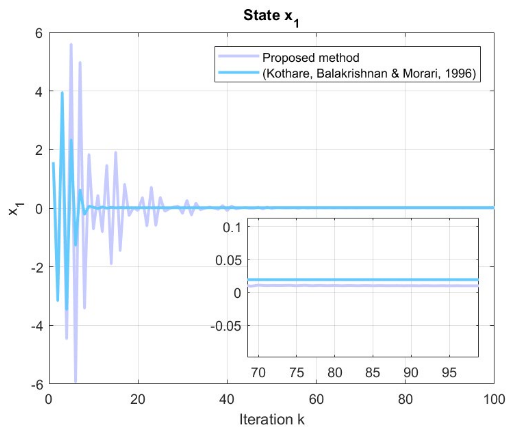


**Figure 4.** Stabilized $x_1$- large disturbance

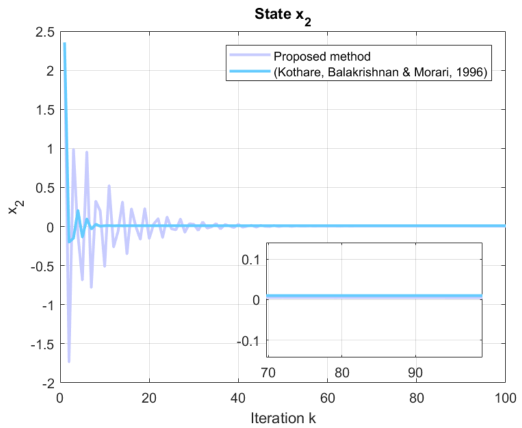


**Figure 5.** Stabilized $x_2$- large disturbance

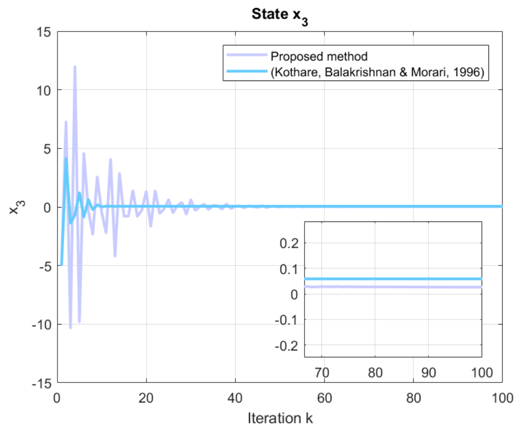


**Figure 6.** Stabilized $x_3$- large disturbance

them reach 0 in steady state. Due to the presence of large disturbances, an error appears. Compared to (Kothare, Balakrishnan & Morari, 1996), the proposed approach has a smaller steady state error. This can be observed by zooming in at the steady state area.

**Example 2:**

The system model is represented by:

$$A_{d1} = \begin{bmatrix} -0.3427 & 0.0368 & 0.2842 \\ 0.0368 & -0.2833 & -0.1226 \\ 0.2842 & -0.1226 & -1.0136 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} -0.3427 & 0.0368 & 0.3 \\ 0.0368 & -0.2833 & -0.1226 \\ 0.3 & -0.1226 & -1.0136 \end{bmatrix},$$

$$B_{d1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -1.9071 & -0.7648 \end{bmatrix}, B_{d2} = B_{d1},$$

The disturbance $\rho$ that affects the system and the other simulation parameters are kept the same as in Example 1.

*Case 1:* Small disturbances:

$$w(k) = 10^{-1} * \begin{bmatrix} 0 & 0 & \sin(x_2(k)) \end{bmatrix}^T \text{ for every } k \geq 1$$

Figures (7)-(9) indicate the simulation results for the proposed approach and for the robust predictive control method in (Kothare, Balakrishnan & Morari, 1996) when small disturbances are present. For both methods, all the states of the system are stabilized and they reach 0 in steady state. The oscillations are damped in a longer period of time for the proposed method, but the advantage comes when large disturbances are present.

*Case 2:* Large disturbances:

$$w(k) = 4 * \begin{bmatrix} 0 & 0 & \sin(x_2(k)) \end{bmatrix}^T \text{ for every } k \geq 1$$

Figures (10)-(12) indicate the simulation results for the proposed approach and for the robust predictive control method in (Kothare, Balakrishnan & Morari, 1996) when large disturbances are present. For both methods, the states of the system are stabilized, but not all of them reach 0 in steady state. Due to the presence of large disturbances an error appears. Compared to (Kothare, Balakrishnan & Morari,

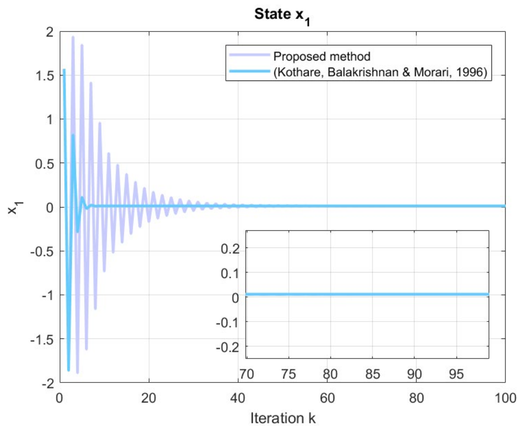


**Figure 7.** Stabilized $x_1$- small disturbance

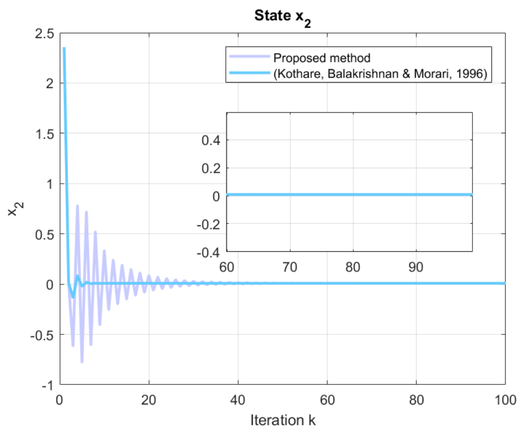


**Figure 8.** Stabilized $x_2$- small disturbance

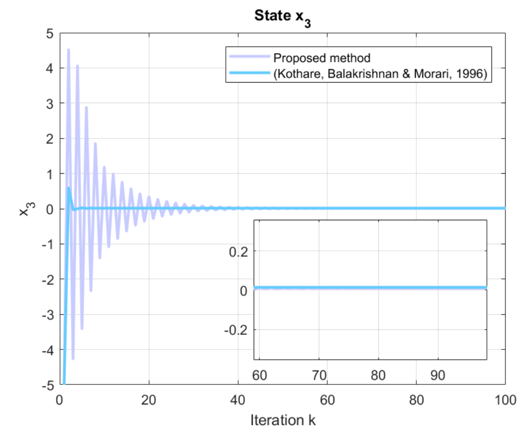


**Figure 9.** Stabilized $x_3$- small disturbance

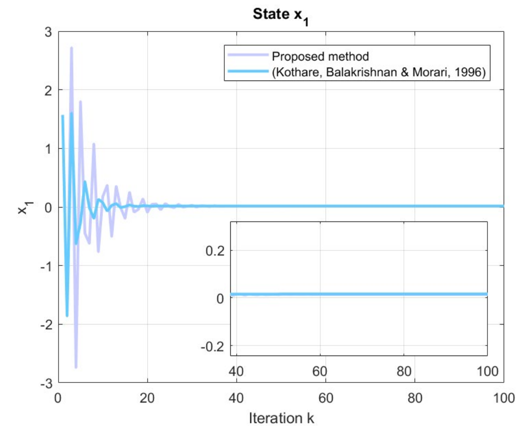


**Figure 10.** Stabilized $x_1$- large disturbance

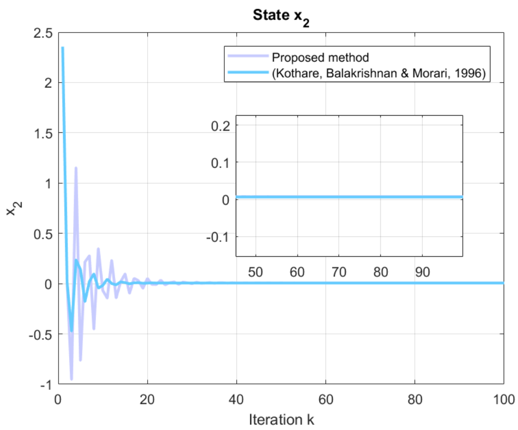


**Figure 11.** Stabilized $x_2$- large disturbance

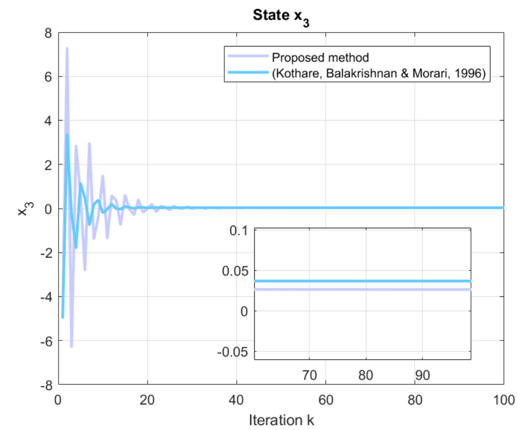


**Figure 12.** Stabilized $x_3$- large disturbance

1996), the proposed approach has a smaller steady state error. This can be observed by zooming in at the steady state area.

## 4. Conclusion

This article proposes a robust MPC approach for systems affected by two disturbances, a constant one and a norm 2 bounded one. Through several mathematical manipulations the norm 2 disturbance is replaced in the computations by its upper bound. The constant disturbance does not create problems in the computations and it appears in the optimization problem constraints. The simulation results using MATLAB indicate a stabilized system using the proposed algorithm. The advantages of the proposed method compared with the method from (Kothare, Balakrishnan & Morari, 1996), are that it applies to a larger class of systems, i.e. systems with disturbances, and it achieves a smaller steady state error in the case of large disturbances. As it can be seen in simulation, a disadvantage of the proposed approach would be the presence of oscillations and a steady state error when large disturbances are present. To overcome these problems, a feedforward component for the control law, which depends on the disturbances, can be introduced. Future work can consider the same scenario, but with the feedforward control law component. This component can better counteract the effect of the disturbance.

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